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Dynamical Behavior of Logistic Map for Different Parameter Values

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Abstract

In this paper we have developed dynamical behavior of logistic map. We have discussed some basic concepts of fixed point, attracting fixed point, neutral fixed point, repelling fixed point and chaos. A proposition, a theorem and a lemma are established. Chaotic behavior of logistic map considering initial seeds, iteration of logistic map, sensitivity to numerical inaccuracies of logistic map, Sensitivity analysis to initial value of logistic map, time series analysis of logistic map are given. Finally we have shown that the attracting fixed point, neutral fixed point and repelling fixed point of logistic map for different parameter values.

Keywords: Logistic Map; Attracting Fixed Point; Neutral Fixed point; Repelling Fixed Point; Chaos.

1. Introduction

The logistic map is a polynomial mapping of degree two, often cited as archetypal example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations [1]. The map was popularized in a seminal 1976 paper by the biologist Robert May [2], in part as a discrete-time demographic model analogous to the logistic equation first created by Pierre François Verhulst [3].

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The logistic function finds applications in a range of fields, including artificial neural networks, biology (especially ecology), biomathematics, chemistry, demography, economics, geosciences, mathematical psychology, probability, sociology, political science, linguistics, and statistics. For any value of r there is at most one stable cycle. If a stable cycle exists, it is globally stable, attracting almost all points [4]. The relative simplicity of the logistic map makes it a widely used point of entry into a consideration of the concept of chaos [1]. A rough description of chaos is that chaotic systems exhibit a great sensitivity to initial conditions—a property of the logistic map for most values of between about 3.57 and 4 [5] . A common source of such sensitivity to initial conditions is that the map represents a repeated folding and stretching of the space on which it is defined. In the case of the logistic map, the quadratic difference equation describing it may be thought of as a stretching-and-folding operation on the interval (0, 1) [6]. In many fields, equilibrium or stability are fundamental concepts that can be described in terms of fixed points. For example, in economics, a Nash equilibrium of a game is a fixed point of the game's best response correspondence. However, in physics, more precisely in the theory of phase transitions, linearization near an unstable fixed point has led to Wilson's Nobel prize-winning work inventing the renormalization group, and to the mathematical explanation of the term "critical phenomenon Programming language compilers use fixed point computations

2. Some important definitions:

2.1. Fixed point

A fixed point of a function is an input the function maps to itself. When we study the fixed points of a function, we can learn many interesting things about the function itself. In mathematics, a fixed point (sometimes shortened to fixed point, also known as an invariant point) of a function is an element of the function's domain that is mapped to itself by the function. That is to say, c is a fixed point of the function $f(x)$ if and only if $f(c)=c.$ This means $f(f(\ldots f(c)\ldots))=f^{n}(c)=c$, an important terminating consideration when recursively computing f.

2.2. Attracting fixed point

The point x_0 is called an attracting fixed point if $|f'(x_0)|<1.$ An attractive fixed point of a function f is a fixed point x_o of f such that for any value of x in the domain that is close enough to x_o, the iterated function sequence converges to x_o. Attractive fixed points are a special case of a wider mathematical concept of attractors.

2.3. Neutral fixed point

The point x_0 is called an neutral fixed point if $|f'(x_0)|=1.$ A fixed point is said to be a neutrally stable fixed point if it is Lyapunov stable but not attracting. The center of a linear homogeneous differential equation of the second order is an example of a neutrally stable fixed point.

2.4. Repelling fixed point

The point x_0 is called an repelling fixed point if $\left| f'(x_0) \right| > 1.$

2.5. Chaos (R. L. Devaney 1989)

Let X be a metric space. A continuous function $f : X \to X$ is said to be *chaotic* on *X* if f has the following three properties:

(C-1) Periodic points are dense in the space *X*

(C-2) *f* is topologically transitive

(C-3) *f has* sensitive dependence on initial conditions

Mathematically,

(C-1) $P_k(f) = \{x \in X : f^k(x) = x\,(\exists k \in \mathbf{N})\}$ is dense in X.

(C-2) For $\forall\ U, V : non-empty\,open\,sets$ of X, $\exists k \in \mathbf{N}$ such that $f^k(U) \cap V \neq \phi.$

(C-3) $\exists \delta > 0$ (sensitive constant) which satisfies:

$\forall\ x \in X\ \ and\ \ \forall N(x,\varepsilon),\ \exists\, y \in N(x,\varepsilon)$ and $\exists\ k \le 0$ such that $d(f^k(x), f^k(y)) > \delta.$

Here, $P_k(f) = \{x \in X : f^{\exists k}(x) = x\}$, $Q_k(f) = \{x \in X : f^k(x) = x : f^i(x) \neq x \text{ for } i = 1,2,\ldots,k-1\}$ and $Per(f) = \bigcup_{k=1}^{\infty} P_k(f)$.

Namely, $P_k(f)$ is the set of all k-periodic points, $Q_k(f)$ is the set of those k-periodic points whose prime period is k and $Per(f)$ is the set of all periodic points. Obviously we have that $Per(f) = U_{k=1}^{\infty} Q_k(f)$ and $\{Q_k(f)\}_{k=1}^{\infty}$ is a family of mutually disjoint subsets of X.

