



---

## Biodiversity Strategy Improved in Ethiopia by Using Optimal Control System

A. N. Mohamad<sup>a\*</sup>, Nigussie Dedefi Waritu<sup>b</sup>, Gizachew kefelew Hailu<sup>c</sup>,  
Mengistu Chalchisa Wakjira<sup>d</sup>

<sup>a</sup>Professor, Department of Mathematics, Debre Berhan University, Debre Berhan & P.O.Box-445, Ethiopia

<sup>b</sup>Lecturer, Department of Mathematics, Debre Berhan University, Debre Berhan & P.O.Box-445, Ethiopia

<sup>c</sup>Assistant Professor, Department of Mathematics, Debre Berhan University, Debre Berhan & P.O.Box-445,  
Ethiopia

<sup>d</sup>Lecturer, Department of Mathematics, Debre Berhan University, Debre Berhan & P.O.Box-445, Ethiopia

<sup>a</sup>Email: dranmd@gmail.com

<sup>b</sup>Email: yusufkhalid75@gmail.com

<sup>c</sup>Email: kgizachewy@gmail.com

<sup>d</sup>Email: chalchisam@gmail.com

### Abstract

Biodiversity Strategy (BS) explains conserving biodiversity is not just a matter of protecting wildlife in nature reserves. It is also about safeguarding the natural systems of the Earth that are our life support systems, purifying the waters, recycling oxygen, carbon and other essential elements, maintaining the fertility of the soil, providing food from the land, freshwaters, and seas, yielding medicines and safeguarding the genetic richness on which we depend in the ceaseless struggle to improve our crops and livestock. Biodiversity can be divided into three hierarchical categories – Genes, species and ecosystems that describe quite different aspects of living systems and that scientists' measure in different ways. We tried to measure BS with suitable parameters by using Optimal Control Systems including Hamiltonian – Jacobi – Bellman Equation (HJBE) and state, costate with control aimed at to improve BS in Ethiopia and explain how to improve and implemented it in Ethiopia in future properly.

**Key words:** Costate; Ecosystem diversity; Optimal Control Systems; HJBE; Species Diversity.

---

\* Corresponding author.

## **1. Introduction**

In the universe, only in earth's plant, animals and microorganisms interacting with one another and with the physical environment in ecosystems form – the foundation of sustainable development. Biotic resources from this wealth of life support human livelihoods and aspirations and make it possible to adapt to changing needs environments. The steady erosion of the diversity of genes, species, and ecosystems taking place today will undermine progress toward a sustainable society. Indeed, the continuing loss of biodiversity is a telling measure of the imbalance between human needs and wants and nature's capacity.

The human race had 850 million members when it entered the in the industrial age, sharing Earth with life forms nearly as diverse as the planet has ever possessed. Today, with population near six times as large and resource consumption proportionally far greater, both the limits of nature and the price of overstepping them are becoming clear. A turning point is upon us. We can continue to simplify the environment to meet immediate needs, at the cost of long – term benefits, or we can conserve life's precious diversity and use it sustainably. We can deliver to the next generation (and the next) a world rich in possibilities or one impoverished of life; but social and economic development will succeed only if we do the first.

### **1.1. Categories of Biodiversity**

Biodiversity can be divided into three hierarchical categories – Genes, Species, and Ecosystems – that describe quite different aspects of living systems and that scientist's measure in different ways.

**Genetic diversity** refers to the variation of genes within species. This covers distinct population of the same species (such as the thousands of traditional rice varieties in India) or genetic variation within a population (which is very high among Indian rhinos, for example, and very low among cheetahs). Until, recently, measurements of genetic diversity were applied mainly to domesticated species and populations held in zoos or botanic gardens, but increasingly the techniques are being applied to wild species.

**Species diversity** refers to the variety of species within a region. Such diversity can be measured in many ways, and scientists have not settled on a single best method. The number of species in a region – its species “richness” – is one often – used measure, but a more precise measurement “**Taxonomic diversity**” also considers the relationship of species to each other:

**For example:** An island with two species of birds and one species of lizard has greater taxonomic diversity than an island with three species of birds but no lizards. Thus, even though there may be more species of beetles on earth than all other species combined, they do not account for the greater part of species diversity because they are so closely related. Similarly, many more species live on land than in the sea, but terrestrial species are so diversity higher in marine ecosystems than a strict count of species would suggest.

**Ecosystem diversity** is harder to measure than species or genetic diversity because the “boundaries of communities – associations of species – and ecosystems are elusive. Besides of ecosystem diversity, many other expressions of biodiversity can be important. These include the relative abundance of species, the age structure

of populations, the pattern of communities in a region, changes in community composition and structure over time, and even such ecological processes as predation, parasitism and mutualism. More generally, to meet specific management or policy goals, it is often important to examine not only compositional diversity – genes, species and ecosystem but also diversity in ecosystem structure and function.

**Human cultural diversity** could also be considered part of biodiversity. Like genetic or species diversity, some attributes of human cultures (say, nomadism or shifting cultivation) represents “solutions” to the problems of survival in particular environments. And, like other aspects of biodiversity, cultural diversity helps people adapt to changing conditions. Cultural diversity is manifested by diversity in language, religious beliefs, land – management practices, art, music, social structure, crop selection, diet, and any number of other attributes of human society.

