# Magnetogravitodynamic Stability of Resistive Streaming Triple Superposed of Fluid Layers

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**Abstract**

Stability of Magnetohydrodynamic streaming resistive triple superposed fluid layers has been studied. The basic equations were obtained by combining ordinary hydrodynamic equations and Maxwells equations related to electromagnetic field theory. The appropriate boundary conditions have been established for this model, then we obtained the dispersion relationship. The behavior of the system in terms of whether stable or unstable has been discussed. The curves are drawn to illustrate the areas of stability and instability. The effect of different parameters on the stability and the instability of this system was studied. It is found in the magnetic field permeability coefficient and the intensity of the magnetic field values has a destabilizing influence. Also, the increase of the fluids density values has a stabilizing influence. The streaming velocity has a destabilizing influence.

*Keywords:* Magnetohydrodynamic; Self-gravitating; Streaming; Resistive; Superposed fluids.

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## 1. Introduction

The study of stability in the fluid mechanics aims to give a physical state stand with a disturbed and still return to original state. Therefore, we have two cases, either stability or instability. Hydrodynamic stability is the field which is described the stability and the instability of fluid flows. Stability theory addresses of trajectories of dynamical systems under small perturbations of initial conditions. This study tells us stability in many areas, for example, biology, geophysics, meteorology, oceanography and engineering. The Magnetohydrodynamic stability of a gravitational medium with streams of variable velocity distribution for a general wave propagation in the presence of the rotational forces has been studied by [1]. The axisymmetric magneto hydrodynamic (MHD) stability of a streaming resistive hollow jet under oblique varying magnetic fields has been discussed by Hasan and his colleagues [2]. The electro gravitational instability of an oscillating streaming fluid cylinder surrounded by a self-gravitating tenuous medium pervaded by transverse varying electric field is discussed under the action of self gravitating, capillary and electro dynamic forces were presented by [3]. The behavior of a dielectric fluid-fluid interface in the presence of a strong electric field of a point charge and the line charge respectively, both statically and in the latter case dynamically, this study was conducted by [4]. Self-gravitation is the gravitational force exerted on a group of bodies, by the bodies that allows them to be held together. Self-gravitation has important effects in the fields of astronomy, physics, seismology, geology, and oceanography. The axisymmetric stability of the interface between two incompressible Selfgravitating non-conducting fluids in the presence of an electric field has been studied by [5]. The magnetodynamic instability of a rotating self-gravitating fluid layer of finite thickness embedded in a fluid of different density was presented by [6]. Reference [7] studied the two superposed fluids and the self-gravitating hydrodynamic basic equations under upon appropriate boundary conditions and general eigenvalue relation. The self-gravitating electrodynamic stability of an annular fluid jet pervaded and surrounded by periodic time dependent electric field has been discussed by [8]. The self-gravitating instability of the present model is discussed by using a simple linear theory. The problem is formulated for a rotating fluid layer, and a dispersion relation valid for all kinds of perturbations is discussed by [9]. The nonlinear stability of electrohydrodynamic of a cylindrical interface separating two conducting fluids of circular cross section in the absence of gravity using electroviscous potential flow analysis has been studied by [10]. The magnetohydrodynamic stability of streaming self-gravitational coaxial fluid cylinders with doubly perturbed interfaces was presented by [11]. The electrogravitational instability of a dielectric fluid cylinder surrounded by medium of negligible motion pervaded by varying transverse oscillating electric field has been investigated in the axisymmetric perturbation has been studied by [12]. The nonlinear capillary instability of the cylindrical interface between the vapor and liquid phases of a fluid is studied when there is heat and mass transfer across the interface, using viscous potential flow theory has been discussed by [13]. The triple-diffusive convection in a micropolar ferromagnetic fluid layer heated and soluted from below is considered in the presence of a transverse uniform magnetic field was presented by [14]. The effect of a uniform magnetic Field on the capillary breakup of a thin cylinder of magnetic liquid at rest, surrounded by an unbounded liquid with other coefficients of viscosity and magnetic permeability, is investigated in the linear formulation is discussed by [15]. The self-gravitating instability of an infinitely extending axisymmetric cylinder of a viscoelastic medium permeated with non uniform magnetic field and the nonuniform magnetic field and rotation is considered to act along the axial direction of the cylinder has been studied by [16]. Also, Reference [17] studied the effects of nonuniform rotation and magnetic

