International Journal of Sciences: Basic and Applied Research (IJSBAR)

ISSN 2307-4531

http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

Casson fluid Model for Blood flow with Velocity Slip in presence of Magnetic effect

*aSarfraz Ahmed, b G.C.Hazarika

[a]*Department: Mathematics, Jorhat Engineering College*
City/country: Jorhat ,Assam. India
[b]*Department: Mathematics, Dibrugarh University*
City/country:Dibrugarh ,Assam India
[b]*Email:gchazarika@gmail.com.*
[a]*Email:sarfrazahmed.jrt@gmail.com*

Abstract

An attempt is made in this analysis to study the flow of blood through a uniform rigid artery with an axial velocity slip condition at vessel wall has been considered by assuming blood a Casson fluid. It has been observed that the effect of the Hartmann number and the Reynolds number on the velocity field as well as on the wall shear stress is very prominent and when the Hartmann number and Reynolds number increases, the fluid velocity decreases. It also include Poiseuille fluid models of blood for both slip and no-slip at wall and one-layered Casson fluid model and no slip at tube wall as its special cases. Role of slip in influencing the flow variables and physiological implications of this theoretical modelling are discussed in brief.

Keywords: Casson fluid model, stenosed vessels, Magnetic effect, Reynolds number, Hartmann number Blood flow.

1.Introduction

Human blood is a suspension of red cells in a continuous and aqueous substance called plasma [4].Plasma behaves as Newtonian fluid whereas whole blood shows non-Newtonian characteristic [9,11] has pointed out that in case of blood vessels with diameter above 250 micrometers, blood may be considered as a Newtonian fluid .Stenosis is formed by substance depositing on vessel walls. Stenosis is formed by substance depositing on vessel walls. A stenosis may lead to partial or total vessel blockage in some instances and therefore poses a serious medical problem. In human body, physiological fluids present in human systems include digestive juices, sweating, blood, saliva, urine etc. Among the body fluids, the most important is obviously blood which is regarded as a suspension of different cells in a continuous aqueous solution called plasma [4]. Plasma behaves as Newtonian fluid whereas, whole blood shows non-Newtonian characteristics [9,11] has pointed out that in case of blood vessels with diameters above 250 micro meters, blood may be considered as a homogeneous Newtonian fluid. But at low shear rates, blood exhibits non-Newtonian behaviour [9]. It is well known that non-Newtonian nature of blood significantly influences the flows, particularly in the cases where blood vessels are curved, branching or narrow etc. Further, blood has a finite yield stress and Casson's equation can take care of this property which has been reported by many investigators [8,12].

---

* Corresponding author.
E-mail address: sarfrazahmed.jrt@gmail.com.

Owing to the intermittent pumping of heart muscles it produces a pressure gradient and as a consequence, blood flows through the circulatory systems [4]. Arterial system is also subject to continuous shear stress changes. One of the circulatory diseases is atherosclerosis which happens due to lipid or lime deposits (atheroma) on the inner coat of the arteries [10]. About 75% of all deaths in industrialized world are caused due to circulatory disease. The accumulation of lipid, fat and other particles along the inside artery wall lead to narrowing of the vessel. In some cases, it is observed that a so-called stenosis of the vessel with age that means elastic fibers in vessel wall are replaced by more rigid collagen fibers and this process is accompanied by widening of the arterial lumen [10]. Also, the diseased areas of blood vessels are characterized by adhesiveness, hardening, loss of elasticity and contraction in the cross-section, found predominantly in the vicinity of beads, bifurcations and other places. As a consequence, there may arise serious complications and disorders in blood supply due to narrowing in the bore of the vessels or widening of the inner wall. Top restore normal blood supply in diseased vessels, it seems, is an introduction of velocity slip at tube wall will be meaningful [13,5,7,3]. In order to study the theology of blood along with its role in the fundamental understanding of many cardiovascular diseases like myocardial infarction, stroke, thrombosis, sickle cell diseases etc. Many investigators have already proposed theoretical [11,14,15,16,17,6] models under various considerations and flow situations. In their models, flow has been considered steady or pulsatile, fluid as a Newtonian one and the boundary condition on no-slip at wall etc. As blood shows some deviations from Newtonian behaviour, it seems consideration of blood, in behaving like a non-Newtonian fluid (a Casson fluid) along with slip at vessel wall etc as is done here, will be appropriate and significant from the physiological point of view.