### **1.2. *The Value of Diversity***

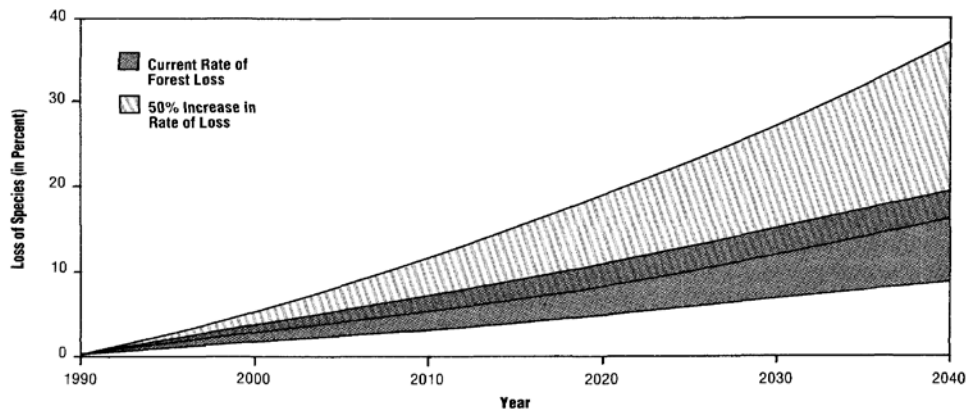
The sheer variety of life has enormous value. The variety of distinctive species, ecosystems, and habitats influence the productivity and services provided by ecosystems. As the variety of species in an ecosystem changes – a legacy of extinction or species introduction – the ecosystem’s ability to absorb pollution, maintain soil fertility and micro – climates, cleanse water, and provide other invaluable services changes too. For example: When the elephant – a voracious vegetarian – disappeared from large areas of its traditional range in Africa, the ecosystem was altered as grasslands reversed to woodlands and woodland wildlife returned.

A diverse array of genes, species and ecosystem is a resource that can be tapped as human needs and demands change. Because biodiversity is so closely intertwined with human needs, its conservation should rightfully be considered an element of national security. It has become increasingly apparent that national security means much more than military might. Ecological dimensions of national security cannot be ignored when countries fight over access to water or when environmental refugees strain national budgets and public infrastructure. A secure nation means not only a strong nation, but also one with a healthy and educated populace, and a healthy and productive environment as well. National security will be strongest in countries that care for their biodiversity and the services it provides. The many values of biodiversity and its importance for development suggest why biodiversity conservation differs from traditional nature conservation [10]. This goal speaks to modified and intensively managed ecosystems as well as natural ones, and it pursued in the human interest and human benefit. In sum, biodiversity conservation seeks to maintain the human life support system provides by nature, and the living resources essential for development.

### **1.3. *What is loss of biodiversity? What are their causes?***

Biological diversity is being eroded as fast today as at any time, since the dinosaurs died out some 65 million years ago. The crucible of extinction is believed to be in tropical forests. Around 10 million species live on earth [11], and scientists estimate that at the rates roughly 5% to 10% of tropical forest species may face extinction within the next 30 years [9]. This estimate may prove conservative, however. Rates of tropical forests loss are accelerating, and some particularly species – rich forests are likely to be largely destroyed in our life time. Some

researchers believe that about 60, 000 of world’s 240, 000 plant species, and perhaps even higher proportions of vertebrate and insect species, could lose their lease on life over the next three decades unless deforestation is slowed immediately [8] See the following figure – 1:



**Figure 1:** Source –Ehrlich and Wilson 2014; Ried 2015

The number of documented species extinctions over the past century is small compared to those predicted for the coming decades. This difference is due, in part, to the acceleration of rates of habitat loss over recent decades but also to the difficulty of documenting extinctions. The vast majority of species has not yet even been described, and many may disappear before they are even known to science.

#### 1.4. *Biodiversity involve in Freshwater Ecosystems*

Ancient world and current world cultures have been centered on freshwater habitats – Babylon between the Tigris and Euphrates Rivers, Egypt on the Nile, Rome on the Tiber, the Aztec capital built on man – made Islands in Lake Tenochtitlan, Paris on the Seine, Kinshasa on the Zaire river, India on Ganga and Krishna rivers. The world’s rivers, lakes, streams, and wed lands provide most of the world’s water for drinking, agriculture, sanitation, and industry as well as huge quantities of fish and shellfish. Fresh waters are also home to a tremendous diversity of fish, amphibians, aquatic plants, invertebrates and micro organisms. Freshwater lakes are classical examples of “Habitat Islands” (In this case, bodies of water surrounded by expenses of land). Like islands in general, the larger, more ancient lakes tend to have high levels of endemism, and in the rift lakes of Africa or **Thana in Ethiopia**, species diversity can be spectacular. With hundreds of species each 90% of them in some case found nowhere else – the East African lake especially in Ethiopia harbor some the world’s greatest concentrations of locally endemic species. The primary method of protecting freshwater biodiversity has been to designate particular species as threatened or endangered, making them subject to national recovery programs or international production. Unfortunately, this approach is failing (Source – Usher 1991) [12]

#### 1.5. *The Strategy for Biodiversity Conservation*

This concept has three elements

- Saving biodiversity
- Studying biodiversity
- Using biodiversity sustainably and equitably

### 1.5.1. Saving biodiversity

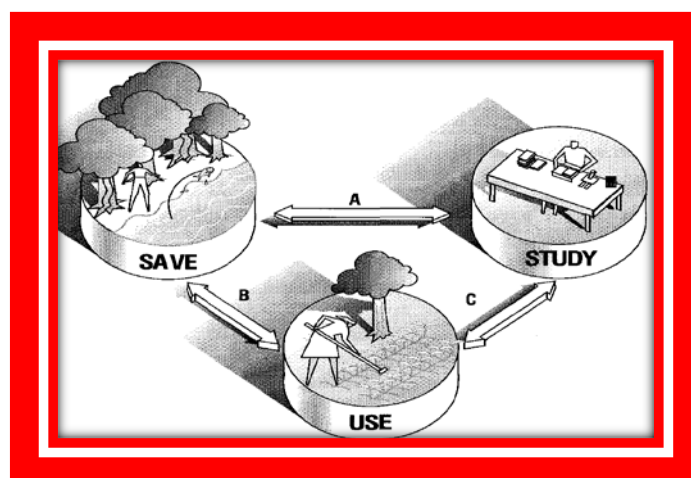
It means taking steps to protect genes, species, habitats and ecosystems. The best way to maintain species is to maintain their habitats. Saving biodiversity therefore often involves efforts to prevent the degradation of key natural ecosystems and to manage and protect them effectively. But since many of the world's habitats have been modified for such human uses as agriculture, the program must include measures to maintain diversity on lands and in waters that have already been disturbed.