field on the instability of a self gravitating infinitely extending axisymmetric cylinder of viscoelastic ferromagnetic medium, the non-uniform magnetic field and rotation are acting along the axial direction of the cylinder and the propagation of the wave is considered along the radial direction. Reference [18] studied the effect of a uniform magnetic field on the capillary breakup of a thin cylinder of magnetic liquid at rest, surrounded by an unbounded liquid with other coefficients of viscosity and magnetic permeability.

## 2. Definition of the problem

### 2.1. Basic equations

We consider three superposed fluids of densities $\rho^{(1)}$, $\rho^{(2)}$, $\rho^{(3)}$ in the regions $(-\infty < z \leq 0)$,

$(0 \leq z < h), (h \leq z < \infty)$ respectively.

We suppose the streaming velocity of fluids as:

$$\underline{u}_0^{(f)} = (U, 0, 0).$$

(1)

The uniform magnetic fields

$$\underline{H}_0^{(f)} = (H_0, 0, 0).$$

(2)

Whereas $f = 1,2,3$. The fluids are considered to be incompressible, no-viscous under the effect of self-gravitation force, electromagnetic force, pressure gradient force and the force due to resistivity.

The basic equations of this model can be written as follows:

$$\rho^{(f)} \frac{d\underline{u}^{(f)}}{dt} = -\nabla P^{(f)} + \mu (\nabla \wedge \underline{H}^{(f)}) \wedge \underline{H}^{(f)} + \rho^{(f)} \nabla \phi^{(f)}.$$

(3)

$$\frac{d\underline{H}^{(f)}}{dt} = (\underline{H}^{(f)} \cdot \nabla)\underline{u}^{(f)} - \nabla \wedge (\eta \nabla \wedge \underline{H}^{(f)}).$$

(4)

$$\nabla^2 \phi^{(f)} = -4\pi G \rho^{(f)}.$$

(5)

$$\nabla \cdot \underline{u}^{(f)} = 0.$$

(6)

$$\nabla \cdot \underline{H}^{(f)} = 0.$$

(7)

Where $\rho$ is the density, $\underline{u}$ is the velocity vector, $P$ is the kinetic pressure, $\mu$ is the magnetic field permeability coefficient, $\phi$ is the self-gravitating potential, $G$ is the gravitational constant and $\eta$ is the coefficient of resistivity, $\underline{H}$ is the magnetic field intensities.

## 2.2. Perturbation analysis

Considers the effect of small disturbances for a small departure from the unperturbed state, then every variable quantity $Y(x, y, z, t)$ can be expressed as follows

$$Y(x, y, z, t) = Y_0(z) + Y_1(x, y, z, t) + ...,$$

(8)

$Y_0$ represent unperturbed quantity and $Y_1$ is a small increment of $Y$ due to disturbances. $Y$ expresses each of the following variables $\underline{u}^{(f)}, \underline{H}^{(f)}, \phi^{(f)}$ and $P^{(f)}$, where f denotes different regions of fluids.

Suppose that the interface can be described by

$$z = z_0 + z_1.$$

(9)

Where

$$z_1 = e^{i(k_x x + k_y y) + \sigma t},$$

(10)

$z_1$ represented by the elevation of the surface wave, $\sigma$ the growth rate, while $k_x$ and $k_y$ (real) are the wave numbers along x and y directions. In the initial state, we can put the basic equations of motion Eq.s (3 - 7) and by using Eq. (8) as follows unperturbed and perturbed systems of equations.

