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Epistemic Objectivity behind Inductive Probability: Beyond Carnap-Popper Controversy on the Problem of Inductive Logic

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Abstract

Science neither aims at having the monopoly over the truth about the world nor establishing a dogmatic knowledge. Natural light of experience is held by empiricists to be the reliable source of human knowledge. Inductive logic has been a leading tool of empirical experiments in justifying and confirming scientific theories with evidence. Science cannot reach where it has reached without inductive logic. Inductive logic has, therefore, played an important role in making science what it is today. Inductive logic helps science to justify its theories not form convictional opinions of scientists but from factual propositions. However, inductive logic has been problematic in the sense that its logic of justification led philosophers of science to demarcation, the distinction of *episteme* from *doxa*. At present, some philosophers of science and scientists attempt to justify why science carries out a reliable knowledge. Some have argued for structuralism and realism of scientific theories rather than believing in the course of miracles and others for their historicity. Both views are explanatories of how science works and progresses. This essay recalls the arguments for structures of scientific theories and their historicity. First, the essay analyses the controversy between Rudolf Carnap and Karl Popper on how the problem of inductive logic in confirming scientific theories can be solved. In so doing, the essay refers to empirical probabilities as well as the limits calculus. Second, the essay merges frequentist and Bayesian approaches to determine how scientific theories are to be confirmed or refuted. Third, the use of a new form of Bayesian Theorem will show how mathematical and logical structures respond to some of the important questions that arise from the historical and realistic views about scientific theories.

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The essay argues for epistemic objectivity behind inductive probability, the key issue of the controversy in question, and proves that the truth about the world is symmetric.

Keywords: Science; Induction; Probability; Demarcation; Deduction; Frequentism; Bayesianism.

1. Introduction

Nobody has experienced what the future is all about. Despite this fact, inductive intuition seems to inform human intellect that the future will resemble the past. Inductive probability will bet that the future will probably resemble the past. Still, inductive logic does not assure us that the future will look like the past. It seems obvious that "we cannot infer the events of the future from those of the present" [1]. There is an ambivalence about predicting the future. In fact, philosophy of science has been aiming to solve such an ambivalence on the ground of inductive logic. Yet, inductive logic has been problematic for science despite the accurate results from its predictions about the future. Induction rocks the work of science and the nature of scientific knowledge. There is an existing debate between Rudolf Carnap and Karl Popper on how the problem of inductive logic in confirmation theory can be solved. In this essay, I will analyse the Carnap-Popper controversy and argue that the controversy is deeper than the problem itself, for it arises not from the problem itself, but from the solutions they have proposed to the problem. I shall prove that the Carnapian logic of justification has significantly contributed to clarifying the methodological and epistemological implications of inductive logic in admitting well-verified scientific theories to be synthetically and analytically true. I will demonstrate that the Popperian logic of discovery has instead exposed the methodological challenges of inductive logic in how science establishes its theories, but it aims at justification as well. As a result, a new form of Bayesian Theorem which I will explain will help me to respond to most of the challenges of inductive logic. The emphasis is on both analytic and synthetic statements that provide certain information about the state and the nature of things out there into the world.

2. The Carnap-Popper Controversy and Science with demarcation

Science is the empirical and systematic inquiry that interprets phenomena of the world. Science relies on inductive logic while justifying and confirming its theories not form convictional opinions but from facts. This exposition of science may sound too simplistic. Although, it is not obvious due to the role played by inductive logic in science. Inductive logic is simply the process of inferring a general law or principle from the observation of particular instances [2]. However, inductive logic has been thought to be problematic since its reasoning process leads philosophers of science to a harsh demarcation between what is a knowledge and what is an opinion, between science and pseudoscience in Popperian perspective. For classical empiricism, only factual propositions would constitute factual knowledge, but what could be thought to be non-factual knowledge became its central problem [3]. Factual knowledge would be generated from a systematic reasoning built upon mathematics and logic. Yet, some scientists were even sceptical about the truth value of factual propositions.

It is true that "if the truth of a proposition does not follow from the fact that it is self-evident to us, then its self-evidence in no way justifies our belief in its truth" [1]. For instance, the truth value of my factual propositions without any proofs would depend on my own interpretation. On the one hand, only with factual propositions, would I be right to claim that there are things I know that I know, and those I know that I do not know. On the other hand, there are things I do not know that I know, and those I do not know that I do not know at all. As a result, my knowledge of things is more or less probabilistic and hence ambivalent. Thus, without proofs, factual propositions would create an ambivalence about the nature of scientific knowledge. This ambivalence led classical empiricism to decline by the fact that the truth value of factual propositions would be probabilistic.

Due to the decline of classical empiricism, two hypothetical perspectives on the nature of science were accredited for two reasons. First, inductive logic problem deprived science of the title of accurate knowledge, while remaining a methodological apparatus for scientific researches. Second, due to the problem of inductive logic, science seemed to have carried out a fallible and conjectural knowledge. These views gave birth to two competing and influential schools of thought in the twentieth century: Neoclassical empiricism and Critical empiricism. The first school arises from the writings of Rudolf Carnap, whereas the second is represented by Karl Popper [3]. Lakatos has paid attention to their controversy, as if the ultimate solution to the inductive logic problem would depend on its resolution.

Lakatos argues that philosophers of science generate problem-shifts while attempting to solve the inductive logic problem. A problem-shift can be either degenerative or progressive. There is a degenerative problem-shift when these philosophers create less complex problems than the original one and pay more attention to the lesser ones. There is a progressive problem-shift when these philosophers create more complex problems than the original one and focus on the complicated ones in order to solve the original one [3]. Lakatos argues that Carnap has made a degenerative problem-shift, while Popper seems to have made a more progressive problem-shift. Lakatos' claim should be assessed by clarifying their understanding of probabilities.

Carnap thinks that the 'degree of evidential support' or 'degree of confirmation' should somehow be equated with probability in the sense of probability calculus in order to classify theories as partially proved or confirmed by some facts to a certain degree. Unlike Carnap, Popper holds that one starts with speculative theories and then tests them severely for only deductive inferences rather than inductive generalisations [3]. Carnap thought that the results of probability calculus would be accurate to some degrees while confirming scientific theories. Principles and theorems of inductive reasoning are "analytic but not synthetic. Inductive logic is non-deductive and non-demonstrative inference. Inductive logic is the same as inductive probability" [4]. Still, Popper thinks that, with probability calculus, a conjectural knowledge would remain conjectural.

Carnap understands probabilities as parts of the system of inductive logic whose project is to predict the future. Frequent observations would help in interpreting the phenomena that take place in the world. For Carnap, probability calculus goes far beyond inductive reasoning with logical language based only on sets of factual propositions. Probabilistic calculus is meant for another propositional basis grounded on logical measures. In fact, Carnapian inductive probability is to be understood within empirical probabilities for a theory of rational belief and action [5]. Given the frequency of pieces of evidence proving a given scientific theory true, and the total number of the experiments conducted, a mathematical figure could show how best such given theory is to be confirmed in order to construct a reliable knowledge.

As per Carnapian inductive probability, inductive logic would not be problematic insofar as the background knowledge constituted by a dataset of the outcomes of a repeating experience infers to the outcomes of the future experience that would depend on the same dataset, and that would constitute a reliable knowledge to be held. For example, if I have been trying to heat metals (metal 1, then metal 2, metal 3, metal 4, metal 5) and ended up realizing that all of them expand, then I can claim that all metals probably expand on heating. It is a probable generalisation because of default of certainty [1]. While carrying out the experiments, I have in mind an alternative hypothesis that there could be some metals that cannot expand on heating. This is an idealisation in theories [6]. Yet, from my dataset, all the 5 metals I heated have expanded. There is need to figure out if an inductive probability can tell us more about the theory. If T stands for the hypothesis that all metals expand on heating, e for the evidences that proves T, n for the total number of the experiments conducted, and P for the probability that T is true given the evidences e, then Carnapian confirmation of T in a probabilistic figure would go as follows: $\mathrm{P(T)} = \frac{\mathrm{e}}{\mathrm{n}} = \frac{5}{5} = 1 = 100\%$. This is only the result based on frequentism in probability.