### 1.5.2. Studying biodiversity

It means documenting its composition, distribution, structure, and function; understanding the roles and functions of genes, species, and ecosystems; grasping the complex links between modified and natural systems; and using this understanding to support sustainable development. It also means building awareness of biodiversity's values, providing opportunities for people to appreciate nature's variety, integrating biodiversity issues into educational curricula, and ensuring that the public has access to information on biodiversity, especially on developments that will influence it locally

### 1.5.3. Using biodiversity sustainably and equitably

It means husbanding biological resources so that they last indefinitely; making sure that biodiversity is used to improve the human condition, and seeing that these resources are shared equitably. "Use" does not, however, automatically imply consumption. Often, the best use of biodiversity may be to maintain it in its natural state for its ecological or cultural values, as in the case of forested watersheds or sacred groves. These three elements diagrammatically shown below:



**Figure 2:** Use – Save – Study Biodiversity

**Explanation – Figure – 2:**

- A. Slowing the loss of biodiversity requires greater understanding of its role in ecosystems and its importance for human life. Conversely, to increase understanding of biodiversity, representative and viable samples of ecosystems, species, and population must be maintained.
- B. Greater incentives will exist to slow the loss of biodiversity if its immediate value to humanity is increased. Conversely, the many current and potential benefits that biodiversity can provide to humanity cannot be sustained unless the biological resource base is maintained.
- C. Developing sustainable uses of biodiversity requires the application of both traditional and modern knowledge of biodiversity and biological resources. Conversely, users' needs should help set biodiversity research priorities.

**1.6. Biodiversity strategy**

**1.6.1. Establishing a national policy framework for biodiversity conservation**

- Reform existing public policies that invite the waste or misuse of biodiversity
- Adopt new public policies and accounting methods that promote conservation and the equitable use of biodiversity
- Reduce demand for biological resources

**1.6.2. Creating an international policy environment that supports national (Ethiopia) biodiversity conservation**

- Integrate biodiversity conservation into national economic policy
- Strengthen the national legal framework for conservation to complement the convention on Biological diversity
- Make the development assistance process a force for biodiversity conservation
- Increasing funding for biodiversity conservation, and develop innovative, decentralized, and accountable ways to raise funds and spend them effectively

**1.6.3. Creating conditions and incentives for local biodiversity conservation**

- Correct imbalances in the control of land and resources that cause biodiversity loss, and develop new resource management partnerships between government and local communities
- Expand and encourage the sustainable use of products and services from the wild for local benefits
- Ensure that those who possess local knowledge of genetic resources benefit appropriately when it is used

**1.6.4. Managing biodiversity throughout the human environment**

- Create the institutional conditions for bioregional conservation and development
- Support biodiversity conservation initiatives in the private sector

- Incorporate biodiversity conservation into the management of biological resources

#### **1.6.5. *Strengthening protected areas***

- Identity national and international priorities for strengthening protected areas and enhancing their role in biodiversity conservation
- Ensure the sustainability of protected areas and their contribution to biodiversity conservation

#### **1.6.6. *Conserving species, populations, and genetic diversity***

- Strengthen capacity to conserve species, populations and genetic diversity in natural habitats
- Strengthen the capacity of off – site conservation facilities to conserve biodiversity, educate the public, and contribute to sustainable development

#### **1.6.7. *Expanding human capacity to conserve biodiversity***

- Increase appreciation and awareness of biodiversity’s values and importance
- Help institutions disseminate the information needed to conserve biodiversity and mobilize its benefits.
- Promote basic and applied research on biodiversity conservation
- Develop human capacity for biodiversity conservation

### **1.7. *Managing Biodiversity through the Human Environment***

“The land is the place where we know to find all that it provides for us – food from hunting and fishing, and forms, building and tool materials, medicine. This land keeps us together within its mountains: we come to understand that we are not just a few people or separate villages, but one people belonging to a homeland” [5].

#### **1.7.1. *Bioregional Management [7]***

Even if most of Earth’s remaining natural ecosystems could be protected from development, they could not adequately maintain biodiversity. The remaining wild is simply not large enough to meet all species’ habitat needs or to provide important ecological services, and many of these still – natural ecosystems will inevitably be transformed by human use in coming decades. Clearly, the success of biodiversity conservation will depend upon how well the overall landscape is managed to minimize biodiversity loss. Human needs and activities must be reconciled with the maintenance of biodiversity, and protected areas must be integrated into natural and modified surrounding. Farms, forests, grazing areas, fisheries, and villages belong on the same planning grid as land restoration projects, protected areas, and species – conservation efforts. The scale of such efforts must be tailored to both ecological processes and the needs and perceptions of local communities. The integrative approach is here termed “Bioregional Management”.

#### **1.7.2. *What is the meaning of Bioregional Management?***

A bioregion is a land and water territory whose limits are defined not by political boundaries, but by the

geographical limits of human communities and ecological systems. Such an area must be large enough to maintain the integrity of the region's biological communities, habitats, and ecosystems, to support important ecological processes, such as nutrient and waste cycling, migration, and stream flow, to meet the habitat requirements of keystone and indicator species, and to include the human communities.