The application of magneto hydrodynamic principles in medicines, engineering is of growing interest. Many investigations have been made on blood flow under the effect of magnetic field. Barnothy [1] has shown that by the application of an external magnetic field the biological systems are greatly affected. Vardanyan [18] showed that the application of magnetic field reduces the flow of blood. Bhuyan and Hazarika [2] have investigated the problem of blood flow with effects of slip in arterial stenosis due to presence of transverse magnetic field and they have also found that the applied magnetic field reduces the velocity of blood through arteries. Here an analysis has been made to describe the behaviour of Casson fluid model for blood flow with velocity slip in the presence of magnetic effect.

1. Mathematical Formulation of the problem:

A two-fluid modelled for blood flow in a uniform tube has been developed in the present problem. The model basically consists of a core of a red cell suspension in the middle layer and the peripheral plasma in the outer layer. The steady laminar flow of an incompressible fluid, assuming it to behave as a Casson fluid [9], through a rigid circular tube is considered. The axial coordinate and velocity are $\hat{z}$ and $\hat{v}$ respectively. B is the applied magnetic field in $\hat{r}$ direction. Flow is governed by the continuity and Navier-Stockes equations and in addition, Casson's constitutive equation the magnetic field applied in $\hat{r}$ direction the equations in the axial and radial directions in dimensionless form. From the report of Young [19] considering the axisymmetric laminar steady flow of blood, the general constitutive equation in the case of mild stenosis subject to the addition may therefore be written as:

$$-\frac{\partial \hat{p}}{\partial \hat{r}} = 0$$

$$-\frac{1}{\hat{r}}\frac{\partial \hat{p}}{\partial \hat{\theta}} = 0 \qquad (1)$$

$$-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{z}} + \frac{\mu}{\hat{r}}\frac{1}{\rho}\frac{\partial}{\partial \hat{r}}\left(\hat{r}\frac{\partial \hat{u}_z}{\partial \hat{r}}\right) + \frac{\sigma}{\rho}\hat{B}^2 = 0 \qquad (2)$$

From which we observed that pressure does not vary in the radial ($\hat{r}$), circumferential ($\hat{\theta}$) and axial $(\hat{z})$ direction and that pressure remain constant across any cross-section of the tube and $\hat{p}$ Is a function of only $(\hat{z})$ that is $\hat{p} = p(\hat{z})$ and so pressure gradient term in the last equation above becomes $\frac{d\hat{p}}{d\hat{z}}$

Then (2) $$-\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{z}}+\frac{\mu}{\hat{r}}\frac{1}{\rho}\frac{\partial}{\partial\hat{r}}\left(\hat{r}\frac{\partial\hat{v}}{\partial\hat{r}}\right)+\frac{\sigma}{\rho}\hat{B}^2=0$$

Non-dimensional form

$$r=\frac{\hat{r}}{R_0},z=\frac{\hat{z}}{R_0},R=\frac{\hat{R}}{R_0},P=\frac{\hat{P}}{\rho U_0^{\,2}},U=\frac{\hat{U}}{U_0},B_0^{\,2}=\frac{\hat{B}^2}{U_0},\mu=\frac{\hat{\mu}}{\mu_0},V=\frac{\mu_0}{\rho}\quad C+\frac{1}{R_e r}\frac{d}{d}\left(r\frac{d}{rd}\right)\frac{v}{r}+\frac{M^2}{R_e}=0 \qquad (3)$$

$$v=\frac{\hat{v}}{U_0}$$

Where M= $\sqrt{\frac{\sigma}{\rho V}}BR_0$, Re= $\frac{R_0U_0}{V}$, $C=-\frac{d\hat{p}}{d\hat{z}}$

At r=R tube wall ,v=o and at r=0 ,v is finite

SOLUTION OF THE PROBLEM:

Equation (3) can be written as

$$r^2\frac{d^2v}{d^2r}+r\frac{d}{d}\frac{v}{r}=k\,^2r$$

Where $K=\left(-C\ _eR-M^2\right)$

Let $r=e^z$,

$$v=\frac{1}{4}K.e^z$$

$$v=\frac{1}{4}\left[-C\ _eR\,M^2\right]r^2 \qquad (4)$$

Also, shear stress component at any distance r from the tube axis is given by

$$\tau_{rz}=\hat{\mu}\frac{d\hat{v}}{d\hat{r}}=\frac{\mu_0U\mu_0}{R_0}\frac{d}{d} \qquad (5)$$