Also, the same procedure is applicable to the Popperian notion of the severe-test-theory for degrees of confirmation regarded as empirical corroboration [7]. For Popper, corroboration is the best explanation we have as we wait for any further evidence to falsify a prior hypothesis. Popper seems to have been concerned with the scientific method that would push us to view the world with a blind eye of the mind through observations without any preconceived notions. However, whenever we get interested in observing things, there are already some kind of preconceptions we tend to have about the things we frequently observe (prior beliefs). For Popper, methods that would only help us to confirm our beliefs are methods of pseudo-sciences because they can help us to prove anything. He argues that scientists do not aim to confirm a theory but to disconfirm it while they are testing it. For Popper, science works with disconfirmation, whereas pseudo-science works with confirmation. However, in the same way, if this Popperian method can serve to disprove anything, it would be therefore classified in the set of pseudo-sciences. Due to this Popperian method, the acceptable scientific theories would be the one that is to be set on testability yet ready for falsifiability and refutability. In other words, aiming to prove scientific hypotheses right would be of no use; all that would be needed is only to prove them wrong. This would, however, sound contradictory. In this case, one would have to ask what would be the scientific knowledge to be held.

As a way of adjusting such contradiction, Popper believes that knowledge is all about probabilities and contingent upon a dataset. In Popperian perspective, we are only justified in believing theories that are most probable given their dataset. Popper equates Carnapian inductive probability to probability logic that determines the degree of probability of a statement [7]. Popper may have misinterpreted Carnapian notion of probability calculus. Carnap talks of an “inductive probability of an argument” in confirming a theory on the basis of probability calculus. A grade confirmation of a theory is induced from the set of pieces of evidence met during the experiments [4]. Hence, such a set is for confirmation of a theory what a set of premises is for the conclusion of an argument. While Carnap recognises an “inductive probability of an argument” within the notion of probability calculus, Popper realises an “inductive probability of a statement”.

There is a difference between an “inductive probability” of an argument and that of a statement. An inductive probability of an argument is “a measure of the strength of the evidence that the premises provide for the conclusion. It is correct to speak of the inductive probability of an argument, but incorrect to speak of the inductive probability of statements" [2]. Carnapian view of inductive probability is rooted in the empirical probability, whereas the Popperian one takes roots in the empirical corroboration. The empirical probability focuses on the set of pieces of evidence to form an argument for a confirmation of a given scientific theory. Yet, the empirical corroboration only targets a particular piece of evidence to get a statement for confirming a given scientific theory. Thus, with the aid of an "inductive probability", the results of an empirical probability might differ from those of an empirical corroboration. While empirical probability aims at reaching certainty, empirical corroboration cannot, in any case, give up to doubt about the hypotheses it aims to confirm or disconfirm with evidence.

The above example can help us to understand how probability and corroboration differ from each other in the empirical approach. If T represents 'all metals expand on heating' as a hypothesis to be tested, s the outcomes of the tests proving T, r the totality of the tests conducted, and C a corroborative confirmation of theory T, then C can be put into a mathematical figure as follows: $\mathrm{C(T)} = \frac{\mathrm{s}}{\mathrm{r}} = \frac{5}{5} = 1$. It follows that $\mathrm{P(T)} = \mathrm{C(T)}$. Let this be case (1). In pushing further this case, we see the core idea of the Carnap-Popper controversy. If at least one metal did not expand while conducting the experiment, then there is a change in the results, and the same would also happen if we are to predict that all metals expand on heating. In this case, 4 metals out of 5 expanded. The results are as follows: $\mathrm{P(T)} = \frac{\mathrm{e}}{\mathrm{n}} = \frac{4}{5} = 0.8$ and $\mathrm{C(T)} = \frac{\mathrm{s}}{\mathrm{r}} = \frac{4}{5} = 0.8 = 80\%$. Let this be case (2). As P(T) decreases, the hypothesis T is likely to be rejected because it is worthy of confirmation if and only if all the gathered pieces of evidence prove it true.

However, there is need to critically and rationally look at the scenarios of the experiment. An evidence that does not confirm the on-test-hypothesis may confirm another hypothesis different from the one on test. The test aims at confirming the hypothesis and if it does not, it might confirm something else. Thus, it would be surprising to wonder if it is really rational to believe that a non-empty set of possible outcomes will not occur exactly in the same way as one believes that the impossible event will not occur [8]. For Jacob Bernoulli, probability is epistemic. Everything is objectively certain in the world, even events of the future. Things always have in themselves the highest certainty [9]. Here comes in a scenario of complement events. A complement of an event stands for all outcomes that are not expected for the event. Together, an event and its complement make all possible outcomes. The Complement Rule, as it is attributed to Bernoulli, states that the sum of the probability of an event and its complement equals 1. Going back to case (2), since $\mathrm{P(T)} + \mathrm{P(T')} = 1$ by the complement rule, then $\mathrm{P(T')} = 1 - 0.8 = 0.2 = 20\%$. This $\mathrm{P(T')}$ is not contradictory to $\mathrm{P(T)}$. Instead, $\mathrm{P(T')}$ is the degree that is lacking in order to complete the truth value of $\mathrm{P(T)}$ at the instance where there are pieces of evidence that do not support the hypothesis T on test. It is as the same as the initial probability $P_i(T)$, not in terms of prior probability but in terms of the historicity of the theory T.

At e = 4 and n = 5, $\mathrm{P(T)} = \frac{\mathrm{e}}{n} = \frac{4}{5} = 0.8$; $\mathrm{C(T)} = \frac{\mathrm{s}}{r} = \frac{4}{5} = 0.8 = 80\% = 4.P_i(T)$ [A]

At e = 3 and n = 5, $\mathrm{P(T)} = \frac{\mathrm{e}}{\mathrm{n}} = \frac{3}{5} = 0.6$; $\mathrm{C(T)} = \frac{\mathrm{s}}{\mathrm{r}} = \frac{3}{5} = 0.6 = 60\% = 3.P_i(T)$ [B]

At e = 2 and n = 5, $P(T) = \frac{e}{n} = \frac{2}{5} = 0.4$; $C(T) = \frac{s}{r} = \frac{2}{5} = 0.4 = 40\% = 2.P_i(T)$ [C]

At e = 1 and n = 5, $P(T) = \frac{e}{n} = \frac{1}{5} = 0.2$; $C(T) = \frac{s}{r} = \frac{1}{5} = 0.2 = 20\% = 1.P_i(T)$ [D]

An interpretation of [A], [B], [C] and [D] lead us to conclude that $P(T) = 5.P_i(T)$, where 5 is the total number of pieces of evidence derived from the 5 experiments carried out. It follows that, at n^{th} experiment, $P(T) = n.P_i(T)$, when T is a truth-preserving hypothesis. Let this be case (3). The scenario shows that zero confirmation is impossible. It follows what would be called Einsteinian confidence in a theory, since he has never assigned a zero prior to his general theory of relativity [10]. In fact, a zero prior would be a total ignorance. Einstein was convinced that only the discovery of a universal formal principle could lead us to assured results [11].

The above example shows that, from case (1) to case (2), the probabilistic confirmation P decreases as the number of pieces of evidence (e) decreases. It follows that the confirmability of T decreases as P decreases. In this sense, it may appear that Popper wins over Carnap since the probabilistic figure 0 would mean 0 confirmation. Apparently, it follows that Carnapian logic of justification equated to probability calculus cannot account for confirmation of a scientific theory. However, that would be a harsh conclusion. It is important to note that even if the mathematical figures tend towards zero, they will never reach an absolute zero. Thus, what is at stake in Carnap-Popper controversy is that nonzero corroboration is as possible as a nonzero degree of confirmation. In any case, from the above example, *e* and *s* do not take a value of zero but a value of 1, because at least the first metal has expanded. All experiments were conducted independently from the observed facts at the first instance.