### **1.7.3. Elements and Dynamics of a Bioregion**

We mention all the elements and dynamics of a Bioregion below in figure – 3a and 3b:

1. A variety of potential area types are used in a bioregion: Strictly protected nature reserves, national or state parks (Zoba wise in Ethiopia) areas for the controlled extraction of non – timber forest products, privately owned conservation areas, and areas of permanent forest estate managed for timber production.
2. Watersheds are managed in their entirety, from ridge top to blue water, and across a range of uses from strictly protected uplands to estuarine fisheries.
3. Degraded lands are restored to a variety of uses, including soil and water conservation, coastal production (Not available in Ethiopia), agriculture, pasture, and protected area expansion.
4. Coastal and marine areas are not available in Ethiopia but we have many big lakes like Thana, Awash etc. they are managed to conserve key lake reefs, mangroves, beaches and other elements maintain fisheries productivity, and provide local economic opportunities through carefully managed tourism development.
5. Rangelands are managed within their carrying capacity to maintain native flora and fauna, raise livestock, and ensure the livelihoods of any nomadic pastoralist peoples.
6. Agricultural lands are managed to optimize long – term productivity and support biodiversity by minimizing use of chemical pesticides and fertilizers, using local as well as introduced crop varieties, and including trees, hedgerows, community woodlots, and wildlife corridors with the agricultural landscape.
7. A range of community – based institutions support biodiversity conservation, including community seed banks agricultural extension services, and biodiversity inventory and research stations
8. Larger towns within the bioregion provide a range of supporting institutions. These include zoos, aquaria, and botanic gardens to conserve endangered species and educate the public, schools, places of worship, and media outlets to build awareness, non – governmental organizations to provide support and information for both communities and government; and biodiversity information centers to serve as a focal point or bioregional dialogue information sharing, and collective action.

We discussed all categories of Biodiversity and Bioregion still now. We stop that work and we will implement the concept of Optimal Control System to improve Biodiversity because we already discussed as above Elements – Dynamics of a Bioregion to improve biodiversity. These concepts exactly accept the rules of optimum control system and will work properly to improve biodiversity in Ethiopia.