Unperturbed system

$$\rho^{(f)} \frac{d\underline{u}_0^{(f)}}{dt} - \mu(\underline{H}_0^{(f)} \cdot \nabla)\underline{H}_0^{(f)} = -\nabla \Lambda_0^{(f)}.$$

(11)

$$\frac{d\underline{H}_0^{(f)}}{dt} = (\underline{H}_0^{(f)} \cdot \nabla)\underline{u}_0^{(f)} - \eta\nabla^2 \underline{H}_0^{(f)},$$

(12)

$$\nabla^2 \phi_0^{(f)} = -4\pi G\rho^{(f)},$$

(13)

$$\nabla \cdot \underline{u}_0^{(f)} = 0,$$

(14)

$$\nabla \cdot \underline{H}_0^{(f)} = 0. \tag{15}$$

Perturbed system

$$\rho^{(f)} \left[ \frac{\partial \underline{u}_1^{(f)}}{\partial t} + \underline{u}_0^{(f)} \cdot \nabla)\underline{u}_1^{(f)} \right] - \mu(\underline{H}_0^{(f)} \cdot \nabla)\underline{H}_1^{(f)} = -\nabla \Lambda_1^{(f)}, \tag{16}$$

$$\frac{\partial \underline{H}_1^{(f)}}{\partial t} = (\underline{H}_0^{(f)} \cdot \nabla)\underline{u}_1^{(f)} - (\underline{u}_0^{(f)} \cdot \nabla)\underline{H}_1^{(f)} - \eta \nabla^2 \underline{H}_1^{(f)}, \tag{17}$$

$$\nabla^2 \phi_1^{(f)} = 0, \tag{18}$$

$$\nabla \cdot \underline{u}_1^{(f)} = 0, \tag{19}$$

$$\nabla \cdot \underline{H}_1^{(f)} = 0. \tag{20}$$

Where

$$\Lambda_0^{(f)} = P_0^{(f)} - \rho^{(f)}\phi_0^{(f)} + \mu(\underline{H}_0^{(f)} \cdot \underline{H}_0^{(f)}). \tag{21}$$

$$\Lambda_1^{(f)} = P_1^{(f)} - \rho^{(f)}\phi_1^{(f)} + \mu(\underline{H}_0^{(f)} \cdot \underline{H}_1^{(f)}). \tag{22}$$

The kinetic pressures and potentials are given by,

$$P_0^{(f)} = \rho^{(f)}\phi_0^{(f)} - \mu H_0^2 + C^{(f)}, \tag{23}$$

$$\phi_0^{(1)} = -2\pi G \rho^{(1)} z^2 + c_1 z + c_2, \tag{24}$$

$$\phi_0^{(2)} = -2\pi G \rho^{(2)} z^2 + c_1 z + c_2, \tag{25}$$

$$\phi_0^{(3)} = -2\pi G \rho^{(3)} z^2 + c_1 z + c_2 - 4\pi G h(\rho^{(2)} - \rho^{(3)})z + 2\pi G h^2(\rho^{(2)} - \rho^{(3)}). \tag{26}$$

Eq.s (24), (25), (26) have been solved in the unperturbed system upon applying the conditions that the self-gravitational and its derivative must continous at the boundaries $z = 0$ and $z = h$, where $c_1$, $c_2$ and $C^{(f)}$ are arbitrary constants of integration. By using the normal mode , we can put $Y_1(x, y, z, t)$ in the following form :

$$Y_1(x, y, z, t) = y_1(z)e^{i(k_x x + k_y y) + \sigma t}.$$

(27)

Substituting from Eq. (27) into Eq.(17), we get

$$\left[ (\sigma + ik_x U) - \eta(k_x^2 + k_y^2 - \frac{\partial^2}{\partial z^2}) \right] \underline{H}_1^{(f)} = ik_x H_0 \underline{u}_1^{(f)}.$$

(28)

By using Eq.(28) in Eq.(16)

$$\rho^{(f)} \left[ (\sigma + ik_x U) - \eta(\sigma + ik_x U)(k_x^2 + k_y^2 - \frac{\partial^2}{\partial z^2}) + \Omega_A^2 \right] \underline{u}_1^{(f)} =$$
$$-\left[ (\sigma + ik_x U) - \eta(k_x^2 + k_y^2 - \frac{\partial^2}{\partial z^2}) \right] \nabla \Lambda_1^{(f)}.$$

(29)