In conducting the experiments, I have at the back of my mind an alternative hypothesis that 'Not all metals expand on heating'. In any case the alternative hypothesis comes true given the results of the experiments, the tendency is to refute the prior hypothesis. This begs a question. A critical scientist would want to know why the results should be taken as true and confirms the alternative hypothesis. There is no evidence that proves the heated metals are real metals as they are assumed to be. There is need to pay more attention on how we name things and what our eye of rationalism sees. They may be metals with impurities or may not be metals at all. If they were purely metals, they would have expanded while on heating. This is the reason why Carnap suggested the idea of probabilities, degrees of evidential support or degrees of confirmation, degrees of rational beliefs, and rational betting quotients. This neoclassical chain of identities has been judged by Lakatos as an implausibility. He argues that "evidential support cannot be the same as degrees of probability in the sense of probability calculus" [3].

Besides, Lakatos supports Popper claiming that he cuts that neoclassical chain by his notion of corroboration. For Lakatos, Popper sets out to prove that the function $C(h, e)$, evidential support, confirmation, or corroboration of h by the evidence e, does not obey the formal calculus of probability in empirical terms [3]. Lakatos goes further to show that Carnap shifts from theories to particular instances, degree of confirmation to degree of rational belief, judging degree of truth to judging coherence. Lakatos believes that Popper's model appears to progressively advance the growth of science. Popper contended that the work of science is to falsify an entire system and that no statement is overturned by falsification. Popper holds that the Logic of falsification needs deductive inferences.

In contrast, Popper does not deny that if a theory T is true, then the observation O must concord with such a theory. However, if O is not observable, then T must be false. This is an entailment with a form of *Modus Tollens*. Here, Popper faces a problem because an entailment takes us back to verify the truth value of premises. Not all entailments are logically true. In fact, Popper makes no progress. To get rid of the problem of induction, he contended that neither a "simple enumerative induction" nor a "sophisticated inference to the best explanation" is necessary to scientific progress. Deduction is sufficient. He paid little attention to how falsification or even corroboration can take place within the set of statements in order to target a hypothesis under test [12].

Furthermore, it is the fact that a "conclusion of a deductive argument" is already set in one of its premises. Popper himself admits that deduction is not used simply to prove conclusions; it is rather used as an instrument of rational criticism [7]. It seems appropriate to claim that deductive logic can no longer be considered as a system that establishes the truth of theories by deducing them from axioms whose truth would be quite certain. Instead, deductive logic would be considered as a system that allows us to rationally and subjectively argue for our various hypotheses. It aims at explaining them systematically and critically. A confirmation of a hypothesis can result from such an explanation [12]. Hence, deductive logic does not tell us the whole truth about a given scientific theory or law. Popperian proposal of deductive logic for confirmation leaves us with an ambivalence so as the inductive logic does. While a deductivist seems to be speculatively persuasive and strives to reach the knowledge through rationalism, an inductivist seems to be realistic with regard to the order of things into the world and strives to gain knowledge as an imitator of the world through empiricism. Hence, science progresses by the fact that induction as its tools requires a planning agenda in order to reach the accurate results.

In trying to solve the inductive logic problem, Popperian notion of deductive logic and corroboration fail to account for further factors that would hinder the best results of experiments. In fact, an *adhocism* perceived in Carnapian approach comes back to Popperian corroboration. Carnap goes further to suggest that there is need to define a measure function in order to satisfy some further factors and conditions that would be associated with the experimentation. Carnap refers to Bayesian approach. For instance, once such a function is defined, the degree to which a given theory h is confirmed by evidence e can be calculated as follows: $\mathrm{P(h \mid e)} = \frac{\mathrm{P(h.e)}}{\mathrm{P(e)}}$, which can also be noted as $\mathrm{P(h \mid e)} = \frac{\mathrm{P(h \cap e)}}{\mathrm{P(e)}}$. While interpreting Carnapian understanding of probabilities, both Popper and Lakatos do not focus on Carnapian Approach to Bayesianism.

3. Bayesian Approach to Confirmation Theory

Carnapian approach to Bayesian Theorem makes a lot of sense in confirmation theory. There is need to evaluate it and see if it matches with the empirical probability already set above. If $\mathrm{P(h \mid e)} = \frac{\mathrm{P(h \cap e)}}{\mathrm{P(e)}}$, then $\mathrm{P(h \mid e).P(e)} = \mathrm{P(h \cap e)}$ i.e. $\mathrm{P(h \mid e)} = \frac{\mathrm{P(h \cap e)}}{\mathrm{P(e)}} \equiv \mathrm{P(h \mid e).P(e)} = \mathrm{P(h \cap e)}$ [E]. In a universal set of events h and e, conditional and joint probabilities allow the rule $\mathrm{P(h \cap e)} = \mathrm{P(e \cap h)}$ [F]. Given [E] & [F]: $\mathrm{P(e \cap h)} =$

$P(e \mid h). P(h)$ in Carnapian approach to Bayesianism. $P(h \mid e). P(e) = P(e \mid h). P(h) \equiv P(h \mid e) = \frac{P(e|h).P(h)}{P(e)}$ [Bayesian Theorem]. Carnap does not mean frequentism in his approach to Bayesian Theorem. Bayesianism may look like frequentism while interpreting the results of empirical probabilities, but they differ in their interpretational use for predictions. There is need to go deeper when it comes to Bayesian interpretation. There is need to deeply look at some scenarios. For $P(h \mid e) = \frac{P(e|h).P(h)}{P(e)}$; $P(h)$ is the prior probability of a given hypothesis h; $P(e)$, the expectedness; $P(e \mid h)$, the likelihood probability provided that h is true; $P(h \mid e)$, the posterior probability of h given the necessary evidence. All these probabilities play a significant role while making rational choices and predictive claims.

Reconsidering the above example, let T replace h. Bayesian Theorem becomes: $P(T \mid e) = \frac{P(e|T).P(T)}{P(e)}$. Let this be case (4). If T is a truth-preserving scientific theory or law, and given that some unexpected outcomes may occur during experimentation, case (3) and case (4) have to be compared: $P_i(T) = P(T \mid e)$, for $P(T \mid e)$ is the posterior probability given the occurrence of any particular piece of evidence e. Since $P_i(T) = P(T|e)$ and $P_i(T) = \frac{1}{n}. P(T)$, then $P(T) = n. P(T|e)$, at n^{th} experiment. Again, since in case (4) $P(T|e) = \frac{P(e|T).P(T)}{P(e)}$, then $P(T) = n. \frac{P(e|T).P(T)}{P(e)}$. The probability of T, given an expected outcome (e) does not change the truth value of the hypothesis T, because a non-expected outcome (~ e) with respect to the hypothesis T confirms something else other than the hypothesis T on test. It would be irrational to think of (~ e) as evidence disconfirming T.

For future predictions, let (e) and (~ e) be sampled in a set E. For any outcome, E will not change the truth value of T. The Bayesian Theorem becomes: $P(T|E) = n. P(E|T). \frac{P(T)}{P(E)}$. P (E) being the marginal probability is equal to 1, for $P(E) = [P(T). P(e|T) + P(\sim T). P(e|\sim T)]_1 + [P(T). P(e|T) + P(\sim T). P(e|\sim T)]_2 + \ldots [P(T). P(e|T) + P(\sim T). P(e|\sim T)]_n$ which can be also understood as $P(E) = C_e^n$ in reference to combinatorics in Bernoulli trials, for all possible outcomes would come from n trials. The result would be "a theorem of a pure probability theory and holds under any interpretation of calculus" [13]. This is so by the fact that either E comes true because T is false or E comes true because of other alternative theories. Hence, $P(E)$ can be reasonably washed out in order to simplify Bayesian Theorem as follows: $P(T \mid E) = n. P(E \mid T). P(T)$, $P(T|E)$ being the posterior probability of a theory T being true given the evidences; $P(E|T)$ being the likelihood probability provided that T is true; $P(T)$ the prior probability of T; n being the number of pieces of evidence derived from experiments that confirm a theory T; and the formula itself being the Bayesian Product Principle. On the one hand, $P(E|T)$ can behave as the prior probability at the first instance while envisaging to keep carrying out various independent experiments. On the other hand, $P(E|T)$ can be equivalent to the complement of $P(T|E)$, because $P(E|T)$ can be some falsehood assigned to T being true in itself. T does not change its truth value. As in case (2), the probability of a complement is the degree that is lacking in order to fully complete the truth value about the hypothesis. Let $P(T \mid E) = n. P(E \mid T). P(T)$, the Bayesian Product Principle, be case (5). There still be a problem with this Bayesian Product Principle, because $P(T)$ being the prior probability does not necessarily equal 1, it must depend on how strongly the experimenter believes in T.