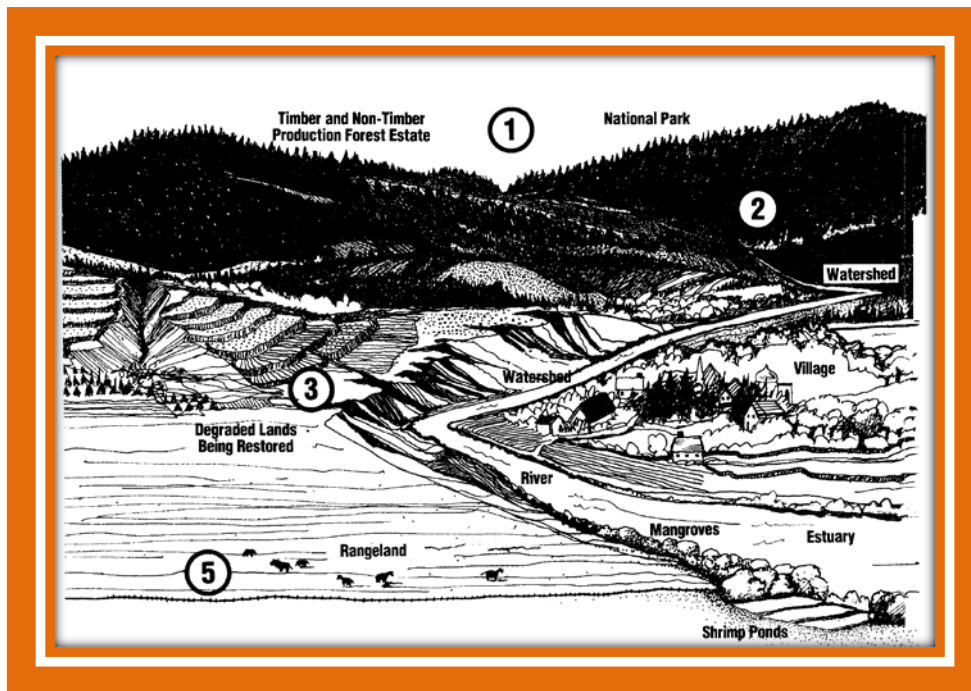


Figure 3a: Elements – Dynamics of a Bioregion

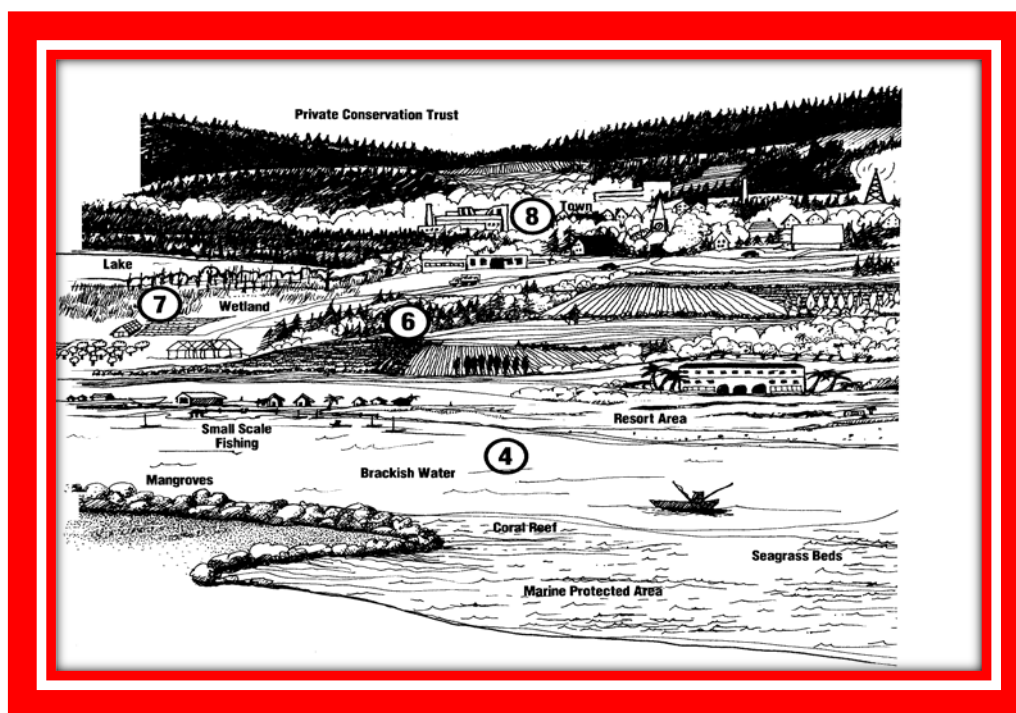


Figure 3b: Elements – Dynamics of a Bioregion

## 2. Using Optimum Control System To Improve Biodiversity In Ethiopia

### 2.1. Problem Formulation

In the concept of 1, we have two categories, they are

- a. Biodiversity types
- b. Bioregion types.

We choose Biodiversity types are **Genetic diversity, Species diversity, Ecosystem diversity and Human culture diversity** that is they are four types we choose here and also

In Bioregion types we discussed 8 categories but we choose only four they are Nature reserves, national or state parks (1), Watersheds and fisheries (2), Agricultural lands (6) and a large town with educational Institution (8) these four types to improve Biodiversity Strategies in Ethiopia. First four things are choose for **State matrix** and remaining Bioregion four suitable things are **control matrix**. These biodiversity strategies depend upon time so we develop our optimal control of Linear Plants or systems with quadratic performance index or measure. This leads to the Linear Quadratic Regulator simply LQR system dealing with state regulation, output region, and tracking. Broadly speaking we are interested in the design of optimal linear systems with quadratic performance indices.

We discuss the plant and the quadratic performance index with particular reference to physical significance. This helps us to obtain some elegant mathematical conditions on the choice of various matrices in the quadratic cost functional. Thus we will be dealing with optimization problem (Best improvement of Biodiversity Strategies) as follows:

Consider a linear, time – varying (LTV) system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{1}$$

$$y(t) = C(t)x(t) \tag{2}$$

With Cost Functional (CF) or Performance Index (PI) or Improve biodiversity strategies functional (IBSF) as

$$J(u(t)) = J(x_0(t), u(t), t_0)$$

$$\Rightarrow J(u(t)) = \left\{ \begin{array}{l} \frac{1}{2} [z(t_f) - y(t_f)]^T F(t_f) [z(t_f) - y(t_f)] \\ + \frac{1}{2} \int_{t_0}^{t_f} [z(t_f) - y(t_f)]^T Q(t) [z(t_f) - y(t_f)] + u^T(t) R(t) u(t) \end{array} \right\} dt \tag{3}$$

$[x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T$ , it is the **State vector**, in our case they are Biodiversity categories, that is

$x_1(t)$  =Genetic diversity

$x_2(t)$  =Species diversity

$x_3(t)$  =Ecosystem diversity and

$x_4(t)$  = Human culture diversity

That is,  $x(t)$  it is the 4 variables State vector;

Then  $x(t)$ , it is the 4 variables vector and  $y(t)$ , it is output vector with four variables  $z(t)$  it is 4 variables references (or 4 variables desired state vector, if the state  $x(t)$ , it is available at any cost)

$u(t) = [u_1(t) \ u_2(t) \ u_3(t) \ u_4(t)]^T$ , it is the **Control vector**, in our case they are elements of Bioregion but we choose only the following elements in bioregion.

$$u_1(t) = (1)$$

$$u_2(t) = (2)$$

$$u_3(t) = (6)$$

$$u_4(t) = (8)$$

(1), (2), (6) and (8), they are elements of bioregion as mention in section – 1

$u(t)$ , it is the 4 variables Control vector and also  $e(t) = z(t) - y(t)$  (Or  $e(t) = z(t) - x(t)$ , if the state  $x(t)$  (biodiversity categories data directly available), in this situation we have

$A(t)$ , it is  $4 \times 4$  State Matrix

$B(t)$ , it is  $4 \times 4$  Control Matrix

$C(t)$ , it is  $4 \times 4$  Output Matrix

And also  $T$ , it means transpose of the matrix or the vector. The cost functional equation (3), contains quadratic terms in error  $e(t)$ , and control  $u(t)$ , and hence called the quadratic cost functional (QCF). We also make certain assumption described below on the various matrices in (3), under these assumptions, we will find that optimal control is closed – loop in nature, that is, the optimal control  $u(t)$ , it is a function of the state  $x(t)$  or output  $y(t)$ . Also, depending on the final time  $t_f$ , it is being finite (infinite), then our system is called finite (infinite) – Time horizon system, we choose finite time only.

## 2.2. Categories of LQR or Time horizon systems

1. If our objective is to keep the state  $x(t) \rightarrow 0$  i.e.  $z(t) = 0$  and  $C = I$  (identity matrix), then we call it State regulator system. In other words, the object is to obtain a control  $u(t)$ , which takes the plant described in (1) and (2) from a nonzero state to zero state. This situation may arise when a plant is subject to unwanted disturbances that perturb the state, for example (Sudden Earth Quake, Storm etc will change our regulator system that is Biodiversity strategies system. For sudden flood and other natural cause will affect our Control system (Bioregion system)

2. If our interest is to keep the output  $y(t) \rightarrow 0$  i.e.  $z(t) = 0$ , then it is termed the Output Regulator system
3. If we try to keep the output or state near a desired state or output then we are dealing with a Tracking System, in both state and output regulator system the error is to be made zero.