Where the Alfven wave frequency is denoted by

$$\Omega_A^2 = \frac{\mu H_0^2 k_x^2}{\rho}.$$

(30)

By using Eq.(19) and taking divergence of the two sides of Eq.(29), we get the following equation

$$\nabla^2 \Lambda_1^{(f)} = 0.$$

(31)

Eq.s (31), (18) leads to second order differential equations, these equations are solved in different regions $(-\infty < z \leq 0)$, $(0 \leq z < h)$, $(h \leq z < \infty)$, then we get:

$$\Lambda_1^{(1)} = B_1 z_1 e^{\ell z},$$

(32)

$$\Lambda_1^{(2)} = (B_2 e^{\ell z} + B_3 e^{-\ell z}) z_1,$$

(33)

$$\Lambda_1^{(3)} = B_4 z_1 e^{-\ell z},$$

(34)

$$\phi_1^{(1)} = D_1 z_1 e^{\ell z},$$

(35)

$$\phi_1^{(2)} = (D_2 e^{\ell z} + D_3 e^{-\ell z}) z_1,$$

(36)

$$\phi_1^{(3)} = D_4 z_1 e^{-\ell z},$$

(37)

where

$$\ell = \sqrt{k_x^2 + k_y^2}.$$

### 2.3. Boundary conditions

(i)  The self-gravitational potential and their derivatives across the fluids interface must be continuous at the boundaries $z = 0$ and $z = h$:

$$\phi_1^{(1)} + z_1 \frac{\partial \phi_0^{(1)}}{\partial z} = \phi_1^{(2)} + z_1 \frac{\partial \phi_0^{(2)}}{\partial z} = \phi_1^{(3)} + z_1 \frac{\partial \phi_0^{(3)}}{\partial z}.$$

(38)

Substituting (10), (24), (25), (26) and (35), (36), (37) into (38), then we can obtain the values of the constants $D_1$, $D_2$, $D_3$ and $D_4$:

$$D_1 = \frac{-2\pi G}{\ell}\left[(\rho^{(3)} - \rho^{(2)})e^{-\ell h} - (\rho^{(1)} - \rho^{(2)})\right],$$

(39)

$$D_2 = \frac{-2\pi G}{\ell}(\rho^{(3)} - \rho^{(2)})e^{-\ell h},$$

(40)

$$D_3 = \frac{-2\pi G}{\ell}(\rho^{(2)} - \rho^{(1)}),$$

(41)

$$D_4 = \frac{-2\pi G}{\ell}\left[(\rho^{(3)} - \rho^{(2)})e^{\ell h} - (\rho^{(1)} - \rho^{(2)})\right].$$

(42)

(ii)  At the boundaries $z = 0$ and $z = h$, the normal component of the velocity vector $\underline{u}$ must be continuous and also suitable with the velocity of the perturbed boundary surface:

$$u_1^{(1)} = u_1^{(2)} = u_1^{(3)} = \frac{\partial z}{\partial t}.$$

(43)

From equations (9), (10), (29), (32), (33) and (34) into (43), then we get

$$B_1 = \frac{\sigma \rho^{(1)}}{2\sinh \ell h}(e^{-\ell h} - 1) - \sigma \rho^{(1)}\left[\frac{1}{2\sinh \ell h}(e^{-\ell h} - 1) + 1\right].$$

(44)

$$B_2 = \frac{\sigma\rho^{(1)}}{2\sinh\ell h}(e^{-\ell h} - 1)\frac{\left[(\sigma + ik_xU) - \eta(\sigma + ik_xU)(k_x^2 + k_y^2 - \frac{\partial^2}{\partial z^2}) + \Omega_A^2)\right]}{\left[(\sigma + ik_xU) - \eta(\ell^2 - \frac{\partial^2}{\partial z^2})\right]}.$$

(45)

$$B_3 = -\sigma\rho^{(2)}\left[\frac{1}{2\sinh\ell h}(e^{-\ell h} - 1) + 1\right]\frac{\left[(\sigma + ik_xU)^2 - \eta(\sigma + ik_xU)(\ell^2 - \frac{\partial^2}{\partial z^2}) + \Omega_A^2)\right]}{\left[(\sigma + ik_xU) - \eta(\ell^2 - \frac{\partial^2}{\partial z^2})\right]}.$$

