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Application of Bayesian Nonlinear Structural Equation Modeling for Exploring Relationships on Residential Satisfaction in Turkey

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Abstract

We introduce a Bayesian Nonlinear Structural Equation Modeling framework to explore the relationships on residential satisfaction in Turkey. The structural equation model (SEM) is a multivariate statistical method that allows assessment of relationships between observed and latent variables. SEM includes methods for regression, path analysis and factor analysis. SEM is widely used to examine the inter-relationships between latent and observed variables in psychological, social and medical research. Generally, linear relationships between observed and latent variables are modeled in SEM. Recent years, modeling of nonlinear relationship in SEM get attract great attention in the literature. A Bayesian approach to SEM may enable models that reflect hypotheses based on complex theory. The Bayesian approach analyses a general structural equation model that accommodates the general nonlinear terms of latent variables and covariates. In this study we make Bayesian non-linear structural equation modeling analysis for Residential Satisfaction.

Keywords: Bayesian Structural Equation Modeling; Latent Variables; Path Analysis; Residential Satisfaction.

1. Introduction

Structural equation modeling (SEM) is a extensive statistical modeling tool for analyzing multivariate data involving complex relationships between and among variables [25].

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SEM is a statistical method designed to test a conceptional or theoretical model. SEM includes confirmatory factor analysis, path analysis, path modeling, and latent growth modeling. One of the main advantages of SEM is that it can be used to examine the relationship between multiple measures and latent structures. It is also applicable to both experimental and non-experimental data, as well as cross-sectional and longitudinal data. SEM implements a confirmatory (hypothesis testing) approach to the multivariate analysis of a structural theory that predicts causal relationships between multiple variables [34].

Although SEMs are most widely used in studies involving latent variables such as quality of life, happiness or stress, they provide a comprehensive framework for covariance structure modeling. SEM's basic advantages are flexibility, latent variable modeling, dealing with error and testing models and theory. Consequently, they have become increasingly used outside of the traditional social science applications [19].

Many theories in social and behavioral sciences not only assume linear effects between variables, but also nonlinear relationships. The nonlinear effects most frequently investigated are interaction effects. These effects indicate that the relationship between a predictor and a criterion variable is weakened or strengthened dependent on the values of the second predictor variable (also called the moderator variable). An important common feature of SEM and many traditional statistical methods are based on linear models. For this reason, the frequently used assumption when using the SEM method; the relationship between observed and latent variables is linear. However, SEM is becoming increasingly popular in the modeling of the nonlinear relationship [42].

The fundamental hypothesis tested for SEM is given by:

$$\Sigma = \Sigma(\theta) \tag{1}$$

where Σ is the population covariance matrix of observed variables, θ is a vector that contains the model parameters, and $\Sigma(\theta)$ is the covariance matrix written as a function of θ. The relationship with $\Sigma(\theta)$ and Σ is the basic identification for model assessment, model estimate and model definition [12].

The iterative methods similar to the factor analysis are applied in the computational process of SEM. The most widely used software in the SEM method are AMOS [6], EQS [8,9] and LISREL [27]. In addition software are CALIS [24], LISCOMP [36], SEPATH (Statistica), Mx [38], Mplus [37], TETRAD [41] and WinBUGS (MRC Biostatistics Unit, 1989). Many Bayesian applications to SEM and factor analysis facilitated by WINBUGS package [16, 29]. We use WinBUGS program in this study. WinBUGS is statistical software for Bayesian analysis using Markov chain Monte Carlo (MCMC) methods. It is based on the BUGS (Bayesian inference Using Gibbs Sampling) project started in 1989.

When the observed variables are multivariate normal and hypothetical model is specified correctly, it can be analytically derived that different estimation methods such as maximum likelihood (ML), generalized least squares (GLS), and weighted least squares (WLS) will produce estimates that converge to the same optimum and have similar asymptotic properties [13,14]. In recent years, the Bayesian method has been applied to analyze many advanced SEMs useful for socio-psychological and practical medical research. Bayesian method is straightforward to diverge from standard assumptions often built into classical estimation methods. The Bayesian method statistical development is based on raw observations rather than the sample covariance matrix. This provides a number of advantages. For instance, the Bayesian method allows the use of genuine prior information in addition to the information that is available in observed data for producing results; it provides better statistics for goodness of fit and model comparison, and also other useful statistics such as the mean and percentiles of the posterior distribution; and it can give more reliable results for small samples [17, 31].

Nowadays Bayesian methods have been proposed by a many of authors. SEMs with mixed continuous and discrete variables [18,46,53], nonlinear SEMs [7,32,50], multilevel SEMs [2,26,31], semiparametric SEMs [46,54], SEMs with missing data [44], longitudinal SEMs [20,47,48,57], among others. Moreover, the Bayesian SEM has actually been applied to substantive real research in various disciplines: such as engineering, marketing research, genetics, ecology, diabetic studies and time series [3,5,23,26,45,56,58].

There are several considerable differences between the Bayesian and frequentist methods. Unlike the classical SEM in which the calculation algorithm is developed based on the sample covariance matrix, Bayesian SEM focuses on the use of raw observations instead of sample covariance matrix and implements some powerful tools for statistical computation [50].