### 2.3. Assumption of various matrices in Cost functional (IBSF)

Certain assumptions as described below on the various matrices in the IBSF in equation (3) and their implications:

#### a. The Error Weighted matrix $Q(t)$

In order to keep the error  $e(t)$ , it is small and error squared non – negative, the integral of the expression  $\frac{1}{2}e^T(t)Q(t)e(t)$ , it should be small and nonnegative. Thus, the matrix  $Q(t)$ , it must be **positive semi definite matrix**. Due to the quadratic nature of the weightage, we have to pay more attention to large errors than small errors.

#### b. The Control Weightage matrices $R(t)$

The quadratic nature of the control cost expression  $\frac{1}{2}u^T(t)R(t)u(t)$  indicates that one (Any one of bioregion variable) to pay higher attention for larger control effort. Since the cost (result) of the control has to be positive quantity, the matrix  $R(t)$ , it should be **positive definite matrix**

#### c. The Control Signal $u(t)$

The assumption is that there are non constraints on the control  $u(t)$ , it is very important in obtaining **the closed loop optimal configuration**

Combining all the above assumptions, we would like on one hand, to keep the error small, but on the other hand, we must not give more attention for larger control other than 4 variables in bioregion.

#### d. The Terminal Cost Weighted Matrix $F(t_f)$

The main purpose of this term is to ensure that the error  $e(t)$ , at the final time  $t_f$ , it is small as possible. To guarantee this, the corresponding matrix  $F(t_f)$ , it should be positive semi definite matrix. Further, without loss of generality, assume that the weighted matrices  $Q(t), R(t)$  and  $F(t_f)$ , they are symmetric

### 2.4. The Attractive Features of QCF

- a. It provides the elegant procedure for the design of closed loop optimal controller
- b. It results in the optimal feedback control that is linear in state function (Biodiversity categories)

That is why we often say that the “Quadratic performance Index fits like Glove [1]

### 2.5. The Procedure for LQR

To keep the state near zero during the interval of interest (In time) we have a linear time varying plant equation (1) with cost functional (3) becomes

$$J(u(t)) = \frac{1}{2}x^T F(t_f)x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}[x^T(t)u^T(t)] \begin{bmatrix} Q(t) & 0 \\ 0 & R(t) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt \quad (4)$$

Where the various vectors and matrices are defined (assumed) as before and note that  $z(t) = 0$

$\Rightarrow e(t) = 0 - x(t)$ , and it is itself is the state thereby implying a state regulator system. We need again various assumptions, they are  $u(t)$ , it is unconstraint, and the initial condition  $x(t = t_0) = x_0$ , it is given (Initial biodiversity in Ethiopia), the terminal time  $t_f$ , it is specified, and the final state  $x(t_f)$ , it is not specified (In which time Biodiversity strategies increases or reduces), the TCM  $F(t_f)$  and  $R(t)$ , they are  $4 \times 4$  positive semi definite matrices respectively, and the control weighted matrix  $Q(t)$ , it is an  $4 \times 4$  positive definite matrix finally the fraction  $\frac{1}{2}$ , in equation (4) is associated to cancel a 2 that would have, otherwise been carried on throughout the result. To obtain optimal solution and then propose the closed loop configuration we list the following steps:

#### Step – 1 – Hamiltonian Concept

Formulate the Hamiltonian as

$$H(x(t), u(t), \lambda(t)) = \frac{1}{2}x^T Q(t)x(t) + \frac{1}{2}u^T R(t)u(t) + \lambda^T(t)[A(t)x(t) + B(t)u(t)] \quad (5)$$

Where  $\lambda(t)$ , it is the **costate vector** of order 4

#### Step – 2 – Optimal Control

Obtain optimal control denoted by  $u^*(t)$ , using the control relation that is

$$\frac{\partial H}{\partial u} = 0 \rightarrow R(t)u^*(t) + B^T(t)\lambda^*(t) = 0 \quad (6)$$

$$\Rightarrow u^*(t) = -R^{-1}(t)B^T(t)\lambda^*(t) \quad (7)$$

Where we used  $\frac{\partial}{\partial u} \left\{ \frac{1}{2}u^T R(t)u(t) \right\} = R(t)u^*(t)$  and  $\frac{\partial}{\partial u} \left\{ \frac{1}{2}\lambda^T(t)B(t)u(t) \right\} = (t)\lambda^*(t)$

Since  $R(t)$ , it is positive definite matrix and then  $R^{-1}(t)$ , it exists.

#### Step – 3 – State and Co – State equation

$$\dot{x}^*(t) = + \left( \frac{\partial H}{\partial \lambda} \right)_* \rightarrow \dot{x}^*(t) = A(t)x^*(t) + B(t)u^*(t) \tag{8}$$

$$\dot{\lambda}^*(t) = + \left( \frac{\partial H}{\partial x} \right)_* \rightarrow \dot{\lambda}^*(t) = -Q(t)x^*(t) - A^T(t)\lambda^*(t) \tag{9}$$

Substitute (7) in (8) we get the state and costate **canonical system** or **Hamiltonian system** of equations

$$\begin{bmatrix} \dot{x}^*(t) \\ \dot{\lambda}^*(t) \end{bmatrix} = \begin{bmatrix} A(t) & E(t) \\ -Q(t) & -A^T(t) \end{bmatrix} \begin{bmatrix} x^*(t) \\ \lambda^*(t) \end{bmatrix} \tag{10}$$