(46)

$$B_4 = \sigma\rho^{(3)}\frac{\left[(\sigma + ik_xU)^2 - \eta(\sigma + ik_xU)(\ell^2 - \frac{\partial^2}{\partial z^2}) + \Omega_A^2)\right]}{\ell\left[(\sigma + ik_xU) - \eta(\ell^2 - \frac{\partial^2}{\partial z^2})\right]}.$$

(47)

(iii) The normal component of the total stresses in the fluid layer of density $\rho^{(1)}$ must be suitable to that of the surrounding fluid of density $\rho^{(2)}$ and $\rho^{(3)}$ across the fluid interface at $z = h$:

$$P_1^{(1)} + z_1\frac{\partial P_0^{(1)}}{\partial z} = P_1^{(2)} + z_1\frac{\partial P_0^{(2)}}{\partial z} = P_1^{(3)} + z_1\frac{\partial P_0^{(3)}}{\partial z},$$

(48)

Eq. (48) can be written as follows:

$$\Lambda_1^{(1)} + \rho^{(1)}\phi_1^{(1)} + \mu(\underline{H}_0^{(1)} \cdot \underline{H}_1^{(1)}) + z_1\rho^{(1)}\frac{\partial\phi_0^{(1)}}{\partial z} = \Lambda_1^{(2)} + \rho^{(2)}\phi_1^{(2)} + \mu(\underline{H}_0^{(2)} \cdot \underline{H}_1^{(2)})$$

$$+ z_1\rho^{(2)}\frac{\partial\phi_0^{(2)}}{\partial z} = \Lambda_1^{(3)} + \rho^{(3)}\phi_1^{(3)} + \mu(\underline{H}_0^{(3)} \cdot \underline{H}_1^{(3)}) + z_1\rho^{(3)}\frac{\partial\phi_0^{(3)}}{\partial z}.$$
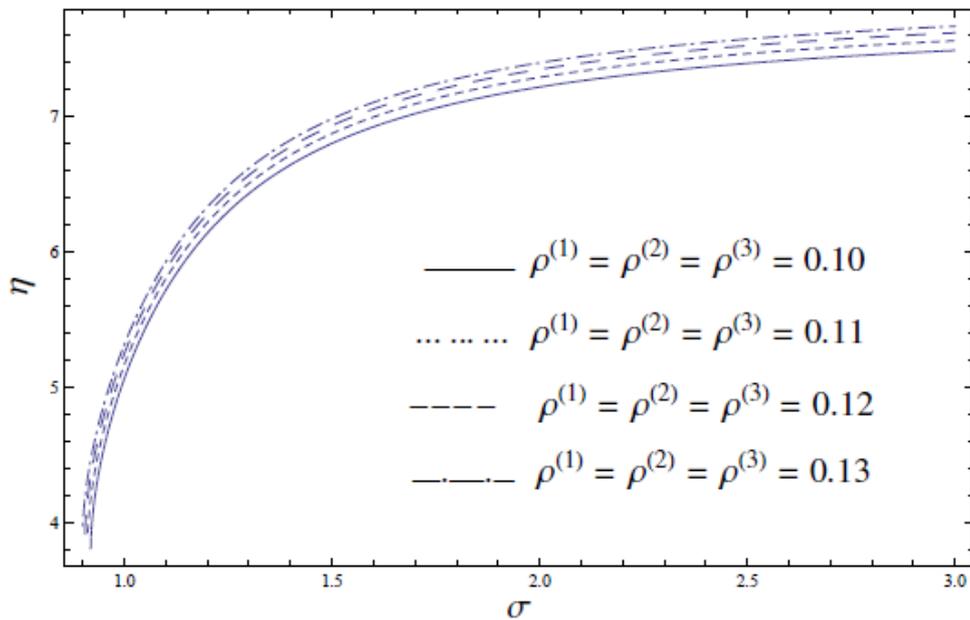
(49)