It appears that there is need of convention among scientists in confirming T since $P(T)$ would be always assigned a prior probability with personalistic opinions. It is true that the claim 'all metals expand on heating' seems to be problematic due to the universal quantifier. Yet, the absence of the universal quantifier makes the claim more problematic when it is negated. In reference to Poincaré, Hattiangadi argues that there is always a choice in taking one hypothesis to be true compared to another and considering it as a principle of physics, for instance. For him, whenever physicists consider a particular principle to be true, its truth value must be approached as a convention among them, for some would choose another principle and declare it to be equally true. He holds that there is no inductive inference that would be valid without appealing to some illicit assumptions, for it is derived from a reasoning method from facts to generalities [14]. However, with inductive probability, there still be a hope of making sense while seeking a rational confirmability of a given hypothesis. It has been already noticed that there is need to reconsider the experiments in any case the evidence does not seem to confirm our hypothesis.

Besides, in some situations where pieces of evidence should have confirmed the hypothesis T, and the case turns to be otherwise, those who are pessimistic about inductive reasoning take such situations as puzzles and give up to solving the problem. However, giving up would not be a rational choice in any case, for such an attitude would reduce our confidence about the hypothesis we believe to be true given the observational facts as well as our power over the state and the nature of things in the world. For the hypothesis T [all metals expand on heating] must not be true and false at the same time, there is need to note that we have a criterion for judging reality as suggested by Hattiangadi. His criterion is that we have "power over nature that gives confidence that we have knowledge, and that we are not merely producing empty words" [14]. Thus, it follows that our interpretation of the results of an experiment must be informed by our factual observations.

Considering once more the above example, the experimenter can either assign T a higher probability of 1 if and only if he believes T to be true or the experimenter can assign T a lower probability of 0.2, which is rational, because that is the lowest probability within the sample of 5 heated-metals. It follows that the posterior probability $P(T|E)$ will be always affected by the prior probability $P(T)$. Still, if $P(T|E)$ has to increase because of the occurrence of pieces of evidence that confirm T, then it is rational to wash out $P(T)$, for $P(T)$ would keep $P(T|E)$ decreasing and hinder the degree of confirmability of T. For an objectivist and rational empiricist committed to verifying T, it is understandable that there must be at least one precedential piece of evidence prior to any other pieces of evidence confirming T in order to assign any prior probability to T. Otherwise, it would be irrational to take $P(T)$ as a prior probability. This being the case, then T entails E but not essentially vice versa. The probability of E and T, as a joint probability, is given by the product of its marginal probabilities; and with n pieces of evidence, $P(T) = n.P(E|T)$. Let this be case (6). The number n plays a role of not having a personalistic interpretation of $P(T)$ because, in any case, $P(T)$ has to come from somewhere, it is not an imaginary probability. Given the cases (5) and (6), the Bayesian Product Principle can be written as follows: $P(T|E) = n.P(E|T).n.P(E|T)$, which is $P(T|E) = \left(n.P(E \middle| T)\right)^2$, for at most two experiments. Let us make a table of results from the above case with a use of logical interpretation of probability calculus and see what the results are like.

Table 1

e_i	$P(E) = C_e^n$	$P_i(T) = \frac{e}{n}$	$P(E\|T) = P(E).P_i(T)$	$P(T) = n.P(E\|T)$	$P(T\|E) = (n.P(E\|T))^2$
0	1	0	0	0	0
1	1	1	1	1	1
2	1	1/2	1/2	1	1
3	1	1/3	1/3	1	1
4	1	1/4	1/4	1	1
5	1	1/5	1/5	1	1
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
n	$\frac{\mathbf{n!}}{\mathbf{e!(n-e)!}}$	$\frac{\mathbf{1}}{\mathbf{n}}$	$\frac{\mathbf{1}}{\mathbf{n}}$	$\mathbf{n.}\left(\frac{\mathbf{1}}{\mathbf{n}}\right)$	$\left(\mathbf{n.}\frac{\mathbf{1}}{\mathbf{n}}\right)^{\mathbf{2}}$

$P(T|E) = (n.P(E|T))^2$ seems to be trivial for it gives us a constant result, which is 1, after some experiments. However, it is obvious that all the pieces of evidence are to be derived from the number of experiments conducted by an experimenter. In fact, evidence and the truth value of T in themselves are also independent of experiments. Hence, since there must be an infinite number of experiments to be conducted, the Bayesian Product Principle can be finally written as follows: $P(T|E) = (n.P(E|T))^n$, for n takes a number of pieces of evidence, and with n experiments, the maximum result will be a constant number, that is 1, at all times. The procedure used in the above table cannot tell us much about $P(T|E) = (n.P(E|T))^n$. However, if the structure $P(T|E) = (n.P(E|T))^n$ is to be taken as a functional structure and as it must equal to a given number within the framework of probability distribution, the range of the results of $P(T|E)$ can be given by the use of limits, for they are fundamental to calculus. The application of limits calculus to the Bayesian Product Principle is of great importance, for it can helps us to understand where should lie our confidence in T.

$$P(T|E) = lim_{n\to[0;\pm\infty]}(n.P(E|T))^n \equiv P(T|E) = \begin{cases} lim_{n\to 0}(n.P(E|T))^n & (1) \\ lim_{n\to\pm\infty}(n.P(E|T))^n & (2) \end{cases}$$

A) From the equation (1), $P(T|E) = \lim_{n\to 0}(n.P(E|T))^n$, and $n.P(E|T)$ can also be written as $e^{\ln(n.P(E|T))}$

$$P(T|E) = \lim_{n\to 0}\left(e^{\ln(n.P(E|T))}\right)^n = \lim_{n\to 0} e^{n.\ln(n.P(E|T))} = e^{\lim_{n\to 0}\frac{\ln(n.P(E|T))}{\frac{1}{n}}}$$

$= e^{\lim_{n\to 0}\frac{\frac{\Delta}{\Delta n}\left(\ln\left(n.P(E|T)\right)\right)}{\frac{\Delta}{\Delta n}\left(\frac{1}{n}\right)}} = e^{\lim_{n\to 0}\frac{\frac{1}{n.P(E|T)}}{-\frac{1}{n^2}}}$ (by differentiation)

$= e^{\lim_{n\to 0} -n^2}$[for $P(E\,|\,T) = \frac{1}{n}$ from the table]

$= e^{-(0)^2} = 1$

From the equation (2), $P(T|E) = \lim_{n\to\pm\infty}\left(n.P(E|T)\right)^n = e^{\lim_{n\to\pm\infty} -n^2}$

$= e^{-(\pm\infty)^2} = 0$

B) In case $P(E|T)$ straightly takes a value of $\frac{1}{n}$, an application of limits calculus to the Bayesian Product Principle, $P(T|E) = \left(n.P(E\,|\,T)\right)^n$, generates a uniform distribution.

$$P(T|E) = \lim_{n\to[0;\pm\infty]}\left(n.P(E\,|\,T)\right)^n \equiv PT|E) = \lim_{n\to[0;\pm\infty]}\left(n.\frac{1}{n}\right)^n \equiv P(T|E) = \begin{cases}\lim_{n\to 0}(1)^n \\ \lim_{n\to\pm\infty}(1)^n\end{cases} = 1$$