$$\tau_{rz}=\frac{\mu_0r\mu_0U}{2R_0}\left[-C\ _e-RM^2\right]$$

Express for wall shear stress τ_w can be obtained from the formula

$$\tau_w=\tau_{rz}(r=R) \qquad (6)$$

$$\tau_w=\frac{\mu_0R\mu_0U}{2R_0}\left[-C\ _e-RM^2\right]$$

Using Equation (6) and $\tau_{Zr}(r_c)=\tau_y$ express for τ_0 will lead to the form

$$\tau_y=-\frac{r_c\mu_0U\mu_0}{2}\left[-C\ _e-RM^2\right]$$

In between τ_Y and τ_w there may arises two cases wall shear stress is greater and that yield stress. In case $\tau_Y \geq \tau_w$ that is if $r_C \geq R$ then there will occur no flow accordingly velocity function will become

$$V_Z = 0$$

$$V_Z = V_Z(r)$$

Again, Casson equation may be reproduce in the following form

$$\dot{v} = f(\tau_{rz}) = \frac{1}{k_c}(\sqrt{\tau_{rz}} - \sqrt{\tau_y})^2$$

$$\tau_{rz} \geq \tau_y \qquad (7)$$

$$\dot{v} = \frac{\left[-C_e - MR^2\right]}{k_c}\frac{\mu_0 U_0 A}{2R_0}\left[\sqrt{r} - \sqrt{r_c}\right]^2$$

$$= 0, \qquad \tau_{rz} \leq \tau_y$$

In the above, vanishing of strain rate that is $\ddot{\gamma}$ implies that

$$\frac{dv_z}{dr} = 0$$

$$v_Z = \text{constant} = v_c \quad \text{When } \tau_{rz} = \tau_y$$

Where v_c is the core velocity at $r = r_c$ (core radius) .As such for blood flow when $r_0 \langle R$ there arises two region $0 \leq r \leq r_c$ and $r_c \leq r \leq R$ and it is clear for region between 0 and r_c equation representing the flow is

$$\frac{dv_Z}{dr} = 0 \quad 0 \leq r \leq r_c$$

Which after integration give rise to the form $v_Z = v_c \quad 0 \leq r \leq r_c$ indicating the velocity profile will become flat in the region and for $r_c \leq r \leq R$ velocity v_Z will show deviation from flat profile and Casson equation has to be applied for this domain of blood flow the same equation it is easily seen that

$$\frac{dv_Z}{dr} = \left[\frac{r - r_c}{2}\left(-C_e - MR^2\right)\right] \frac{dv_Z}{dr} = \left[\frac{r_c - r}{2}\left(C_e + MR^2\right)\right] \quad r_c \leq r \leq R \qquad (8)$$

In solving equation (8) we have used the following velocity slip condition at vessels

$$v_Z = v_S \text{ at r=R} \qquad (9)$$

Where v_S is the constant slip velocity at tube wall in axial distance .As a result of integration between r and R we have

$$\int_r^R \frac{dv_Z}{dr}dr = \int_r^R \frac{r_0 - r}{2\mu}\left(C_e + MR^2\right)dr \quad u_z = u_s + \frac{\left(C_e + MR^2\right)(R - r)}{4\mu}\left[(R + r) - 2r_0\right]$$

$$r_0 \le r \le R \quad (10)$$

At $r_0 = r$ expression for core velocity can be obtained from equation (10)

$$u_o = u_s + \frac{\left(C_e + M^2\right)}{4\mu}\left(R - r_0\right)^2 \quad (11)$$

And for all values of r between 0 and r_0 velocity function is $u_o = u_s \quad 0 \le r \le r_0$

Thus from above expression and consideration velocity distribution u_z can be re-written in the following manners

$$v_z = \begin{cases} v_z(r) & r_c \le r \le R \\ v_c & 0 \le r \le r_c \\ 0 & r_c \rangle R \end{cases} \quad (12)$$

Where $u_z(r)$ and u_0 are given in equation (10) and (11) respectively

The rate of volume flow can be found from

$$Q = \int_{r=0}^{R} 2\pi r\, v_z\, dr \quad (13)$$

2. Results and discussion:

The objective of this analysis is to study the flow characteristics of a Casson fluid model for blood flow with velocity slip in presence of magnetic effect. The problem is solved numerically using Shooting method. Numerical calculations have been done for various considerations of parameters i.e., the Hartman Number M and the Reynolds number Re Numerical results are shown graphically. It has been observed that the effect of the Hartmann number M and the Reynolds number Re on the velocity field as well as on the wall shear stress is very prominent. If shear stress τ_{rz} is greater than yield stress τ_y then flow of blood is possible otherwise, there will be no blood flow.

