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Determining the Coefficients of Storing Information in the Human Memory

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Abstract

The authors introduce a methodology for approximate finding the coefficient of storing information in the human memory. The method is realized in one specific variant. For this purpose, many tests are implemented and analyzed in a suitably selected student's group.

Keywords: amount of information; coefficient storing information; approximations.

1. Introduction

The following terms are introduced in article [1]:- *Quantity of information* - a set of knowledge related to the same science or a section of human knowledge that can be measured, i.e. it possesses a measure;- *Coefficient storing information* - the change in the amount of information at any moment is proportional to the volume of information at that moment. The coefficient of proportionality is called the coefficient storing information and it is a function of time.It is clear that these concepts have an individual character. They are specific even for different individuals placed under the same external conditions. In order to use analytical mathematical methods and especially the results to be applicable to a wide range of people placed under different external conditions, it is assumed that the above-mentioned concepts have averaged character and relate to the qualities of an idealized abstract individual. Under certain conditions, we can assume that the results obtained for this idealized individual can also be transferred to the representatives of the large groups of people which have the similar positions in society.Such generalizations are characteristic of the applications of mathematics.

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As the examples, we will point out analogous mathematical methods in population dynamics (see [2,3]), queuing theory (see [4,5]), control theory (see [6]), mathematical methods in the economy (see [7]) etc.

2. Preliminary Notes

We will use the following notations:

- t_0 - the initial moment (point) of investigation the dynamics of the amount of information;
- I_0 - the amount of information in the human memory at the initial moment t_0;
- $I(t;t_0,I_0)$ - the amount of information in the memory at the moment $t \geq t_0$, provided that at the initial moment t_0, the amount of information is I_0. There is $I(t;t_0,I_0) \geq 0$, $t \geq t_0$. It is clear that $I_0 = I(t_0;t_0,I_0)$;
- $\alpha = \alpha(t)$ - the coefficient of storage information for $t \geq 0$;
- $\beta = \beta(t, I(t))$ - the coefficient of proportional completing information at each training for $t \geq 0$.

In the above mentioned paper [1], the model equation for storing the information in the memory in the absence of additional training is obtained. The corresponding initial problem has the form

$$\frac{dI}{dt} = \alpha(t-t_0)I, \quad I(t_0) = I_0. \tag{1}$$

The solution of the above initial value problem (1) is the function

$$I(t;t_0,I_0) = I_0 \exp\left(\int_{t_0}^{t} \alpha(\tau - t_0)\, d\tau\right). \tag{2}$$

Therefore, if the initial moment t_0 of the process and the initial amount of information I_0 are known, then by equality (2), we can easily conclude that the coefficient of storage information $\alpha = \alpha(t)$ and the amount of information $I = I(t;t_0,I_0)$ define mutually. For practice, it is more useful to know the amount of information I. Moreover, it is clear that:

- From these two magnitudes, it is much easier to determine the amount of information through an experiment. Then, as a consequence, we can find the coefficient of proportionality;

- In the absence of a short-term training, there is no practical significance in finding the coefficient α. Therefore, in this case, finding a coefficient of information storage is meaningless.

In the article [8], the dynamics of the amount of information was studied in the presence of the short-term intensive trainings. Hypothetically, It can be assumed that these trainings take place instantaneously in the form of impulsive increasing the memory information. In this case (see [8]), the general problem modeling the dynamics of the amount of information in memory has the form

$$\frac{dI}{dt} = \alpha(t - t_i)I, \quad t_i < t \le t_{i+1}; \tag{3}$$

$$I(t_i + 0) = \left(1 + \beta(t_i, I(t_i))\right).I(t_i), \ i = 1, 2, ...; \tag{4}$$

$$I(t_0) = I_0, \tag{5}$$

where the filling of information take place at the moments $t_1, t_2, ..., \ t_0 < t_1 < t_2 < ...$. We introduce the notations

$$I_i^+ = \left(1 + \beta(t_i, I(t_i; t_0, I_0))\right).I(t_i; t_0 I_0), \ i = 1, 2, ...$$

Recall that, in paper [8] and [9], the form of the solution of impulsive system (3), (4), (5) is determined

$$I(t; t_0, I_0) = \begin{cases} I_0 \exp\left(\int_{t_0}^{t} \alpha(\tau - t_0)\, d\tau\right), & t_0 \le t \le t_1; \\ I_1^+ \exp\left(\int_{t_1}^{t} \alpha(\tau - t_1)\, d\tau\right), & t_1 < t \le t_2; \\ \text{..............................} & \\ I_i^+ \exp\left(\int_{t_i}^{t} \alpha(\tau - t_i)\, d\tau\right), & t_i < t \le t_{i+1}, \ i = 2, 3, ...; \\ \text{..............................,} & \end{cases} \tag{6}$$

Clear that, the solution is a discontinuous function and its points of discontinuity coincide with the impulsive moments $t_1, t_2, ...,$ at which the amount of information is continuous function on the left - hand side. In order to find the approximate function $I(t; t_0, I_0)$ in this case, it is necessary to conduct the experiments (observations) in each interval of continuity $[t_0, t_1], \ (t_1, t_2], \ (t_2, t_3], ...$ The large number of experiments in the general case significantly complicate to find numerically $I(t; t_0, I_0)$. Thus, in order to obtain approximately the amount of information in the presence of short-terms intensive training, it is necessary to realize the following plan

1. We find approximately the function $I = I(t; t_0, I_0)$ in the arbitrary interval without external intensive filling the information in the memory;
2. Finding the function $\alpha = \alpha(t)$ using formula (2);
3. Obtaining the function $I = I(t; t_0, I_0)$ in the presence of impulsive effects by formula (6).