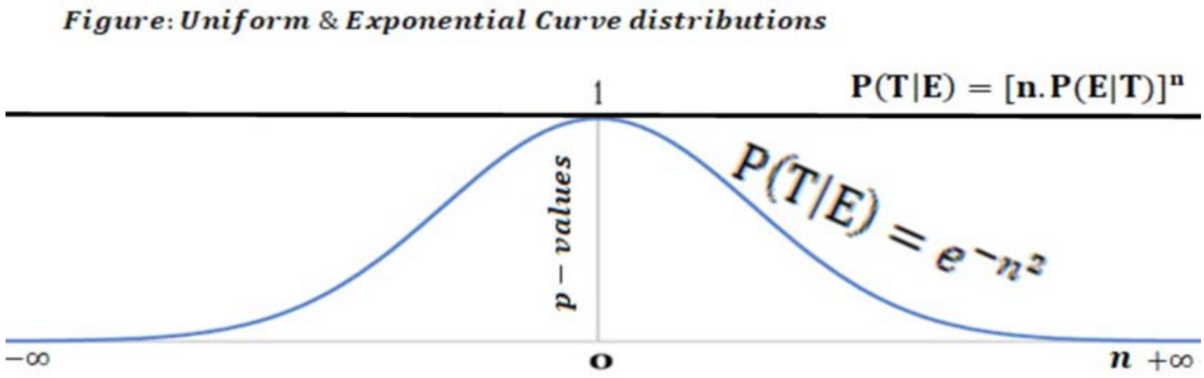

Figure 1

From the equation (1) and (2), the Bayesian Product Principle is to be understood within the framework of probability distribution as $P(T|E) = \lim_{n\to[0;\pm\infty]}\left(n.P(E|T)\right)^n$. With n pieces of evidence to be met, the results of $P(T|E)$ remain between zero and 1, which can also be written as $0 \le P(T|E) \le 1$. It may be difficult to understand how $P(T|E)$ should be 1 with zero evidence in probability distribution. The fact is that computational mathematics has access to the sense of what the value of zero as a number is all about. In the sense of probability calculus, zero evidence is not interpreted in the sense of nothingness *per se*. This is understood in twofold. First, zero evidence is the degree given by the ratio of that single particular or targeted piece of evidence over the totality of all possible pieces of evidence summed together. Second, zero evidence is that absolute proportion of each and every single piece of evidence in the totality of all outcomes of the possible experiments to be conducted. In this sense, all particular pieces of evidence are equally proportional within their universal set of evidence. In a uniform distribution, each and every particular piece of evidence makes the calculated probability 1 when its own value is well understood in its functional contribution to the absolute total value.

The concept of zero evidence can also mean that initial position of an experimenter who starts to question whether what is believed to be true is really true or not. This can be interpreted in two related ways. Either an observation proves a theory true and an experimenter begins to falsify it, or an observation does not prove a theory true and an experimenter strives by all means to reach at the truth value of a theory. For instance, T to be true at the first instance, the experimenter does not set any pieces of evidence, only experiments come to meet them, for they are already out there in the world. In addition, once the pieces of evidence are known from the course of observations before they prove true a given theory, predictions can be made with no puzzles. All observations are to be detected within the framework of probability distribution from a dataset with n experiments. The concept of zero evidence is just rationalized, it can be in the mind of the experimenter with no scrutiny, but it does not exist on its own. Zero evidence is the rate provided by a single piece of evidence over the totality of all pieces of the necessary evidence. If the p-value is low, the prior hypothesis is either rejected or accepted and if the p-value is high, the alternative hypothesis is either accepted or rejected or vice versa. Here comes in a dilemma that is to be only solved by the objectivism of the experiments noting that all pieces of the necessary evidence are uniformly proportional and converge to the same absolute value.

The application of limits calculus to the Bayesian Product Principle proves how and why the past and the future are symmetrical with respect to the present, because the results of the before and the after some pieces of evidence converge into those of the present. The past and the future are, in fact, embedded into the present. The past is made present by reminiscence and the future by the awaiting. The same application also proves that zero knowledge about the world is impossible.

The Bayesian Product Principle $\mathrm{P(T|E)} = \left(\mathrm{n.P(E}\,\big|\,\mathrm{T)}\right)^{\mathrm{n}}$ is a uniform distribution as a mathematical structure that provides a necessary truth but empty of sense without the mind of a truth-seeker that makes use of it in order to reach at the accurate results derived from facts. This is the nature of mathematics. In my case, it is a mathematical structure that is designed for analytic statements about the nature of things in the world; an application of limits calculus to it generates a uniform distribution and proves that the truth about the world is intrinsically uniform. It is also designed for synthetic statements about the state of things out there in the world. In this case, an application of limits calculus to it generates an exponential curve of probability distribution which can help us to justify how the truth about the nature of things in the world is symmetric and give us the interval of confidence we have in a given scientific theory after some tests or experiments.

Given the conditions, the state of things into the world can change with time, whereas their nature is inherent. Hence, the nature of a thing is intrinsic while its state is extrinsic. Logics has been employed in coming up with the Bayesian Product Principle, which is the method of logical positivism as well. An attempt to analyse mathematics and logics with the purpose of constructing a factual knowledge from observational and theoretical statements has been the work of logical positivism. In as much as the sense-based statements and theoretical entities prove and say something about the world, the mathematical and logical structures derived from them should be analytically applicable to the descriptive aspect of things in the world. Logical positivism takes seriously inductive logic for the fact that an inductive argument proves itself to be ampliative since the content of its conclusion goes far beyond the content of its premises.

4. Science with Structuralism and its Historicity

In logical positivism perspective, stating that ‘All metals expend on heating [T]’ would be as meaningful as stating that ‘Not all metals expand on heating [∼T]’. Both [T] and [∼T] are verifiable, and even the above Bayesian Product Principle can also prove them to be analytically true. An approach like this would be, however, irrational because there is no use of facts in stating [T] and [∼T], for there seems to be no relation between the two; each of them is on its own. Such a scenario is due to the inductive dilemma. Since logical positivism leaves the meaning of a statement into dilemma, a new school of thought attempted to get rid of that dilemma. The attempt was done by separating the logical methodology of science from its history and distinguishing the theoretical aspects of language from the observational ones based on the analysis of facts. This has also been the Popperian approach to Carnapian approach with regard to Bayesianism. Logical empiricism came in as the redeemer of the scientific knowledge left dilemmatic, but it could not account for theories made about the unobservable entities. Hence, there seems to be no rationality and objectivity in confirming scientific theories.

In showing how science works with irrationality and subjectivity, Thomas Kuhn argues that science works with paradigm-shifts. He demonstrates how scientific revolution takes place in a way that is similar to what arises out of political revolution in the political community. Consistent with Kuhn, due to the paradigm-shifting occurrences in the scientific community, scientific revolution is a “noncumulative developmental episode in which an older paradigm is replaced in whole or in part by incompatible new one” [15]. Kuhnian idea on the scientific revolution is that it is analogous to political revolution in the sense that in both types of revolution newer ideas come to replace either wholly or partly the older ones.

In political revolutions, institutional changes happen when new institutions come to replace old ones. In the same way, scientific revolution occurs when a new paradigm replaces in whole or in part an old paradigm. In assessing the nature of the scientific revolution, Kuhnian contribution needs to be appreciated, for the fact that a scientific revolution seems to be more accurate in the sense of non-cumulativeness than its sense of cumulativeness. For instance, in my case, it is not reasonable to accept [T] and [∼T] at the same time as theories that describe the same reality. The hypothesis ‘all metals expand on heating’ survives to some extent its negation ‘it is not the case that all metals expand on heating’. On the contrary, the hypothesis ‘some metals expand on heating’ does not survive the implications of its negation ‘it is not the case that some metals expand on heating’. Yet, the removal of the universal and existential quantifiers makes the hypothesis ‘metals expand on heating’ worthless when it is negated. All these scenarios show that there is a need to assess how accurate is Kuhn’s contribution to science.