Where  $E(t) = B(t)R^{-1}(t)B^T(t)$

The general boundary condition is produced here as

$$\left[ H^* + \frac{\partial S}{\partial t} \right]_{t_f} \delta t_f + \left[ \left( \frac{\partial S}{\partial x} \right)_* - \lambda^* \right]_{t_f}^T \delta x_f = 0 \tag{11}$$

Where,  $S$  equals entire terminal cost in the CF (4) and for our present system  $t_f$  it is specified and it makes  $\delta t_f = 0$ , in (11) and  $x(t_f)$ , it is not specified and it makes  $\delta x_f$  it is arbitrary in (11) and hence we get, the coefficient of  $\delta x_f = 0$ , in (11) that is

$$\lambda^*(t_f) = \left( \frac{\partial S}{\partial x(t_f)} \right)_* = \frac{\partial \left[ \frac{1}{2} x^T(t_f) F(t_f) x(t_f) \right]}{\partial x(t_f)} = F(t_f) x^*(t_f) \tag{12}$$

This final condition on the costate  $\lambda^*(t_f)$ , together with the initial condition on the state  $x_0$  and the canonical system of equations (10) form a Two – Point Boundary Value Problem (TPBVP). The state – space representation of the set of relations for the state and costate system (10) and the control (7) is shown in

figure –4 (Source: Optimal Control Systems – Desineni Subbaram Naidu – 2015, Idaho State University, Pocatello, Idaho, U. S. A- CRC Press)[13]

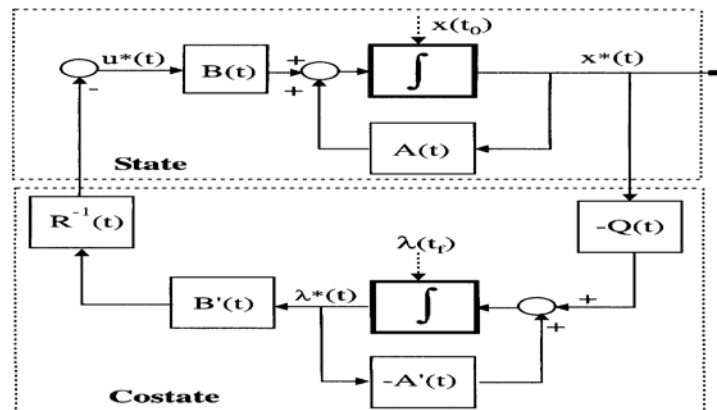


Figure 4: State and Costate System

#### Step – 4 – Closed Loop Optimal Control

The state space representation as shown in figure – 4: prompt us to think we can obtain the optimal control  $u^*(t)$  as a function of (negative feedback) of the optimal state  $x^*(t)$ , now we formulate a closed loop optimal control, that is, to obtain optimal control  $u^*(t)$  it is a function of the costate  $\lambda^*(t)$  as seen in equation (7) as a function of the state  $x^*(t)$ , let us examine the final condition on  $\lambda^*(t)$  given by equation (12). This in fact relates the costate in terms of the state at the final time  $t_f$ . Similarly, we may like to connect the costate with the state for the complete interval of time  $[t_0, t_f]$ , and therefore, let us assume a transformation

$$\lambda^*(t) = P(t)x^*(t) \tag{13}$$

Where  $P(t)$ , it is to be determined, and then, we can easily see that with (13) and (7) becomes

$$u^*(t) = -R^{-1}(t)B^T(t)P(t)x^*(t) \tag{14}$$

Equation (14) is now a negative feedback of the state  $x^*(t)$ , **note that this negative feedback resulted from our “theoretical development” or “Mathematics” of optimal control procedure and not introduced internationally.**

Differentiate (13) w. r. to time  $t$ , we get

$$\dot{\lambda}^*(t) = \dot{P}(t)x^*(t) + P(t)\dot{x}^*(t) \tag{15}$$

Using (14) in the control (7), state (8) and costate (9) system of equations as shown above we get

$$\dot{x}^*(t) = A(t)x^*(t) - B(t)R^{-1}(t)B^T(t)P(t)x^*(t) \tag{16}$$

$$\dot{\lambda}^*(t) = -Q(t)x^*(t) - A^T(t)P(t)x^*(t) \tag{17}$$

Substituting the state and costate relations (16) and (17) in (14) we get

$$\begin{aligned} -Q(t)x^*(t) - A^T(t)P(t)x^*(t) &= P(t)x^*(t) + P(t)[A(t)x^*(t) - B(t)R^{-1}(t)B^T(t)P(t)x^*(t)] \\ &\Rightarrow -Q(t)x^*(t) - A^T(t)P(t)x^*(t) \rightarrow \\ [ \dot{P}(t) + P(t)A(t) + A^T(t)P(t) + Q(t) - P(t)B(t)R^{-1}(t)B^T(t)P(t) ] x^*(t) &= 0 \end{aligned} \tag{18}$$

Essentially, we eliminated the costate function  $\lambda^*(t)$ , from control (7), the state (8) and costate (9) equations by introducing the transformation (13)

**Step – 5 – Matrix Differential Riccati Equations [2]**

The relation (18) should be satisfied for all  $t \in [t_0, t_f]$ , for any choice of the initial state  $x^*(t_0)$  (In Biodiversity strategies improving starting time), and also  $P(t)$ , it is not depend on the initial state. Therefore, equation (18) should hold good for any value of  $x^*(t)$  and also it is a clear meaning that the function  $P(t)$  it should satisfy the matrix differential equation

$$\dot{P}(t) + P(t)A(t) + A^T(t)P(t) + Q(t) - P(t)B(t)R^{-1}(t)B^T(t)P(t) = 0 \tag{19}$$