2.5.1. Proposition

Tent map $\tau : X = [0,1] \to X$ *is chaotic dynamical system.*

Proof: we show that the map τ satisfies the chaotic three conditions (C-1), (C-2) and (C-3) in the following (1), (2) and (3) respectively. We again note that

$$P_n(\tau) = \{x \in X : \tau^n(x) = x\},$$

$$Per(\tau) = \bigcup_{n=1}^{\infty} P_n(\tau).$$

(1) First we note that for any $x \in X$ and $n \in N$, $P_n(\tau) \cap (x - \frac{1}{2^n}, x + \frac{1}{2^n}) \neq \phi$.

Let $x \in X = [0,1]$ and open set $U(\ni x)$ be given. Then there exists $\varepsilon > 0$ such that $U(x,\varepsilon) \subset U$. If we assume $\varepsilon < 1$, then there exists $n \in N$ such that $\frac{1}{2^n} < \varepsilon$.

Thus $(x - \frac{1}{2^n}, x + \frac{1}{2^n}) \subset U(x,\varepsilon) \subset U$

Moreover there exists x_n such that $x_n \in P_n(\tau) \cap (x - \frac{1}{2^n}, x + \frac{1}{2^n})$. Since it follows that $x_n \in P_n(\tau) \cap U$ i.e. $P_n(\tau) \cap U \neq \phi$. Hence $Per(\tau)$ is dense in X. therefore τ satisfies (C-1).

(2) Here we first note than+1t for any $x \in X$ and $n \in N$,

$$(x - \tfrac{1}{2^n}, x + \tfrac{1}{2^n}) \supset (\tfrac{k}{2^{n+1}}, \tfrac{k+1}{2^{n+1}}), k \in \{0,1,\ldots,2^{n+1}-1\}$$

Let U and V are two non-empty sets in X. then there exists $x \in U$ and $\varepsilon > 0$ such that $(x-\varepsilon, x+\varepsilon) \subset U$ and since $\frac{1}{2^n} < \varepsilon$, we have

$$(x - \tfrac{1}{2^n}, x + \tfrac{1}{2^n}) \subset (x-\varepsilon, x+\varepsilon) \subset U,$$

Moreover there exists k such that $(\frac{k}{2^{n+1}}, \frac{k+1}{2^{n+1}}) \subset (x - \frac{1}{2^n}, x + \frac{1}{2^n}) \subset U$.

We also have that $$\tau^{n+1}([\tfrac{k}{2^{n+1}}, \tfrac{k+1}{2^{n+1}}]) = [0,1].$$

Thus $\tau^{n+1}(U) \supset \tau^{n+1}([\frac{k}{2^{n+1}}, \frac{k+1}{2^{n+1}}]) = [0,1]$

Therefore, $\tau^{n+1}(U) \cap V = [0,1] \cap V \neq \phi$.

That is $\tau^l(U) \cap V \neq \phi$

Where $n+1 = l \in N$. Hence τ is one-sided topologically transitive. Therefore τ satisfies (C-2).

(3) Let $x \in X = [0,1]$ and $\varepsilon > 0$ be given. Then there exists $n \in N$ such that $\frac{1}{2^n} < \varepsilon$. We assume that $x \in [\frac{k}{2^n}, \frac{k+1}{2^n}]$ for some $k \in \{0,1,...,2^{n+1} - 1\}$. Then

$$\tau^n([\tfrac{k}{2^n}, \tfrac{k+1}{2^n}]) = [0,1].$$

There exists $y \in [\frac{k}{2^n}, \frac{k+1}{2^n}] \subset U(x, \in)$ and we have $d(\tau^n(x), \tau^n(y)) \geq 1/2$. Therefore τ satisfies (C-3).

2.5.2. Theorem

Let X and Y be metric space and (X, f) and (Y, g) be topological dynamical systems. Then if f is chaotic then g is chaotic.

Proof: Suppose f satisfies $(C-1)$ and $(C-2)$ and we show that g satisfies these two conditions. Since

$$Per(f) = \{x \in X : f^n(x) = x \text{ for some } n \in N\}. \tag{2.1}$$

and $$Per(g) = \{y \in Y : g^n(y) = y \text{ for some } n \in N\} \tag{2.2}$$

then $$Per(g) = \{h(x) : x \in Per(f)\} \tag{2.3}$$

First we show (2.3) Let $y \in Per(g)$. Then $g^n(y) = y$ for some $n \in N$. We put $x = h^{-1}(y)$. Then we have, $f^n(x) = f^n(h^{-1}(y)) = h^{-1} \circ g^n \circ h(h^{-1}(y)) = h^{-1} \circ g^n(y) = h^{-1} \circ y = h^{-1}(y) = x$

for some $n \in N$. Then by (2.1) we have, $x \in Per(f)$. Since $x = h^{-1}(y)$, then

$y = h(x) \in \{h(x) : x \in Per(f)\}$. Thus $Per(g) \subset \{h(x) : x \in Per(f)\}$

Next let $y = h(x)$ for $x \in Per(f)$. Then $g^n(x) = x$ for some $n \in N\}$.