## 3. Dispersion relation

Eq. (49) leads to the dimensionless dispersion relation:

$$\frac{-\mu H_0^2 k_x^2}{2\pi G\left[(\sigma + ik_x U)^2 - \eta\ell(\sigma + ik_x U) + \frac{\mu H_0^2 k_x^2}{\rho^{(1)}}\right]} + \frac{\mu H_0^2 k_x^2 \rho^{(2)}}{2\pi G\left[(\sigma + ik_x U) - \eta\ell\right]} \times$$

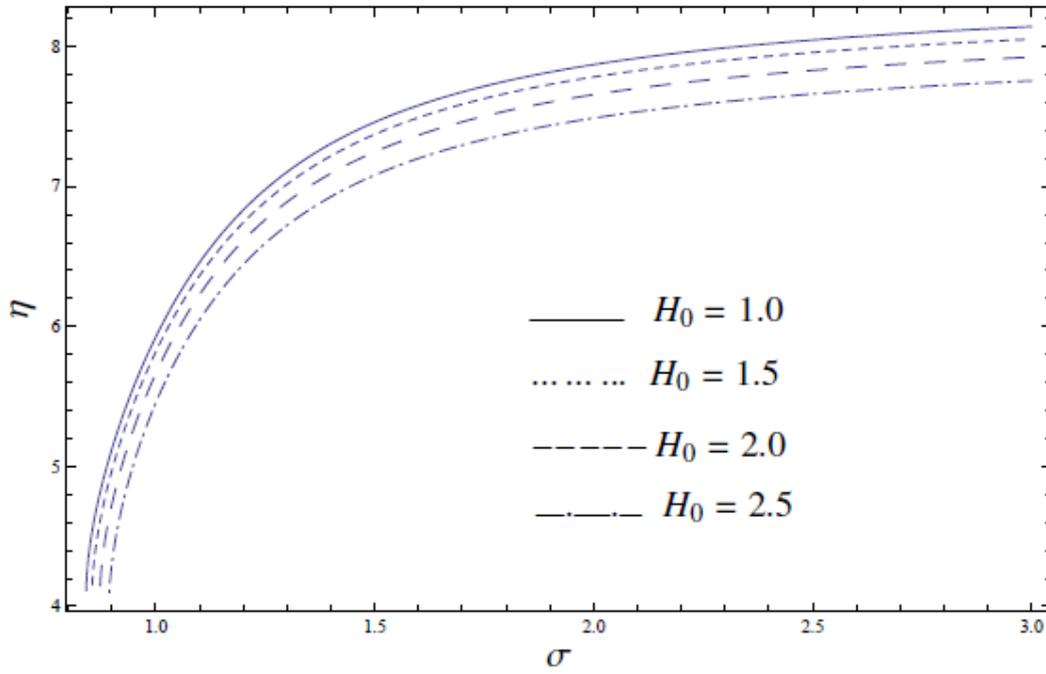$$\left[\frac{1}{\sinh \ell h}(e^{-\ell} - 1) + 1\right] + \frac{\rho^{(2)}}{2\pi G}\left[(\sigma + ik_x U)^2 - \eta\ell(\sigma + ik_x U) + \frac{\mu H_0^2 k_x^2}{\rho^{(1)}}\right] \times$$