Equation (19) is the matrix differential equation of the Riccati type, and often it is called the matrix differential Riccati equation (DRE) and also the transformation (13) is called Riccati Transformation,  $P(t)$ , it is called the Riccati Coefficient matrix or simply Riccati matrix or Riccati coefficient, and the equation (14) is the Optimal Control (feedback) law. The matrix DRE (19) it can also be written in the compact form as

$$\dot{P}(t) = -P(t)A(t) - A^T(t)P(t) - Q(t) + P(t)E(t)P(t) = 0 \tag{20}$$

Where  $E(t) = B(t)R^{-1}(t)B^T(t)$

Comparing the boundary condition (12) and the Riccati transformation (13), we get the final condition on  $P(t)$  as

$$\lambda^*(t_f) = P(t_f)x^*(t_f) = F(t_f)x^*(t_f) \rightarrow P(t_f) = F(t_f) \tag{21}$$

Therefore, the matrix DRE is to be solved **backward in time** using the final condition (21) to obtain the solution  $P(t)$  for the entire interval  $[t_0, t_f]$

**2.6. The Result of Optimal Control or IBS**

We have arisen one question that is the optimal control  $u^*(t)$  it is either maximum or minimum?

This question is answered by considering the Second Partial of the Hamiltonian. That is done by examining the Second Variation of the CF, that is examining the nature of optimal control is that the matrix

$$\Pi = \begin{bmatrix} \frac{\partial^2 H}{\partial x^2} & \frac{\partial^2 H}{\partial x \partial u} \\ \frac{\partial^2 H}{\partial u \partial x} & \frac{\partial^2 H}{\partial u^2} \end{bmatrix} \tag{22}$$

Equation (22) it must be positive definite (negative definite) for minimum (maximum). In most of the cases this reduces to the condition that

$$\left( \frac{\partial^2 H}{\partial u^2} \right)_* \tag{23}$$



It must be positive definite (negative definite) for minimum (maximum). Now using the Hamiltonian (5), and calculating the various partials

$$\left(\frac{\partial^2 H}{\partial x^2}\right)_* = Q(t); \left(\frac{\partial^2 H}{\partial x \partial u}\right)_* = 0; \left(\frac{\partial^2 H}{\partial u \partial x}\right)_* = 0 \text{ and } \left(\frac{\partial^2 H}{\partial u^2}\right)_* = R(t) \quad (24)$$

Substituting (24) in (22) we get

$$\Pi = \begin{bmatrix} Q(t) & 0 \\ 0 & R(t) \end{bmatrix} \quad (25)$$

Since  $R(t)$ , it is positive definite and  $Q(t)$ , it is positive semi definite and then  $\Pi$ , it is positive semi definite, however, the condition that the second partial of  $H$  w. r. to  $u^*(t)$ , it is  $R(t)$ , it is positive definite and this is enough to guarantee that the control  $u^*(t)$ , it is minimum.

### 3. Conclusion and Future Implements

In our case we choose Bioregion in the place of Control that is why we must check maximum value of  $u^*(t)$ , and also we will get surprising result for biodiversity strategies improvement that is if we interchange biodiversity categories as control and bioregion elements as state at that time we need  $u^*(t)$ , it must be minimum. Surprisingly both results are accurately correct. Secondly, we will not take any data, because we give and explain as above general TPBVP problem in the closed loop optimal control, if any researchers, they may choose time variant data for all biodiversity categories, and bioregion elements week wise, month wise and even year wise simply calculate only the value of *Matrix*  $\Pi$  with any dimensional like as  $n \times n$  matrix form and we can solve by using any software either MATLAB or OCTAVE, and we will get excellent result numerically with negligible or less error approximation. Thirdly, the researcher or scholars they can extend variables either in biodiversity or bioregion elements suitable for Ethiopia that also will be possible to get relevant result. Fourthly the result is obtained by interchanging state and control variables vice versa, we get the same result.

### References

- [1]. Athans, M. and Falb, P. Optima Control: An Introduction to The Theory and Its Applications, McGraw Hill Companies, Inc., New York, NT, 1997
- [2]. Bellman, R. E. Introduction to Matrix Analysis. McGraw Hill Book Company, New York, NY, Second edition, 1971.
- [3]. Bridgewater, Peter. 1988. Biodiversity and landscape. Earth – Science Reviews 25: 486 – 491
- [4]. Bittanti, S., Laub, A. J, and Willems, J. C. editors: The Riccati Equation. Springer – Verlag, New York, NY, 1991.
- [5]. Commission of Inquiry into Aspects of the Timber Industry in Papua New Guinea. 1989. Interim

Report No. 4. Vol. 1. Government of Papua New Guinea, Port Moresby, Papua New Guinea.

- [6]. Ehrlich, Paul R., and Edward O. Wilson. 1991. Biodiversity studies: science and policy. *Science* 253: 758 – 762
- [7]. Lang, R. (Eds.). 1986. *Integrated Approach to Research Planting and Management*. University of Calgary Press. Calgary, Alberta, Canada
- [8]. Raven, P. H. 1988. Biological resources and global stability. PP 3 – 27 In: S. Kawano, J. H. Connell, and T. Hidaka (eds.), *Evaluation and Codaptation in Biotic Communities*. University of Tokyo press, Tokyo, Japan.
- [9]. Reid, W. V. 1992. How many species will there be? In: T. Whitemore and J. Sayer (eds.) *Tropical Deforestation and Species Extinction*. Chapman and Hall, London, U. K.
- [10]. The rationale for nature conservation is summarized in the World Charter for Nature of the United Nations General Assembly, reprinted in McNeely et al., 1990
- [11]. Thomas, C. D. 1990. Fewer Species. *Nature* 347:237
- [12]. User A. D. 1991. *Inhabitants of the Mool River*. The Nation, March 31. Bangkok, Thailand.
- [13]. DesneniSubbaram Naidu: **Optimal Control System**, CRC Press, Boca Raton, London, New York , 2012.