Thus we have $g^n(y) = h \circ f^n \circ h^{-1}(y) = h \circ f^n \circ h^{-1}(h(x)) = h \circ f^n(x) = h(x) = y$ for some $n \in N$. Then by (2.1) we have, $x \in Per(g)$ for some $n \in N$. Thus $\{h(x) : x \in Per(f) \subset Per(g)\}$. Therefore $Per(g) = \{h(x) : x \in Per(f)\}$.Now we show that *g* satisfies $(C-1)$, namely we show that if for any $x \in X$ and any $\varepsilon > 0$, there exists $w \in Per(f)$ such that $d(x, w) < \varepsilon$. Then for any $y \in Y$ and any $\varepsilon > 0$, there exists $w \in Per(g)$ such that $d(y, z) < \varepsilon$. Let $y \in Y$ and $\varepsilon > 0$ are given. Since $h(x)$ is surjective, there exists $x \in X$ such that $h(x) = y$ and since $h(x)$ is continuous, for any $\varepsilon > 0$ there exists $\delta > 0$ such that $d_X(x, x') < \delta$ Implies $d_Y(h(x), h(x')) < \varepsilon$. That is we have shown that for given

$y \in Y$ and $\varepsilon > 0$, there exists $h(w) \in Per(g)$ such that $d_Y(h(x), h(w)) < \varepsilon$. Thus $Per(g)$ is dense in Y , therefore g satisfies $(C-1)$.

Next we show that g satisfies $(C-2)$. Namely we show that for arbitrary two non-empty open sets U and V , there exists $k \in N$ such that $f^k(U) \cap V \neq \phi,$ then $g^k(U) \cap V \neq \phi$.

Let U and V are non-empty open sets in Y . Then there exists $y_1 \in U$ and $\varepsilon_1 > 0$ such that $U_1(y_1, \varepsilon_1) \subset U$ and $y_2 \in V$ and $\varepsilon_2 > 0$ such that $U_2(y_2, \varepsilon_2) \subset V$. Since h is surjective, there exists $x_1 \in X$ such that $h(x_1) = y_1$ and there exists $x_2 \in X$ such that $h(x_2) = y_2$. Since $h: X \to Y$ is continuous, there exists $\delta_1 > 0$ such that $d_X(x_1, x') < \delta_1$ implies $d_Y(h(x_1), h(x')) < \varepsilon_1$ and there exists $\delta_2 > 0$ such that $d_X(x_2, x') < \delta_2$ implies

$d_Y(h(x_2), h(x')) < \varepsilon_2$. We take $U_1(x_1, \delta_1)$ and $U_2(x_2, \delta_2)$ as open set in X., hence by assumption $f^k(U_1(x_1, \delta_1) \cap U_2(x_2, \delta_2) \neq \phi.$ That is, there exists $z \in f^k(U_1(x_1, \delta_1)$

and $z \in U_2(x_2, \delta_2)$ .Then there exists $z \in U_1(x_1, \delta_1)$ and $f^k(z) \in U_2(x_2, \delta_2)$. Therefore, $d_X(x_1, z) < \delta_1$ and $d_x(x_2, f^k(z) < \delta_2$ .Thus

$$d_y(h(x_1, h(z)) < \varepsilon_1 \Rightarrow h(z) \in U_1(h(x_1), \varepsilon_1) \subset U \Rightarrow g^k(h(z) \in g^k(U_1(y_1, \varepsilon_1)) \subset g^k(U)$$

and $d_y(h(x_2, f^k(z)) < \varepsilon_2 \Rightarrow h(f^k(z)) \in U_2(h(x_2), \varepsilon_2) \Rightarrow g^k(h(z)) \in U_2(y_2, \varepsilon_2) \subset V$

Therefore, we have $g^k(h(z)) \in g^k(U_1(y_1, \varepsilon_1)) \cap U_2(y_2, \varepsilon_2) \subset g^k(U) \cap V.$ That is, $g^k(U) \cap V \neq \phi.$ Therefore g satisfies $(C-2)$.

2.5.3. Lemma

Let X= [0, 1]. Logistic map $\lambda: X \to X$ is topologically conjugate to tent map $\tau: X \to X$ by the homeomorphism $h: X \to X$ defined by $h(x) = \sin^2 \frac{\pi}{2} x$.

Indeed we have

$$(h \circ \tau \circ h^{-1})(x) = (h(\tau(h^{-1}(x))$$

$$= (h(\tau(\tfrac{2\arcsin\sqrt{x}}{\pi})), 0 \leq \tfrac{2\arcsin\sqrt{x}}{\pi} \leq 1, (i.e, 0 \leq x \leq 1)$$

$$= (h(2\tfrac{2\arcsin\sqrt{x}}{\pi}), 0 \le \tfrac{2\arcsin\sqrt{x}}{\pi} \le 1/2, (i.e, 0 \le x \le 1/2)$$

$$= \sin^2 \tfrac{\pi}{2}(\tfrac{4}{\pi}\arcsin\sqrt{x}) \qquad (h^{-1}(x) = \sin^2 \tfrac{\pi x}{2})$$