The problems of determining:

- The impulsive jumps I_i^+, $i = 1, 2, \ldots$, which take place at the impulsive moments $t_1, t_2, \ldots$;

- Numerical approximation of the coefficient of proportional filling the information $\beta = \beta\left(t, I\left(t\right)\right)$, $t \geq 0$ at each training is beyond the scope of this study. We will repeat once again that the objective of this work is to find approximate coefficient $\alpha = \alpha\left(t\right)$.

3. Method of determining the Storage Coefficient of Information

We will present the methodology in several successive steps.

3.1. Selection of the object of study

First, we determine the type and composition of the group which will be surveyed, i.e. how to find the coefficient of storing information). It is important to observe the following requirements:

- Homogeneity: Studies are applicable to a group of people placed under the same essential external conditions. The absence of additional training is the most important;
- Target: The survey refers to the one and same type of information;
- Mass participation: Research targets relatively large groups of people;
- The sample of members of the observation group should adequately reflect the distribution of the major types of subgroups in the study group.

3.2. Determination of the numerical parameters of observations

The following observation parameters should be specified here:

- Number and times of test implementation (experiments or observations): Observations should be fairly evenly distributed, at a time interval for which it has been previously specified that there is no further training on the subject of the survey. Observations must be a sufficient number in order to ensure the relevance of the results using regular mathematical methods. Such method is the least squares, which occupies a central role in the realization of the task. According to the remarks above, we determine the number of observations n and the moments $\tau_1, \tau_2, \ldots, \tau_n$, $0 < \tau_1 < \tau_2 < \ldots < \tau_n$ of the experiments.
- Number of participants in each experiment: The participants must be representatives of each subgroup composing the group profile (subject of study). Their number may vary in different experiments, since the results are averaged. Let the number of participants in the observations be $k_1, k_2, \ldots, k_n$, respectively.
- Weighting factors: It is possible to enter weighting factors for each participant in every experiment. In general, the coefficients correspond to the participant's frequency of occurrence in the study group. Assume that for the first experiment (held at the moment τ_1) the weight factors related to the participants are $\lambda_{11}, \lambda_{21}, \ldots, \lambda_{k_1 1}$; for the second experiment (carried at the moment τ_2), the weight factors are $\lambda_{12}, \lambda_{22}, \ldots, \lambda_{k_2 2}$, and for the last experiment, the weight factors are $\lambda_{1n}, \lambda_{2n}, \ldots, \lambda_{k_n n}$. If in the

i-th experiment, the first participants and only they are representative of the same subgroup, then:

- All of these participants have the same weighting factor, i.e. $\lambda_{1i} = \lambda_{2i} = ... = \lambda_{ki}$;

- Sum of the weight coefficients of representatives of the same subgroup is equal to the frequency of subgroup, i.e. $\lambda_{1i} + \lambda_{2i} + ... + \lambda_{ki} =$ frequency of the representatives of subgroup.

It is clear that the next equalities are valid

$$\lambda_{11} + \lambda_{21} + ... + \lambda_{k_1 1} = 1; \quad \lambda_{12} + \lambda_{22} + ... + \lambda_{k_2 2} = 1; \quad \lambda_{1n} + \lambda_{2n} + ... + \lambda_{k_n n} = 1.$$

3.3. Determination of the amount of information in the observations

Modern reporting for the information in memory is based on the tests.

The participants in the given survey analyze the test and then share the available information in their memory.

It is written too much by different authors about the content, form, scope, method of compiling the tests and so on.

In our opinion, this question is a question of methodology and therefore, it is far from the subject of this study. For this reason, we will not dwell on shaping the tests.