In agreement with Kuhn, it seems obvious that scientific revolution can be seen as one kind of revolution that takes place in a way political revolution does. However, if I am to look at the process of these types of revolutions, there are more dissimilarities than similarities that make one different from the other in nature and procedure. It is noticeable that, in politics, there is no rational or logical basis in favouring one institution over another, because the values of each institution are different from those of the other. In a political community, each and every group argues for its own values, there is no objective reason in adopting one institution and leaving behind others. Everything is done in a persuasive manner. For instance, dictatorship and democracy do not share the same values. Once one wins over another, it cuts off itself from the political ideas of the other. In this respect, any winning institution practically tries to persuade the political community. Thus far, in questioning Kuhnian idea on scientific revolution, one may want to know whether the case of institutional changes in the political community is the same as in the scientific community.

To disagree with Kuhn, it is obvious that the processes of revolution in science are not the same as in politics. I am convinced that the scientific community works with objective reason, rational and logical basis in picking one paradigm over another. When it comes to the problem-solving of the paradigms-shifting, there is no such a thing as persuasion in science as it is the case for the institutional changes in politics. In picking one paradigm over another, the scientific community refers to its value and promising solutions to the prevailing scientific problem. When a paradigm replaces another, not everything from an old paradigm collapses, the new one has to have certain truth values and reliable views in connection with those of the old one. For instance, Cartesian explanation of gravitation, Newtonian Formulation of gravitation and Einstein theory of gravitation are commonly grounded on the force that draws objects towards their center. This being the case, my question is to know whether Kuhn suggested that scientists are guided by paradigm shifts without rules and if his idea of non-cumulativeness still holds.

In the scientific community, there are shared values in adopting paradigms either old or new ones, such as accuracy, consistency, scope, simplicity, and fruitfulness or progressiveness. These are five characteristics of a good theory later suggested by Thomas Kuhn himself in his response to the criticisms of the portrayal he gave to the scientific revolution. For a newer paradigm to be adopted by the scientific community, it needs to be consistent with and relevant to the views about an old paradigm on a given investigated scientific theory or principle within the domain to which it is more appropriate. If this is the case, then a question that comes in here is that one may want to know why Kuhnian portrayal of the scientific revolution is non-cumulative. In the end, efficiency and convenience have to complete the five values for the sake of scientific evolution. Thus, science does not need a revolution, but an evolution.

Unlike Thomas Kuhn, the logical empiricists believed that science was objective and rational. The only accurate method in choosing a reliable theory among competing theories was to use the lab and observe the results of their experiments for a better interpretation. The problem of justification could be sorted out through mathematics and logic. The language could be sorted out and all unobservable facts should be relegated and oriented to the realm of metaphysics. In this sense, science progresses cumulatively, because the results of theories could complement one another for the growth of science. However, in [16], Kuhn challenges the picture of science painted by the logical empiricists. Kuhnian approach is that the true picture of the way science works can be seen by studying the history of science. Such a study would show that science does not progress cumulatively but through various revolutions. In time of revolutions, experiments, observations, mathematics, logic, or language itself cannot help scientists to choose between paradigms because paradigms are incommensurable. Again, this view seems to have painted the picture of science as being non-objective and nonrational.

Additionally, Bayesian Theorem has been applied to how the logic and history of science can be merged. For instance, Wesley Salmon argues that Bayesian Theorem provides some aid in bridging the gap between Logical empiricism and Kuhn's historical approach on Theory Choice. Salmon suggests that there is need to possibly take an algorithm from logical empiricism and interpret its values with the aid of the historical approach. For him, an algorithm is simply a set of rules or formula. For instance, if logical empiricism has come up with a bold theory like 'all metals expand on heating', the algorithm that may confirm the degree of our beliefs in such a theory can be given in a probabilistic figure. Thus, we could use the tools of historical approach to tell ourselves what conditions *sin qua none* should be envisaged for a particular metal to expand on heating. In this sense, scientists can restore elements of rationality and objectivity in science.

Salmon maintains that an objectivist interpretation would consider the history of science and make use of rational arguments while interpreting Bayesian Theorem with regard to Theory Choice. As Salmon suggests, three criteria can account for objectivity while evaluating prior probabilities: pragmatic, formal and material. Pragmatic criteria would look at the circumstances under which a given theory is adopted, formal criteria would consider its consistency and material criteria would evaluate the content of a theory to be sure that there is no contradiction. These criteria also incorporate Kuhn's criteria of simplicity and consistency as well as the account of the history of science.

Salmon takes logical empiricism to be dependent on algorithms in the form of Bayesian Theorem. The idea is that the values of the algorithm could be well interpreted with the aid of Kuhn's criteria and reference to history and experience. Nevertheless, due to the threshold of shifting-ratios as well as the catchall hypothesis for alternative hypotheses, the Bayesian algorithm that Salmon came up with cannot accurately account for a theory which is to be held as true compared to others. An objective interpretation of Bayesian Theorem seems to contain both rationality and objectivity. I agree with Salmon that if the criteria for objectivity are followed and reference is made to scientific experience and frequency, then different scientist can act rationally and objectively even if they might have different prior probabilities. Ultimately, as Salmon observes, a 'washing out' may occur when the differences in prior probabilities are resolved by increasing confidence in a given scientific theory. This is in the same line as what I have done in order to came up with what I have called the Bayesian Product Principle. Salmon holds that Bayesian Algorithm can help scientific community to rationally and objectively choose one theory among competing theories. However, there is still the question of how to evaluate the likelihoods.

The likelihood checks the impact of the evidence against the truth of the theory. For a deterministic theory, the evidence must concur with the theory. From the above example, if the theory 'all metals expand on heating' is proven true, then the evidence from the experiment on metal 1 must be that metal 1 expands on heating. Thus $P(E|T)$ must be 1. If the evidence from the experiment is that metal 1contracts on heating, then the theory must be false since it contradicts clear evidence. In this case, $P(E|T)$ must be 0. But what if the result of the experiment is false. This would make our calculation of the likelihood more complicated. Salmon suggests that when such a scenario happens, we shall allow for what he calls plausible scenarios. Scientists should look for plausible reasons why evidence E is contradicting the theory T. If they can find such plausible reasons, then they can raise the likelihood of the theory from 0, thus saving the theory. These plausible scenarios in themselves have the potential to increase our knowledge of the world. In seeking for the proof of these plausible scenarios, we may arrive at information which we lacked before. Hence, this method accounts for Kuhn's criteria of fruitfulness.

It is important to note that Kuhn's criteria of simplicity and consistency have also been touched upon in the discussion on prior probabilities. This leaves behind the criteria of scope and accuracy which are not clarified enough to be used as criteria in making judgements on theory choice. As response, Salmon proposes three grades for rationality [17]. First, there is need to avoid logical contradictions and incoherent probabilities, for the presence of either constitutes a form of irrationality: "static rationality". Second, there is a "kinematic rationality" that merges static rationality and the rule of Bayesian conditionalization for a stronger type of rationality. Bayesian conditionalization requires a person to revise her prior probabilities in the light of new evidence. But the personalist can simply avoid this revision by choosing to change the values of prior probability. Third, there is a "dynamic rationality" that fuses kinematic with objectivist interpretation. This is the highest grade of rationality given that it is based on some form of objectivity. With this "dynamic rationality", Salmon appears to have partially responded to Kuhn's question on the role of rationality in the choice between competing scientific theories.

As a result, Salmon recommends an objective interpretation of Bayesian Theorem which seems to contain both rationality and objectivity. Salmon, however, admits that there are problems which his approach does not fully address. These include the question of incommensurable paradigms and the sameness of a background knowledge. Despite these persisting challenges, he believes that Bayesian Theorem can at least help both parties to realise that they share some common ground. Salmon's plan is to build a bridge between logical empiricism and Kuhn's historical approach, which is more or less a Popperian rationalistic approach.