$$= \{2\sin(\arcsin\sqrt{x}).\cos(\arcsin\sqrt{x})\}^2$$

$$= 4\{\sin(\arcsin\sqrt{x})\}^2.\{1-\sin^2(\arcsin\sqrt{x})\}$$

$$= 4x(1-x)$$

$$= \lambda(x)$$

And in the other case we have $(h \circ \tau \circ h^{-1})(x) =$

$$= h(2-2(\tfrac{2\arcsin\sqrt{x}}{\pi})), 1/2 \le \tfrac{2\arcsin\sqrt{x}}{\pi} \le 1, (i.e, 1/2 \le x \le 1$$

$$= \sin^2 \tfrac{\pi}{2}(2-\tfrac{4}{\pi}\arcsin\sqrt{x})$$

$$= \{\sin(\pi - 2\arcsin\sqrt{x})\}^2$$

$$= \{\sin(2\arcsin\sqrt{x})\}^2$$

$$= \{2\sin(\arcsin\sqrt{x}).\cos(\arcsin\sqrt{x})\}^2$$

$$= 4x(1-x)$$

$$= \lambda(x).$$

Since by 2.5.1 Proposition, τ is chaotic, therefore by 2.5.2 theorem, the logistic map λ is chaotic.

3. Chaotic Behavior of Logistic Map Considering Initial Seeds

We take the Logistic Map is $x \to rx(1-x)$ where $r = 4$, so $x \to 4x(1-x)$.To find the fixed points of this iteration we must solve this $x = 4x(1-x)$.The fixed points are $x = 0$ and $x = \dfrac{3}{4}$.

If we consider the orbit $x_0 = 0.5$ we find $0.5 \to 1 \to 0 \to 0 \to ....$.

which is an example of an eventually fixed orbit. The points $x_0 = 0.5$ is not itself fixed, but it second iterate is fixed. Again we have found a very special orbit.

Now let's try a few more. The chart on the following page is given the first 10 iteration with four decimal places taking initial seeds $x_0 = 0.2,\ 0.11,\ 0.3,\ 0.34,\ 0.65$.

Table 1: Chaotic behavior of logistic map considering initial seeds

Iteration Number	Initial Seed $x_0 = 0.2$	Initial Seed $x_0 = 0.11$	Initial Seed $x_0 = 0.3$	Initial Seed $x_0 = 0.34$	Initial Seed $x_0 = 0.65$
0	0.2	0.11	0.3	0.34	0.65
1	0.64	0.3916	0.84	0.8976	0.91
2	0.9216	0.9530	0.5376	0.3677	0.3276
3	0.2890	0.1791	0.9943	0.9299	0.8811
4	0.8219	0.5883	0.0225	0.2606	0.4190
5	0.5854	0.9688	0.0879	0.7708	0.9738
6	0.9708	0.1208	0.3208	0.7068	0.1022
7	0.1133	0.4248	0.8716	0.8289	0.3669
8	0.4020	0.9774	0.4476	0.5671	0.9292
9	0.9616	0.0884	0.9890	0.9820	0.2630
10	0.1478	0.3222	0.0434	0.0707	0.7753

Do we see the pattern? Definitely not. These orbits do not appear to be tending to be a fixed points or cycling. Instead we seem to jump all over the place to behave very differently from orbits of linear iterations. These seem to be chaotic.

4. Iteration of Logistic Map

Let the Logistic Map is $f(x) = -r(x-1)$ where r is the parameter bounded by $(0,4$). Then we iterate the function in the following s using MATHEMATICA programming.

$$f(x) = -r(x-1)x,$$

$$f^2(x) = -r^2(x-1)x\left(r\,x^2 - r\,x + 1\right)$$

$$f^3(x) = -r^3(x-1)x\left(r\,x^2 - r\,x + 1\right)\left(r^3x^4 - 2\,r^3x^3 + r^3x^2 + r^2x^2 - r^2x + 1\right)$$

$$f^4(x) = -r^4(x-1)x\left(r\,x^2 - r\,x + 1\right)\left(r^3x^4 - 2\,r^3x^3 + r^3x^2 + r^2x^2 - r^2x + 1\right)\ (r^7x^8 - 4\,r^7x^7 + 6\,r^7x^6 - 4\,r^7x^5 +$$
$$r^7x^4 + 2\,r^6x^6 - 6\,r^6x^5 + 6\,r^6x^4 - 2\,r^6x^3 + r^5x^4 - 2\,r^5x^3 + r^5x^2 + r^4x^4 - 2\,r^4x^3 + r^4x^2 + r^3x^2 - r^3x + 1)$$

As in the above process, we can iterate the map up to n times. Depending on the parameter value r, we have shown the graph after some iterations of Logistic Map as follows:

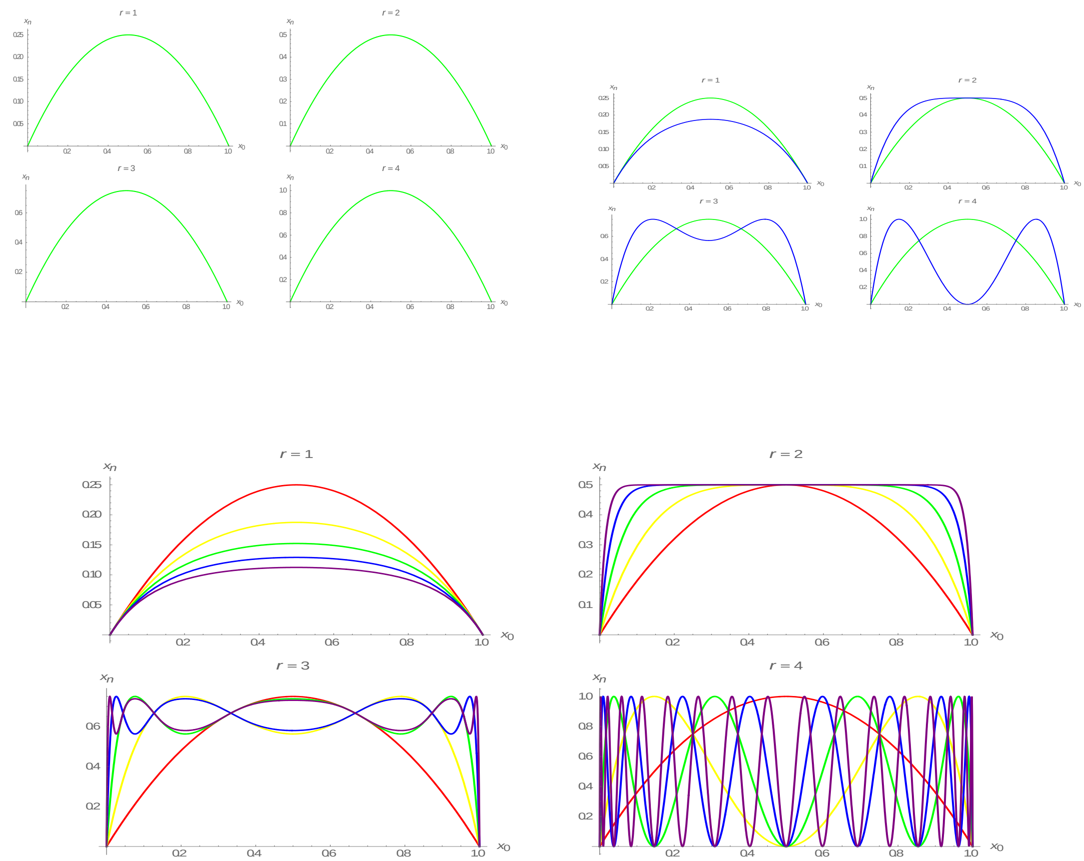


Figure 4.1: Graph of the Several iteration Logistic Map depending on r

The above figures represent the 1st, 2nd and 5th iterations of the Logistic Map.

5. Sensitivity to Numerical Inaccuracies of Logistic Map

For some values of the parameter r, the logistic model $y_{n+1} = ry_n\left(1 - y_n\right)$ is very sensitive to numerical inaccuracies. To see this, we calculate 100 values from the model with r = 4, first by using normal decimal numbers and then by using high-precision numbers. In the latter case, we start with numbers that have a precision of 65 digits:

```
vals1 = NestList[4#(1 − #)&, 0.01, 100]; vals2 = NestList[4#(1 − #)&, 0.01`65, 100];
```

Values corresponding to vals2 are thick. From approximately iteration 50 on, the values differ greatly. In calculating vals2, we started with numbers having 65 digits of precision. During the calculation, many digits were lost so that the last value 0.598853 only has a precision of approximately 6.6. Look at some elements of vals2.

Thus, we know that all the digits of vals2 are correct. This means that the values in vals1are incorrect from approximately iteration 50 on. This demonstrates the sensitivity to numerical inaccuracies of the logistic model for some values of the parameter *r*. Thus, if we calculate long sequences from the logistic model, it is important to use a high enough precision during the calculation. From the plot of vals2, we see that the series behaves quite chaotically. It is known that *chaotic* models are very sensitive to numerical inaccuracies. It can be shown that the logistic model is chaotic for *r* from approximately 3.57 to 4, although inside this interval there are also some small non chaotic intervals.

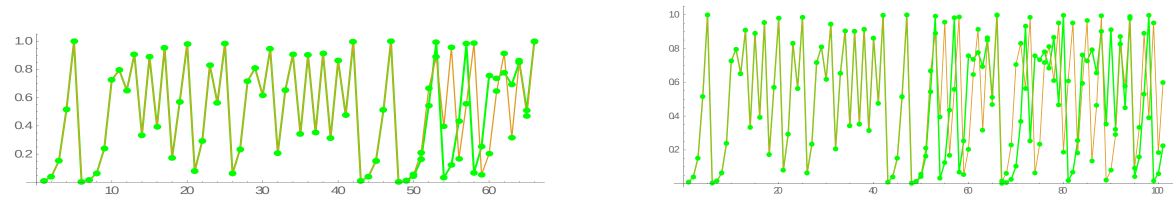


Figure 5.1: Sensitivity to Numerical inaccuracies of the Logistic Map

6. Sensitivity Analysis to Initial Value of Logistic Map

Chaotic models are also very sensitive to the initial value. To show this, compute, with $r = 4$, 50 iterations using starting points $0.02 + 10^{-i}$,$i = 1, \ldots, 25$. Then plot the 20th value of each of the 25 series. Also plot the 50th value of each of the 25 series

```
vals = Table[NestList[4#(1 − #)&, 0.02 + 10^−i, 50], {i, 25}];

{ListPlot[vals[[All, 20]], PlotRange → {−0.05, 1.05}, PlotLabel → "20th value"],

ListPlot[vals[[All, 50]], PlotRange → {−0.05, 1.05}, PlotLabel → "50th value"]}
```