$$\left[\frac{1}{\sinh \ell h}(e^{-\ell} - 1) + 1\right] = \frac{\rho^{(1)}}{2\pi G} + \frac{1}{\ell}\left[(\rho^{(2)} - \rho^{(3)})(\rho^{(2)} - \rho^{(1)})e^{-\ell} - (\rho^{(1)} - \rho^{(2)})^2\right].$$
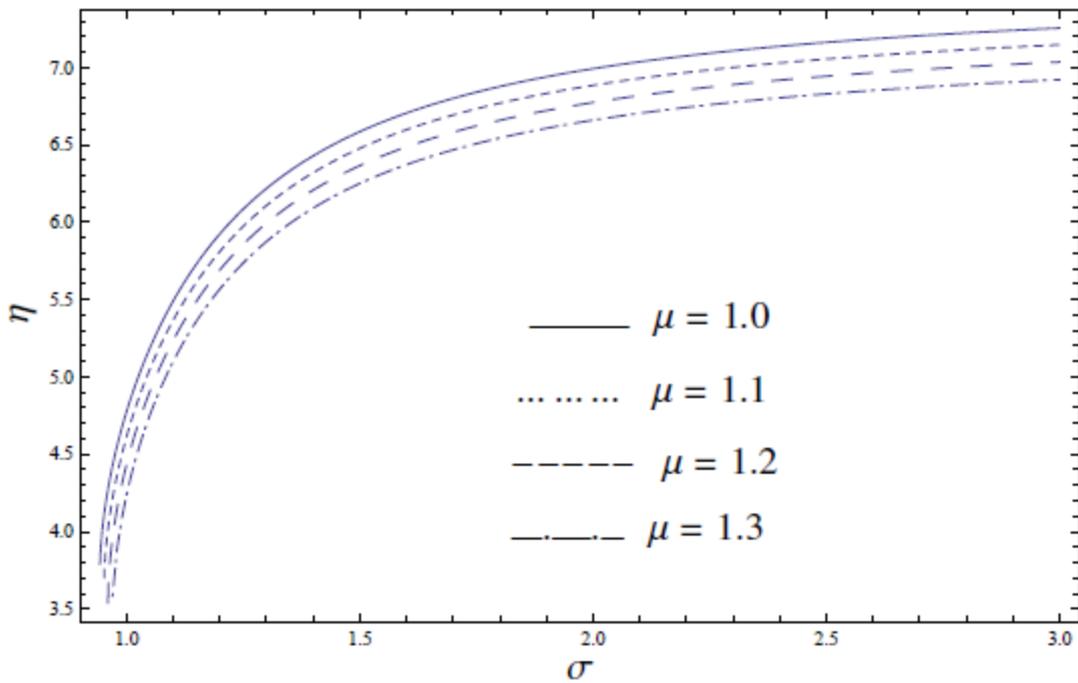
(50)

Now, we are studying the effect of the different variables on the stability and drawing the relation between the growth rate and the coefficient of resistivity
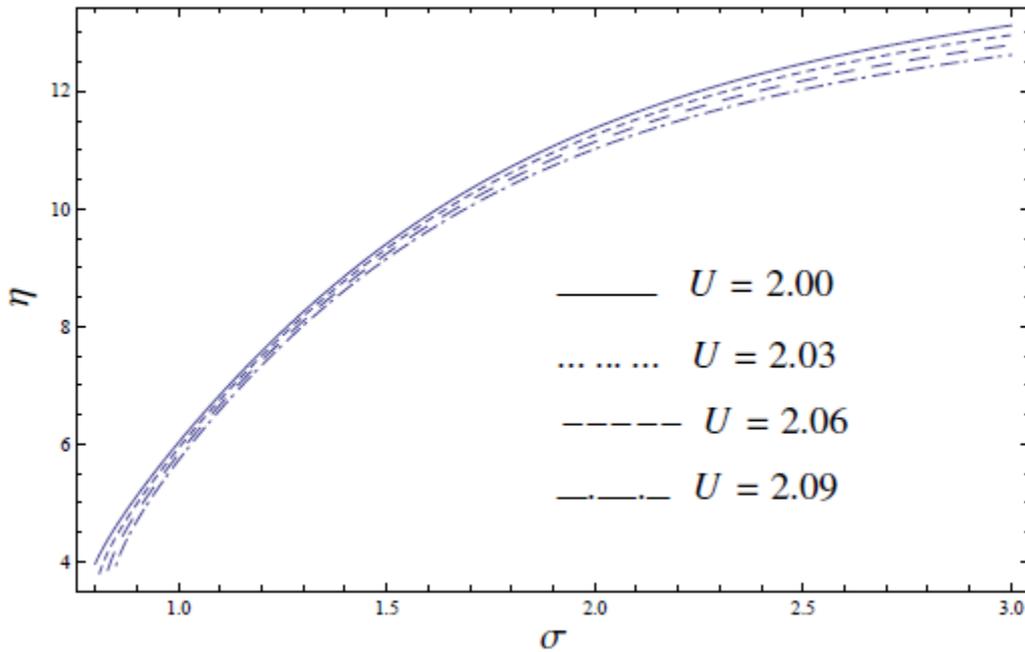


**Figure 1:** stability diagram for a system having the particulars: $U = 0.8, H_0 = 1, \mu = 0.01$.

**Figure 2:** stability diagram for a system having the particulars: $\rho^{(1)} = \rho^{(2)} = \rho^{(3)} = 1, U = 1, \mu = 0.01$.