We will specify few details related to the reporting of information that is important for accurately determining the coefficient of storage information:

- The tests must contain information that has been provided to the students in one or more previous training. For convenience, assume that the information given in advance to the participants in the experiment has a measure *1*, i.e. $I_0 = 1$;
- The main purpose of each specific observation i $\left(i = 1, 2, ..., n\right)$ on every specific participant in the i-th experiment with the number *ij* $\left(j = 1, 2, ..., k_i\right)$ is to estimate approximately the amount of information stored in its memory at the moment τ_i.
- In other words, we find the constants $p_{11}, p_{12}, ..., p_{1k_1}$, $p_{21}, p_{22}, ..., p_{2k_2}, \ldots,$ $p_{n1}, p_{n2}, ..., p_{nk_n}$, for which $0 \le p_{ij} \le 1,\ \ i = 1, 2, ..., n,\ \ j = 1, 2, ..., k_i$ is satisfied. The constant p_{ij} shows the portion of information (which we investigate) stored in the memory of j-th participant in i-th experiment. Approximate determination of the information amount I_i at the moment τ_i in the memory of an abstract generalized individual, we find the formula

$$I_i = \lambda_{i1} p_{i1} + \lambda_{i2} p_{i2} + ... + \lambda_{ik_i} p_{ik_i},\ \ i = 1, 2, ..., n. \qquad (7)$$

3.4. Approximate determination of the amount of information

In the preceding paragraphs, we obtain the tabulated function

Table 1

τ_1	τ_2	$\cdots$	τ_n
I_1	I_2	$\cdots$	I_n

Using these numerical data, we define (intuitively) the parameters $a, b,...$ and parametric family of the functions $\Phi(a,b,...)$. The requested function $I(t;a,b,...)$ belongs to this family and approximates the amount of information $I(t;t_0,I_0)$, i.e., we have:

$$I(t;a,b,...) \in \Phi(a,b,...), \quad I(t;a,b,...) \approx I(t;t_0,I_0).$$

Using the least squares method, we will specify that the function $I(t;a,b,...)$ is the best average quadratic approximation of the amount of information $I(t;0,1)$). We assume that the initial moment is $t_0 = 0$ and the initial amount of information is $I_0 = 1$.

3.5. Approximately determination of the coefficient of storing information

In general, according to formula (2), the approximation $\alpha(t;a,b,...)$ of the coefficient of storage information $\alpha = \alpha(t)$ satisfies the equality

$$I(t;a,b,...) = \exp\left(\int_0^t \alpha(\tau;a,b,...)\,d\tau\right)$$

$$\Leftrightarrow \ln I(t;a,b,...) = \int_0^t \alpha(\tau;a,b,...)\,d\tau$$

$$\Leftrightarrow \alpha(\tau;a,b,...) = \frac{1}{I(t;a,b,...)}.\frac{d}{dt} I(t;a,b,...).$$

3.6. Particular case

In particular, let be given the type of function that is a quantitative dimension of the information (see again formula (2)), it is natural to assume that the parametric family of the middle-quadratic approximations has the type

$$\Phi(a,b,...) = \{\exp(A(t;a,b,...));\ a,b,... \in R\}.$$

Using this type of parametric family of approximations, it is reasonable to convert Table 1 in Table 2, respectively.

Table 2

τ_1	τ_2	$\cdots$	τ_n
$A_1 = \ln I_1$	$A_2 = \ln I_2$	$\cdots$	$A_n = \ln I_n$

Then we define the type of parametric family $\varphi(a,b,...)$ to which this function belongs i.e., $A(t;a,b,...) \in \varphi(a,b,...)$.

Using the least square method, we specify the parameters $a,b,...$ i.e., we find the function $A(t;a,b,...)$. It is clear that this is the best mean-square approximation of the class $\varphi(a,b,...)$ for the integral function $\int_0^t \alpha(\tau)\,d\tau$.

In this case, the approximation $\alpha(t;a,b,...)$ of the coefficient of storage information satisfies the equality

$$A(t;a,b,...) = \int_0^t \alpha(\tau;a,b,...)\,d\tau \ \Rightarrow\ \alpha(\tau;a,b,...) = \frac{d}{dt}A(t;a,b,...).$$

4. Algorithm for determining the coefficient of storage information

Using the considerations in the previous paragraph, we reach the following algorithm:

Algorithm

4.1. Determine the number of observations n;

4.2. Define the times of the experiments $\tau_1, \tau_2, ..., \tau_n$;

4.3. Determining the number of participants in the observations $k_1, k_2, ..., k_n$;

4.4. Determination the relative weights of the participants in the observations $\lambda_{11}, \lambda_{12}, ..., \lambda_{1k_1}$, $\lambda_{21}, \lambda_{22}, ..., \lambda_{2k_2}, \ldots, \lambda_{n1}, \lambda_{n2}, ..., \lambda_{nk_n}$;

4.5. Determination of the parts of the stored information by the participants in the experiments $p_{11}, p_{12}, ..., p_{1k_1}$, $p_{21}, p_{22}, ..., p_{2k_2}, \ldots, p_{n1}, p_{n2}, ..., p_{nk_n}$;

4.6. Evaluation the constants $I_1, I_2, ..., I_n$ using formula (7);

4.7. Calculating the constants $A_1 = \ln I_1$, $A_2 = \ln I_2, ...,$ $A_n = I_n$;

4.8. For the table function set by Table 2, we define the type of approximating parametric family $\varphi(a,b,...)$

;

4.9. Define the function $A(t;a,b,...) \in \varphi(a,b,...)$ (finding the parameters) using the least square method;

4.10. Finding the estimate of the function $\alpha(t;a,b,...) = \frac{d}{dt} A(t;a,b,...)$.

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