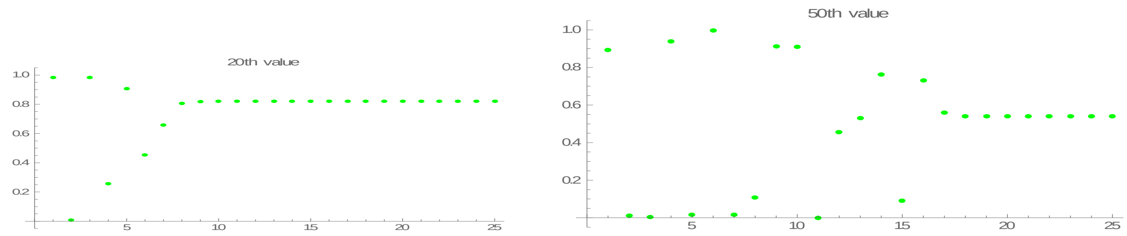


Figure 6.1: Sensitivity to initial Value of Logistic Map

From the first plot, we see that even if the starting point differs from 0.02 by 10^{-7} or more (see the first seven points in the plot), the value of y_{20} significantly differs from the value that results when starting from 0.02. From the second plot, we see that if the starting point differs from 0.02 by 10^{-16} or more, the value of y_{50} differs significantly from the value that results when starting from 0.02.

8. Time Series Analysis of Logistic Map

The orbits seem to be wandering around the interval $0 \le x \le 1$ rather aimlessly. Let's see if we can detect a pattern from the time series for one of these orbits. Here is the time series graph for the seed $x_0 = 0.65, 0.2, 0.2001$ with iteration 60,100 respectively

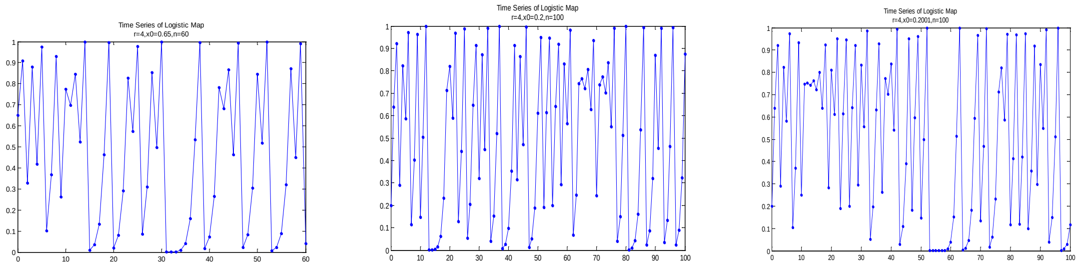


Figure 8.1: Time Series of Logistic Map

Just when we think we are beginning to see a pattern in the above picture, the time series graphs begins to do something else and a new pattern emerges. After some iteration we observe that there is no pattern in the above picture. This is our glimpse of what Mathematicians called Chaos.

9. Dynamical Behavior of Logistic Map

Logistic model is $\frac{dx}{dt} = rx(1-\frac{x}{k}),$ where r is a parameter and k is carrying capacity. If $k=1,$ then $\frac{dx}{dt} = rx(1-\frac{x}{k}),$ is called logistic function and logistic model becomes as

$$f_r(x) = rx(1-x).$$

$$\Rightarrow rx - rx^2 = x$$

$$\Rightarrow rx^2 - rx = -x$$

$$\Rightarrow rx^2 - rx + x = 0$$

$$\Rightarrow x(rx - r - 1) = 0$$

Either $x = 0$ or $rx - r - 1 = 0$

$$\Rightarrow r(x-1) = -1$$

$$\Rightarrow (x-1) = \frac{-1}{r}$$

$$\Rightarrow x = \frac{-1}{r} + 1$$

$$\Rightarrow x = \frac{r-1}{r}$$

The fixed points of f_r can be found by solving the equation $rx(1-x) = x \Rightarrow x = (p_0 = 0), \left(\frac{r-1}{r} = p_r\right).$

For r=1, r=3, r=3.8, r=4; the fixed points are the points of intersection of $f_r(x) = rx(1-x)$ and $y = x$ as shown in Figure 2.12.