Besides the work of Salmon, a historical approach to science has been evaluated in comparison with Scientific Realism. John Worrall argues that structural realism is the best way to go in bridging the gap between Scientific Realism and the historical view of radical change in theories [18]. Unlike Worrall, Psillos argues that despite the challenges posed to Scientific Realism over the years, it is still a tenable position to hold onto and its epistemic optimism is justified [19]. Their arguments are closely related to some extent and their aim is to come up with a more unfailing structure of the scientific realism that would be a reliable ground of scientific theories. They have tried to establish such a kind of structure, but it may raise some questions.

With structures only, scientific realism would be problematic insofar as it cannot be trusted as a source of true scientific theories that can tell us the truth about the world. Structures do not necessary "conform to the linguistic habits inferred from the evidence, and even stable linguistic habits are not automatically boundaries of sense, because long-lasting empirical beliefs can be and often have been corrected by new discoveries" [20]. We cannot know whether the unobservable posited by science actually exist or not. For instance, Einstein's Theory of space and time, String Theory and Theories about DNA are of the unobservable. We may not know that they really exist. It is unreasonable to base our explanation of all those theories on miracles or to believe them as true scientific theories. Yet, because we cannot easily accept miracles, believing in true predictive theories is the best way to go. It is better to believe, with scientific structures, that theories are true rather than believing in the idea of miracles.

However, if we are to look at the course of the history of science, scientific theories have not always been true. Ptolemian System has been replaced by Copernican System. Newtonian gravitation has been challenged by Einsteinian notion of gravity. It is reasonable to believe that even newer theories will be proved false in the future. Thus, we should not take scientific theories to be true, because there may be proven false at a particular time. This is, in fact, a result of the separability assumption which is "not only part of science but also of non-scientific traditions" [20]. The fact is that we cannot know the truth content of the world, but we can only posit that which helps us to make sense out of what we experience. Whatever the aim of the scientist, he cannot attain the whole truth about the world. Yet, it has been experienced that newer theories take some elements from old ones. Paul Feyerabend seems to have agreed with that by claiming that "theories, facts, and procedures that constitutes the scientific knowledge of a particular time are assumed to be the results of specific and highly idiosyncratic historical developments" [20]. There must be a kind of continuity between theories and a relation between phenomena. All we need is the structure of science.

As stated by Stathis Psillos, scientific realism is currently constituted of three main theses that seem to make it "attractive due to the balance of feasibility and dignity it offers to our quest of knowledge" [19]. First, the metaphysical thesis (metaphysical realism) states that "the world has a definite and mind-independent structure" [19]. If this were the case, what would be the rationale of holding onto scientific theories? What would be the role of human intellect in search of knowledge? What would be knowledge? The idea behind this first thesis is to "distinguish scientific realism from all the anti-realist accounts of science" [19]. Second, the semantic thesis (semantic realism) states that "scientific theories must be taken at face value as they are truth conditioned descriptions of their intended domain, both observable and unobservable. Hence, they are capable of being true or false". If this were the case, how come some scientific predictions come true? Where lies the problem? This second thesis intends to "make scientific realism different from eliminative instrumentalism and reductive empiricist accounts" [19] or pragmatic anti-realism. Third, the epistemic thesis (epistemic optimism) states that "mature and predictively successful scientific theories are well confirmed and approximately true of the world" [19]. If this were the case, could not there be some other ways of coming up with the truth about the world without relying on scientific theories? The third thesis distinguishes "scientific realism from agnostic or sceptical accounts of empiricism" [19], which is the Popperian perspective as well. The scientist aims at getting to truth about the world, but he may never know if he has attained such a truth.

As Psillos, Worrall believe that the "best defense of realism is to try to synthesize the historical record with some form of realism" [19]. Worrall's way of doing this synthesis is to posit "Structural Realism". Though, Psillos does not believe that structural realism is a tenable position. He seems to hold onto some optimism that there is a way to show that radical change in theories does not compromise the continuity of the truth about the world. Worrall does not seem to believe that any position other than the one on structural realism is tenable. He calls into question the concepts used by optimistic realists such as approximation towards truth, cumulativeness. It would make some sense to bring into question the optimism of Psillos and the synthesis of Worrall as a way of responding to questions that arise from the notion of structural realism in science. It seems that a number of questions still needs to be responded to before any form of realism can be legitimately held. Even at present, scientific theories can either succeed or fail. There is still a long way to go in order to have unfailing structures of science. Despite this fact, through logical empiricism, structures are possible.

Inductive probability, or induction in general, still has a role to play in the work of science. In fact, "laws and theories that make up scientific knowledge are derived by induction from a factual basis supplied by observations and experiments. Once such general knowledge is available, it can be drawn on to make predictions and offer accurate explanations" [21]. In other words, scientific theories and laws are induced from facts gathered with observations while predictions and explanations are deduced from scientific theories and laws generally well-accepted. Facts are prior to theories and independent of theories. They are reliable foundations of scientific knowledge. The Bayesian Product Principle, $\mathrm{P(T|E)} = \big(\mathrm{n.P(E|T)}\big)^{\mathrm{n}}$, has been induced from facts and it proves inductive probability to be reliable foundations of predictions and hence scientific knowledge. It helps to weigh the truth value of theories while picking one over another. It can be part of universal principles the science contains in its restricted universality framework, even if the universality of science does not mean that all scientific theories or laws are universally true and all its methods universally applicable [18]. Its objective interpretation requires a clear understanding of joint and conditional probabilities. There is need to note that $\mathrm{P(T|E)}$ does not necessarily mean $\mathrm{P(T \cap E)}$ in Bayesian approach to probabilities. $\mathrm{P(T \cap E)}$ is calculated within the framework of coherent probabilistic structures. $\mathrm{P(T|E)}$ is calculated within the framework of a systematisation of certain normative principles of rationality. It follows that scientific theories cannot be objects of beliefs, but only formal arrangements for organising events and their degrees of beliefs [5]. The above Bayesian Product Principle satisfies the concomitant claims of Bernoulli and Einstein. Bernoulli's claim about the objective certainty of things in the world is confirmed by Einstein who claimed that the discovery of a universal formal principle can lead us to assured results. By arguing in the same line with Bernoulli and Einstein, I am claiming that all metals that participate in the same reality [let it be called Metal-ness] should expand on heating, for each metal heated under normal conditions should expand and by expanding would reflect the same reality: the nature of metals and their physical reactions while on heating. Inductive probability has a lot to say about the world. Things are uniformly ordered into the world. There is no need of fracturing the knowledge we get from the observational facts of a repeated experiment. It is up to us to revise the experiment whenever we get the unexpected results. What is true remains true no matter what the results from experimentation might be, meaning that truth remains truth no matter what a human intellect can achieve. There is no problem with inductive probability and, hence, with induction in general. The problem of induction lies in our convictions and not in our logical argument about the truth of the nature of things into the world. If it is the nature of metals to expand on heating, human intellect cannot change that reality. Our observations inform our experimentations while confirming or disconfirming scientific theories. Scientific theories are explanations of laws of nature. Science succeeds where it works in accordance with the laws of nature. Science should revise its methods where it has failed. The role of inductive probability is to bring about simplicity in human intellect for an epistemic objectivity. I define epistemic objectivity as a relational understanding of observational facts and their rational and objective interpretations.

5. Conclusion

In this essay, I have discussed that the Carnapian understanding of probabilities was misunderstood and unvalued by Popper and then misjudged by Lakatos. Their arguments are both for justification and confirmation of scientific theories. There is no such thing as discovery in Popperian perspective. I have demonstrated how the logic of discovery goes back to look at how scientific theories come about, which is inadequate to solving the problem of induction. The Popperian perspective is closely related to Kuhnian perspective, they are both parts of the historical view of change in scientific theories. The problem of inductivism is equated to the problem of demarcationism. It is, in fact, the problem of drawing a line of demarcation between those hypotheses which could be properly described as belonging to empirical science, and others which might possibly be described as pseudo-scientific" [22]. Popper's concern is due to the subjectivity and objectivity of science that Michael Polanyi has even referred to in his arguments for personal and tacit knowledge [24].