**Figure 3:** stability diagram for a system having the particulars: $\rho^{(1)} = \rho^{(2)} = \rho^{(3)} = 0.8, U = 1, H_0 = 1$.

**Figure 4:** stability diagram for a system having the particulars: $\rho^{(1)} = \rho^{(2)} = \rho^{(3)} = 0.03, H_0 = 1, \mu = 0.1$ .

From Figure 1, as ( $\rho^{(1)} = \rho^{(2)} = \rho^{(3)} = \mathbf{0.10}, \mathbf{0.11}, \mathbf{0.12}, \mathbf{0.13}$), we find the following domains **(0 < σ < 0.919), (0 < σ < 0.912), (0 < σ < 0.905), (0 < σ < 0.90)** unstable states. The neighboring stable domains are **(0.919 < σ < ∞), (0.912 < σ < ∞), (0.905 < σ < ∞), (0.90 < σ < ∞).** From Figure 2, as $(H_0 = \mathbf{1.0}, \mathbf{1.5}, \mathbf{2.0}, \mathbf{2.5})$, we find the following domains **(0 < σ < 0.841), (0 < σ < 0.855), (0 < σ < 0.873), (0 < σ < 0.894)** unstable states. The neighboring stable domains are **(0.841 < σ < ∞), (0.855 < σ < ∞) , (0.873 < σ < ∞), (0.894 < σ < ∞).** From Figure 3, as $(\mu = \mathbf{1.0}, \mathbf{1.1}, \mathbf{1.2}, \mathbf{1.3})$, we find the following domains **(0 < σ < 0.804), (0 < σ < 0.823), (0 < σ < 0.835), (0 < σ < 0.841)** unstable states. The neighboring stable domains are **(0.804 < σ < ∞), (0.823 < σ < ∞), (0.835 < σ < ∞), (0.841 < σ < ∞).** From Figure 4, as $(U = \mathbf{2.00}, \mathbf{2.03}, \mathbf{2.06}, \mathbf{2.09})$, we find the following domains **(0 < σ < 0.850), (0 < σ < 0.830), (0 < σ < 0.810), (0 < σ < 0.790)** unstable states. The neighboring stable domains are **(0.850 < σ < ∞), (0.830 < σ < ∞), (0.810 < σ < ∞), (0.790 < σ < ∞).** From what we had before, We observe that the increase of the fluids density values has a stabilizing influence (Fig.1). We conclude that the increase of the intensity of the magnetic field values, the system gives a stable situation (Fig.2). We conclude that the increase of the magnetic field permeability coefficient values, the system gives a stable situation (Fig.3). Finally, we found that the streaming velocity has a destabilizing influence (Fig.4), which is consistent with all previous studies.

**4. Conclusions**

In this study, we have examined the influence of the existence of self-gravitating, magnetic field, streaming resistive triple superposed fluid layers. After we obtained the dispersion relation, we plotted $(\sigma - \boldsymbol{\eta})$ plane and studying the effect of different variables on the process stability.

**The following is a general summary of the study in this paper:**

1- The streaming velocity has a destabilizing influence.
2- We observe that the increase of the magnetic field permeability coefficient values, the system gives a stable situation and with the continuous increase of the magnetic field permeability coefficient values, the system is more stable and the stability zone increases.
3- We observe that the increase of the intensity of the magnetic field values, the system gives a stable situation and with the continuous increase of the intensity of the magnetic field values, the system is more stable and the stability zone increases.
4- The increase of the fluids density values has a stabilizing influence.

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