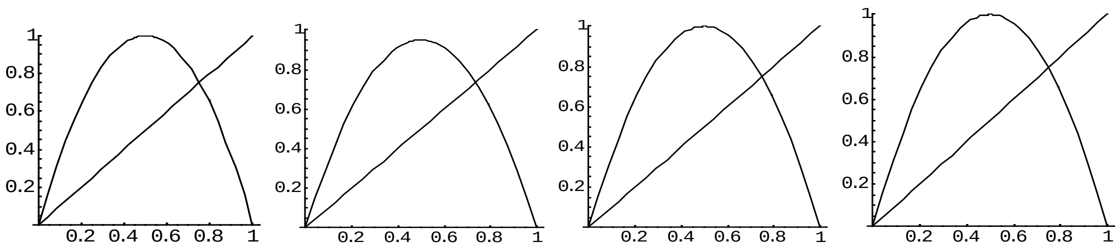


Figure 9.1 (a) when $r = 1$ (b) when $r = 3$ (c) when $r = 3.8$ (d) when $r = 4$

Case ($r = 1$) : When $r = 1$ then $x = 0$ is the only fixed point for $f_r(x)$.

Case ($r = 3$) : When $r = 3$ then $x = 0, \frac{2}{3}$ are the fixed points for $f_r(x)$.

Case ($r = 3.8$) : When $r = 3.8$ then $x = 0, 0.74$ are the fixed points for $f_r(x)$.

Case ($r = 4$) : When $r = 4$ then $x = 0, \frac{3}{4}$ are the fixed points for $f_r(x)$.

Now we show that whether the fixed points are attracting, repelling, neutral or not.

We note that $f_r'(x) = r(1-2x) \Rightarrow | f_r'(x) | = r(1-2x).$

(i) When $0 < r < 1$, then $|f_r'(0)| = |r| < 1$. So by definition, 0 is an attracting fixed point.

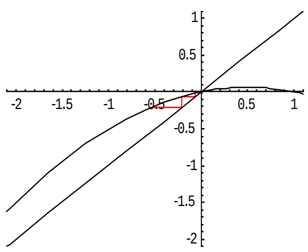
When $0 < r < 1 \Rightarrow 2 > 2 - r > 1$, then $|f_r'(\frac{r-1}{r})| = |r(1 - 2.\frac{r-1}{r})| = |2 - r| > 1$. So by definition, $\frac{r-1}{r}$ is a repelling fixed point.

(ii) When $r = 1$, then $|f_r'(0)| = |r| = 1$ and $|f_r'(\frac{r-1}{r})| = |r(1 - 2.\frac{r-1}{r})| = |2 - r| = 1$. So by definition, both 0 and $\frac{r-1}{r}$ are neutral fixed point.

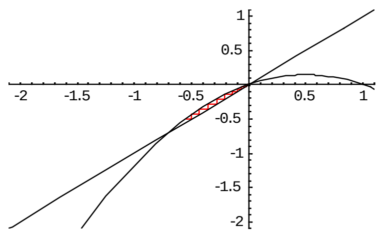
(iii) When $1 < r < 3 \Rightarrow |f_r'(0)| = |r| > 1$. So by definition, 0 is a repelling fixed point in this case and $|f_r'(\frac{r-1}{r})| = |2 - r| < 1$. So by definition, $\frac{r-1}{r}$ is a attracting fixed point.

(iv) When $r = 3 \Rightarrow |f_r'(0)| = |r| > 1$. So by definition, 0 is a repelling fixed point and $|f_r'(\frac{r-1}{r})| = |2 - r| = 1$. So $\frac{r-1}{r}$ is neutral fixed point.

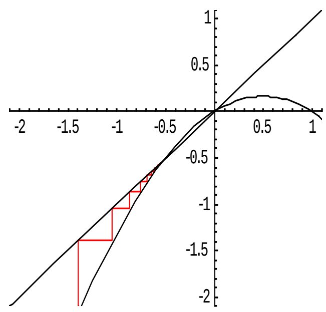
(v) When $3 < r < 4 \Rightarrow |f_r'(0)| = |r| > 1$ and $|f_r'(\frac{r-1}{r})| = |2 - r| > 1$. So both 0 and $\frac{r-1}{r}$ are repelling fixed points. Now we illustrate the above cases for some particular values of r by the “cobweb” graphical analysis:



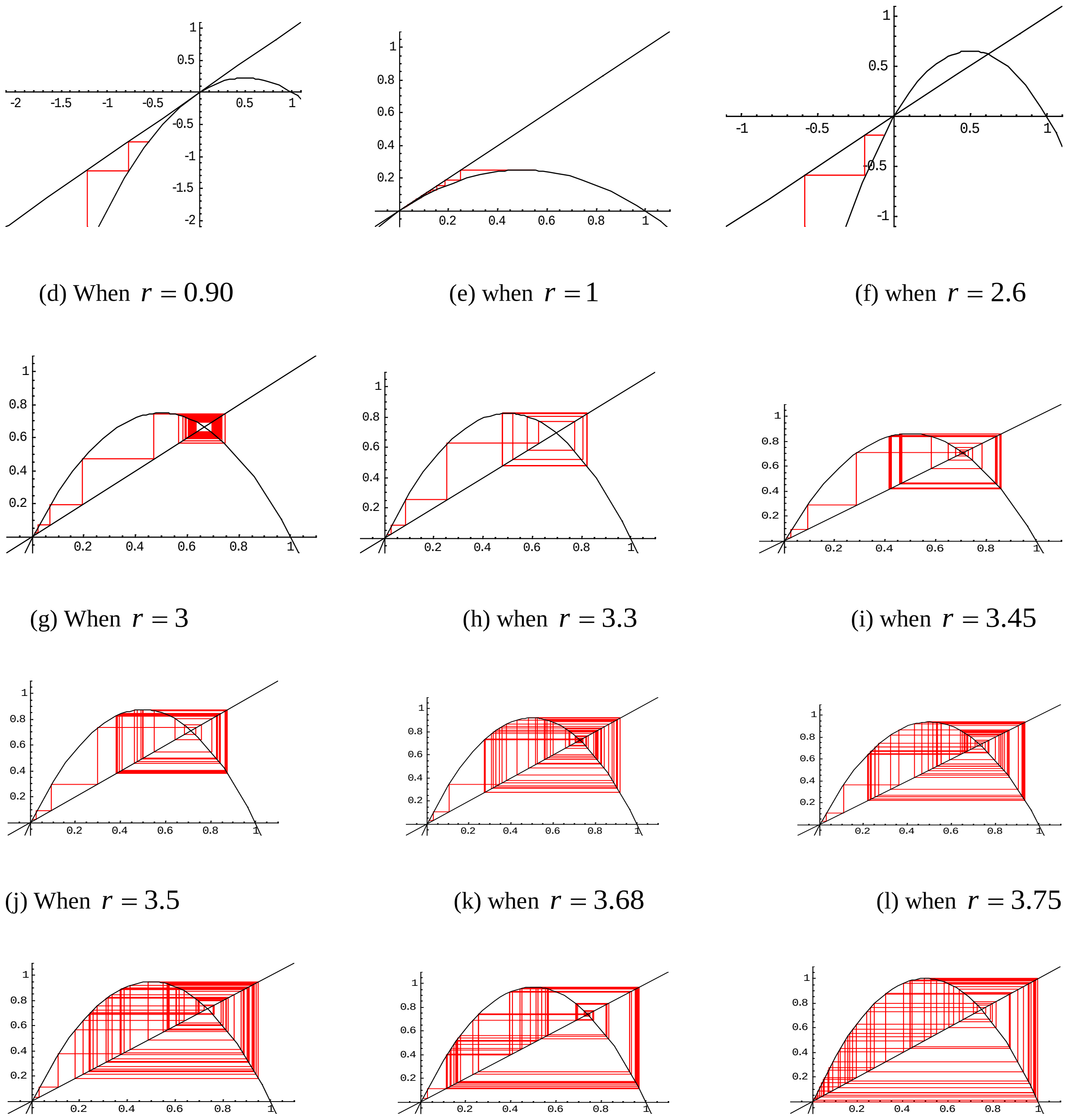
(a) When $r = 0.25$



(b) when $r = 0.58$



(c) when $r = 0.65$



(d) When $r = 0.90$ (e) when $r = 1$ (f) when $r = 2.6$

(g) When $r = 3$ (h) when $r = 3.3$ (i) when $r = 3.45$

(j) When $r = 3.5$ (k) when $r = 3.68$ (l) when $r = 3.75$

(m) When $r = 3.80$ (n) when $r = 3.88$ (o) when $r = 4$

Figure 9.2: Dynamical behavior of logistic map

(a) When $r = 0.25$, then the following "cobweb" graph shows that 0 is attracting fixed point and $\frac{r-1}{r}$ is repelling fixed point.

(b) When $r = 0.58$, then the following "cobweb" graph shows that 0 is attracting fixed point and $\frac{r-1}{r}$ is repelling fixed point.

(c) When $r = 0.65$, then the following "cobweb" graph shows that 0 is attracting fixed point and $\frac{r-1}{r}$ is repelling fixed point.

(d) When $r = 0.90$, then the following "cobweb" graph shows that 0 is attracting fixed point and $\frac{r-1}{r}$ is repelling fixed point.

(e) When $r = 1$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both neutral fixed points.

(f) When $r = 2.6$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(g) When $r = 3$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(h) When $r = 3.3$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(i) When $r = 3.45$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(j) Suppose $r = 3.5$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(k) Suppose $r = 3.68$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(l) Suppose $r = 3.75$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(m) When $r = 3.80$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(n) When $r = 3.88$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

(o) When $r = 4$, then the following "cobweb" graph shows that 0 and $\frac{r-1}{r}$ are both repelling fixed points.

11. Conclusion

We have discussed some basic concepts of logistic map. We have also discussed, fixed point, attracting fixed point, repelling fixed point, neutral fixed point and chaos easily. The logistic function is the inverse of the natural logistic function and so can be used to convert the logarithm of odds into a probability. Dynamical behavior of logistic map has been given for different parameter values . To illustrate logistic map diagram we have used MATHEMATICA program appropriately. We analyzed the attracting fixed point, repelling fixed point and neutral fixed point from the logistic map diagram. The realistic mathematical logistic model is useful in many applications from experimental instruments to rigorous mathematical analysis techniques. Finally we have noticed that the chaotic behavior occurs for most values of between about 3.57 and 4.

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