Besides, the problem of demarcation goes even backwards to Aristotle's credentials on whether a statement is scientific or not [23]. Popper thinks that the criterion of verifiability, adopted by logical positivists, including Rudolf Carnap, was inadequate for demarcation [22]. He appropriately intended to contrast it with his own criterion, but he failed to do so due to his denial of inductive reasoning. Even his appeal to some sort of scientific convention for the short-term acceptance (prior acceptance) of theories, based on their corroboration (posterior acceptance) with our understanding of the world (reliability) through probabilistic hypotheses does not solve the problem. Popper indicates that "every test of a theory, that results either in its corroboration or falsification must stop at some basic statement which we decide to accept" [22]. For him, if a statement or theory has not been falsified, our tendency is to accept it as convention for the sake of scientific growth.

The Popperian approach prioritises growth over reliability. It shifts the emphasis from 'acceptance to reliability' and from 'reliable' to 'more reliable than' [3]. Is growth without reliability a good measure of scientific progress? Lakatos pointed out that "the most rigorous observance of the Popperian method may lead us away from the truth, accepting false and refuting true laws" [3]. Given this assertion, would it really be logical to assume that corroboration is directly proportional to the truth content of scientific hypotheses? Are there no other ways of solving the problem of induction without making problem-shifts? The logic of justification and that of discovery as well as the historical approach face that same problem. But The methods of justification differ from the methods of discovery. It is not unreasonable to believe that Carnapian inductive logic is empirically progressive. Structures that make science stands for reliable knowledge are possible.

The new form of Bayesian Theorem−the Bayesian Product Principle− is the structure of its own kind in Theory Choice. For if metals are as they are in themselves and by their nature, it is not understandable and clear to justify why and how we cannot hold that 'all metals expand on heating', once verified but not falsified, is true. And from this assertion, every other verified scientific theory or law follows. It is true that "there is a wider area of personal judgment in every verification of a scientific theory. Contrary to current opinion, it is not the case that a proven discrepancy between theoretical predictions and observed data suffices in itself to invalidate a theory" [24]. Our understanding of the world depends in no way on the falsifiability of our beliefs, but only on the accuracy of their justification. In fact, the Popperian perspective is a reductionist, which turns popper's deductive inferences into circular reasoning, because any unfalsifiable scientific theory would not come true. True scientific theories are true, not because they are falsifiable, but because they work in accordance with the natural order of things into the world.

Carnap-Popper controversy has been animated by the difference of the Carnapian understanding of the epistemic objectivity embedded in the inductive probability and the Popperian corroboration approach. While for Carnap the truth about the world seems to be analytic, for Popper it is asymptotic. Yet, the analytic and asymptotic conceptions of the truth about the world do not contradict one another for an objectivist truth-seeker, they rather complement one another. Both views give room to epistemic objectivity in science through empirical experiences. Had it not been the epistemic objectivity behind the predictions of science through the so-called inductive logic problem, science would not have made progress. As Ayer put it “the very way we go about making basic observations is deeply rooted in induction, noting that we acquire the habit of accepting certain statements as the result of having the appropriate experiences” [25]. The controversy in question cannot be of no use, it is a part of a reappraisal of scientific theories and that is what makes scientific knowledge factual knowledge, a knowledge that may be approached under various angles with an objectivist eye of the mind, a nondogmatic knowledge but truth-preserving given the domain in which it is applied. To complete both Carnap and Popper, within the framework of probability distribution, the new form of Bayesian Theorem proves that truth about the world is symmetric, meaning that the future looks like the past in virtue of the natural and relational order of things into the world.

Confirmatory approach, discovery approach, and historical approach to scientific theories cannot fully overcome the mistakes of the past for future predictions. None of these three approaches can accurately account for the best theory, only an epistemic justification can do so. Therefore, a methodological prospective through empiricism is far better than a critical retrospective through rationalism, even if they may inform one another. Carnapian approach to Bayesian Theorem is successful in how scientific theories can be confirmed or refuted. The Bayesian Product Principle proves Carnap successful in trying to overcome the problem of induction. It deepens the optimism of Psillos and Worrall about scientific realism and structural realism respectively. It proves that numberless pieces of evidence are not necessarily important while holding onto a given scientific theory, because as we increase the number of experiments in order to meet as many as possible pieces of evidence, we may tend to deny those we have already met by relying on those which do not confirm a given scientific theory under test. In fact, we may also confuse true pieces of evidence and false ones. Popper is not completely wrong when he thinks that knowledge is all about probability and contingency upon a dataset.

The Bayesian Product Principle does not demonstrate that probability is deterministic, but that what we ought to know about the nature of things in the world is deterministic. Nor does the Bayesian Product Principle intend to show that our knowledge of the world is deterministic but that what there is to know about the world is already set out there into the world. The Bayesian Product Principle proves that we are only justified in believing things that are most probable given the dataset. Truth may not be possible at the first place, but all we need is to always revise our beliefs and improve our knowledge of things into the world. The Bayesian Product Principle may also conform with the related predictions of Bernoulli and Einstein about the “objective certainty of things in the world” and the “discovery of a universal formal principle” that can lead us to assured results. It proves that things are as they are in themselves, before we even discover them out there in the world.

The Bayesian Product Principle proves that the overgeneralized problem of inductive logic is both a methodological and epistemological problem of our quest of certainty about our knowledge of the world. The use of both empiricism and rationalism does not address such a problem. It is obvious that one may only want to be empiricist and yet remains uncertain about the knowledge we ought to have about the world. In the same line, one may only want to be rationalist and yet again remains sceptical about the necessary knowledge of what there is to know about the world. The problem with both empiricism and rationalism is that they pay inadequate attention to the objective aspect of scientific findings, both tend to remain at the level of phenomenalism and psychologism. Hence, we do not only need to be empiricists and rationalists but also and above all objectivists.

Both empiricism and rationalism are grounded on Objectivism. This being the case, it is obvious that one may fix biased objectives to be reached at either through empiricism or rationalism. In this sense, all that is to come as results may depend on what the senses may be able to capture and what the reason may be able to tell about the same results through interpretation. In essence, both empiricism and rationalism are intertwined and inform each other in virtue of essential objectivism. By this notion of essential objectivism, I mean, in rigorous use of the term, that notion of the necessary aspect of objectivity of every kind of reasoning process free from any sort of subjective objectivity in our quest of truth and knowledge about the world. Truth in the sense of what there is to know and knowledge in the sense of what is known about the world should be objectively approached with no hunches of both empiricists and rationalists. Truth and knowledge about the world are essentially unique. Therefore, in its very sense, objectivism is to be essentially spoken of both that non-trivial reasoning process and non-differential interpretation of the evidence in guaranteeing the accuracy of findings in our quest of truth and knowledge about the world. Therefore, in verifying the truth value of scientific theories or laws, neither empiricism nor rationalism sets out evidence on their own. The evidence is met through experimentation and testing founded on Objectivism.

Acknowledgement

The main constraint of this study, conducted within the field of philosophy of science, is due to the fact that the Bayesian Product Principle does not respond to the challenges which might be raised from inductive reasoning that cannot be formalised. For instance, it does not address the questions that arise from the statements to be made about our probable knowledge of God in virtue of Its nature and existence through inductive reasoning. Nor does this study respond to the equivocality of coins-tossing or dices-tossing claimed to be fair and any other forms of chance-game of the kind due to their lack of objectivity within the framework of probability calculus. It is, then, suggested that further studies about the objectivity that may eventually be found in the inductive reasoning with statements only based on unobservable entities in the world need to be conducted. This study only puts an emphasis on verifiable analytic and synthetic statements about the nature and the state of things that are to be found out there in the world. An understanding of the results of this study as well as their application to other possible queries on the problem of inductive logic must not be overgeneralised but can still be more explored for further researches on the generalised problem of induction, particularly within the domain of philosophy of science.

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