Fig (1) illustrate that the velocity profile v_z decreases with increases of Hartmann number M in presence of slip. It is also observed that Hartmann Number increases the effect of magnetic field is decreases.

Fig (2) shows that the nature of velocity profile is same if there is no slip.

Fig (3) Illustrate that the velocity profiles with effect of magnetic field for various values of Reynolds number. It is seen that the velocity profile decreases as the Reynolds number increases with slip. Fig (4).Illustrate that the velocity profile v_z is also same in no slip case.

Analysis developed here is based on certain assumptions which may lead to some physiological implications viz., (a) Flow is assumed steady which is indeed true for very thin arteries in CVS where the pulsatility effects are small. (b) Assumption that velocity variation in axial direction is negligible as compared to its variation in radial direction, may lead to the implication that the length of the artery is too large as compared to the radius.

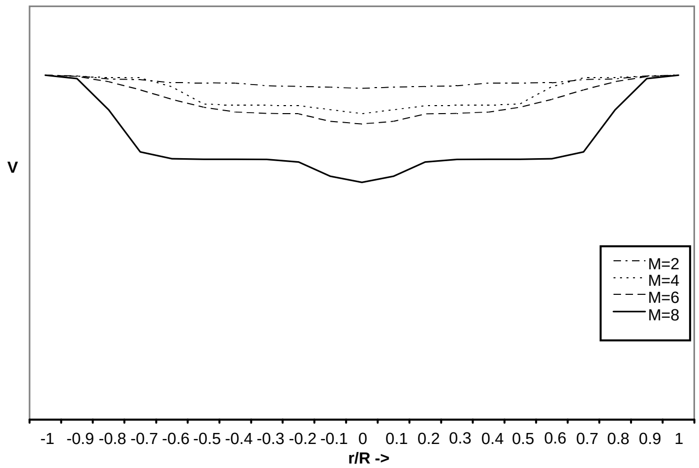

Fig. 1. Variation of Velocity profile for different Hartmann number at Re=2 with slip

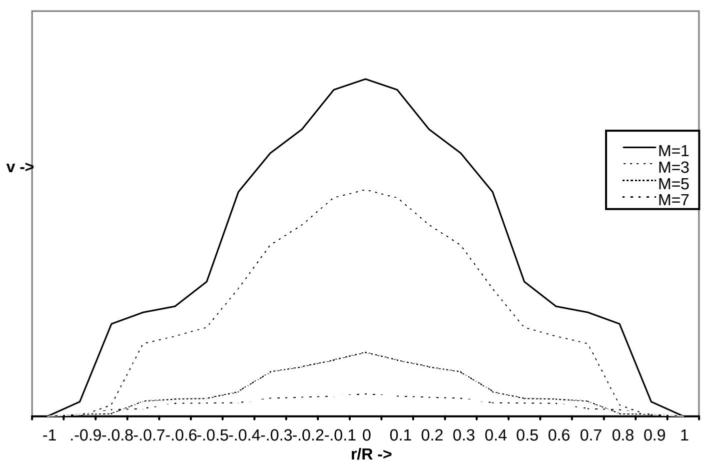

Fig. 2. Variation of Velocity profile for different Hartmann number at Re=2 with no slip

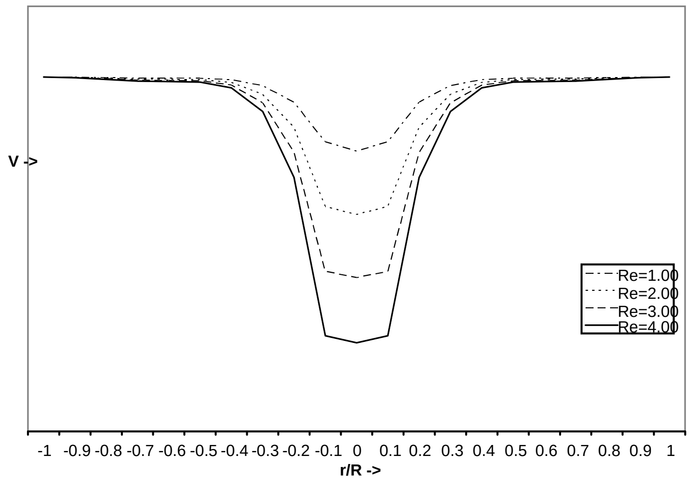

Fig. 3 Variation of velocity profile for different Reynolds number at M=2 with slip

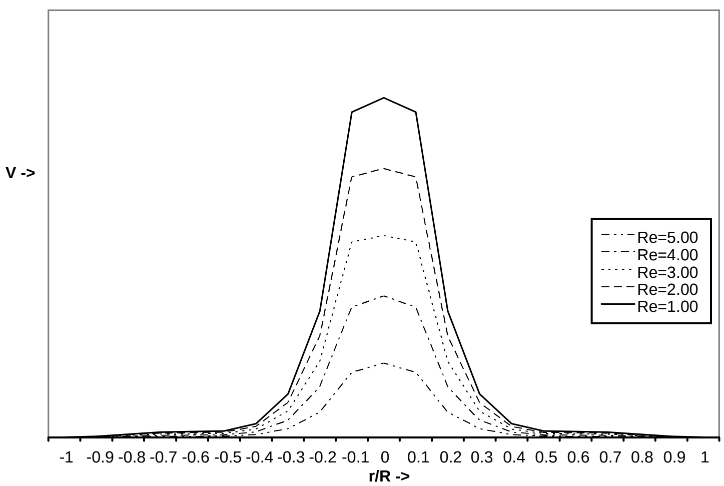

Fig4. Variation of Velocity profiles for different Reynolds number at M=2 with no slip

References:

[1] Barnothy,M.F.(ed),Biological effect of Magnetic field, Vol-I&II, Plenum Press(1969)

[2] Bhuyan, B.C. and Hazarika, G.C., *Bio-Science Research Bulletin*, Vol. 17(2), pp. 105-112 (2001)

[3] Biswas, D. and Nath, J., Oscillating Blood Flow Through a Uniform Artery with Wall Slip, Journal of Assam University, 4(1), pp. 142-154 (1999).

[4] Boyd, W., Text Book of Pathology: *Structure and Functions in Diseases*, Lea and Febigar, Philadelphia (1963).

[5]Brunn, P., The Velocity Slip of Polar Fluids, Rheol. Acta., 14, pp. 1039-1054 (1975).

[6] Bugliarello, G. and Sevilla, J., Velocity Distribution and Other Characteristics of Steady and Pulsatile Blood Flow in Fine Glass Tubes, Biorheology, 7, pp. 85-107 (1970).

[7] Chaturani, P and Biswas, D., *Resistance to Blood Flow Through a Steosed Tube with Axial Velocity Slip at Wall*, Proc. 12th NC-FMFP, NFM-10, pp. 400-405, IIT-Delhi (1983)

[8]Cocklet, G.R. *The Rheology of humand blood*, Biomechanics ed. Y.C. Fung, pp. 63. (1972)

[9]Fung, Y. C. *Biomechanics : Mechanical Properties of Living Tissues*, Springer-Verlag, New York Inc. (1981).

[10] Guyton, A.C., Text Book of Medical Physiology, Igakushoin International, Philadelphia (1970).

[11] MacDonald, D.A., On Steady Flow Through Modelled Vascular Stenoses, *J. Biomechanics*, 12, pp. 13-20 (1979)

[12] Merrill, F.W., Rheology of Human Blood and Some Speculations on its Role in Vascular Homeostatics Biomechanical Mechanisms in Vascular Homeostatics and Intravascular Thrombosis, ed. P.N. Sawyer, pp. 127-137, Appleton Century Crofts, New York (1965)

[13] Nubar, Y., Blood Flow, Slip and Viscometry, *Biophys., J.*, 11, pp. 252-264 (1971).

[14] Oka, S., Pressure Development in a Non-Newtonain Flow Through a Tapered Tube, Biorhelogy, 10, pp. 207-212 (1973).

[15] Singh, N.L., A Theoritical Approach to the Effect of Wall Layer Thickness on Blood Rheology, Proc. Math. Soc., BHU, Vol. 3, pp. 27-30 (1987).

[16]Srivastava,V.P.,Particulate Suspension Blood Flow Through Stenosed Arteries:Effects of Hematocrit and Stenosis Shape,*Indian J.Pure Appl Math.,*33(9),pp.1353-1360 Sept(2002)

[17] Sud, V.K. and Sekhon, G.S., *Phy. Med. Biol.*, Vol. 34, pp. 795-805, (1989)

[18] Vardanyan, V.A., Biophysics, Vol. 18 (3), pp. 491-496 (1973)

[19] DF Young and Tsai FY,Flow characterstics in model of arterial stenosis-I,Steady flow.J.Biomech.,Vol.6,pp.395-410(1973).