The Bayes method requires that prior distributions be specified for each of the model unknowns, including the latent variables and the parameters from the measurement and structural models. Frequentist, generally assume normal distributions for the latent variables, but do not specify priors for mean or covariance parameters. Prior plays an important role since the posterior distributions on which the Bayesian inferences are based depend on the likelihood of both previous distribution and the data. Especially, specification of the prior ensures for the incorporation of substantive information about structural relationships, which may be available from previous studies or social science theory. If this information is not available, vague priors can be selected. While the sample size increases, the posterior distribution will be driven less by the prior, and frequentist and Bayesian estimates will tend to agree closely [19]. The main objective of this study we illustrate the value of the Bayesian SEM for developing a model which describes the Residential Satisfaction of an individual living in all provinces of Turkey.

2. Data and Instrument

The analysis is applied on a data set obtained from the Life Satisfaction Survey (LSS) that took place in Turkey. LSS was organized by the Turkish Statistical Institute in the year 2013. This survey is the most recent LSS covered all cities in Turkey.

LSS emerged for the first time as a module of the Household Budget Survey in 2003 in order to measure individuals' perceptions of general happiness and social values, measure their satisfaction with the main components of life and public services, and identify changes over time [52].

All private households of the Republic of Turkey have been included in Turkish citizens and foreign citizens aged 18 years and over. Institutional population (dormitories, rest homes for elderly persons, special hospitals, military barracks and recreation quarters for officers etc.) are not covered [52]. The survey is designed to give estimates in Turkey-rural-urban level in previous years. For the first time in 2013, it is designed to give estimates at the provincial level. The sample size of the survey in 2013 are calculated to estimate based on Nomenclature of Territorial Units for Statistics (NUTS) Level 3 (81 provinces) [52]. In this study, Bayesian SEM analysis has been done to the Residence Satisfaction (RS) using LSS. Bayesian SEM analysis was performed this survey to a random sample of 2124 individuals. The concept of Residential satisfaction is related to the qualities of housing and neighborhood, the accessibility of the household to different services and facilities, and economical opportunities and social networks. This concept has been an important research topic in disciplines such as geography, sociology, psychology and planning for a long time. There are two main reasons why this topic is popular. First, residential satisfaction is considered an important component of the overall quality of life of individuals. Second, subjective evaluations of individuals' homes and neighborhoods determine how they respond to the housing environment and form the basis of public action demands. If the individual is not satisfied with their current home or neighborhood, their relocation can be considered often and they can be actually moved to a different one. For this reason, information about the factors that shape residential satisfaction is critical for a better understanding of the household mobility decision process. Satisfaction with one's residential situation shows that there are no complaints and there is a high degree of agreement between real and desirable situations [28,39]. The RS which is assumed to be related to Work Life and Gain Satisfaction (WLGS), Personal Relationship Satisfaction (PRS) and Public Utility Satisfaction (PUS) could also be measured directly based on certain indicators. RS scale scores which measures two dimension: residence apartment, residence district; WLGS scale scores which measures six dimension: satisfaction of job, salary, monthly household income, social life, individual free time and duration to go to work; PRS scale scores which measures four dimension: relationships of relatives, friends, neighbors and colleagues; PUS scale scores which measures six dimension: services of medical, security, forensic, educational, Social Security Institution and transportation.

3. Linear Structural Equation Models

Structural equation modeling is a multivariate statistical analysis method used to analyze structural relationships. This method is a combination of factor analysis and multiple regression analysis. In the Confirmatory Factor Analysis (CFA) model, correlations between latent factors can be assessed by their covariance matrix; nevertheless, latent variables are never regressed on other variables. The main purpose of linear SEMs is to generalize the CFA model to assess how latent variables affect each other in various ways [30]. SEM consists of two parts, the measurement equation and the structural equation.

3.1. Measurement Equation

The measurement equations test the accuracy of the proposed measurements by evaluating the relationship between latent variables and their respective indicators. Let $\boldsymbol{y} = \left(y_1, \dots, y_p\right)^T$ be a $p \times 1$ vector of observed variables that have been selected for the analysis, and let $\omega = \left(\omega_1, \dots, \omega_q\right)^T$ be a $q \times 1$ vector of latent variables that are related to the observed variables in $\boldsymbol{y}$. The link between the observed variables and all the latent variables in ω is defined by the following measurement equation:

$$\boldsymbol{y} = \Lambda\omega + \varepsilon \tag{2}$$

where Λ is a $p \times q$ matrix of unknown factor loadings, and ε is a $p \times 1$ random vector of measurement (residual) errors. Here, ε is distributed as $N\left[0, \Psi_{\varepsilon}\right]$ where Ψ_{ε} is a diagonal matrix. Let $\omega = \left(\eta^T, \xi^T\right)^T$, where η and ξ are $q_1 \times 1$ and $q_2\left(= q - q_1\right) \times 1$ random vector which respectively contain the outcome and explanatory latent variables in ω [50].

Therefore, linear SEMs are formulated as a CFA model that provides a linear structural equation of latent factors.

3.2. Structural Equation

Structural equations provide an assessment of hypothesized relationships between latent variables, thus allowing statistical hypotheses to be tested for study. The effects of $\xi = \left(\xi_1, \dots, \xi_{q_2}\right)^T$ on $\eta = \left(\eta_1, \dots, \eta_{q_1}\right)^T$ are assesed by the following structural equation:

$$\eta = \Pi\eta + \Gamma\xi + \delta \tag{3}$$

where Π and Γ are matrices of unknown coefficients and δ is a residual vector which is distributed as $N\left[0, \delta\right]$. It is assumed that ξ is distributed as $N\left[0, \Phi\right]$ and that ε and δ are independent. Moreover, it is assumed that $\left|I - \Pi\right|$ is a positive constant which does not involve elements in Π. Eqs. (1) and (2) define the most basic linear SEM [29]. A linear SEM diagram is shown in Figure 1.

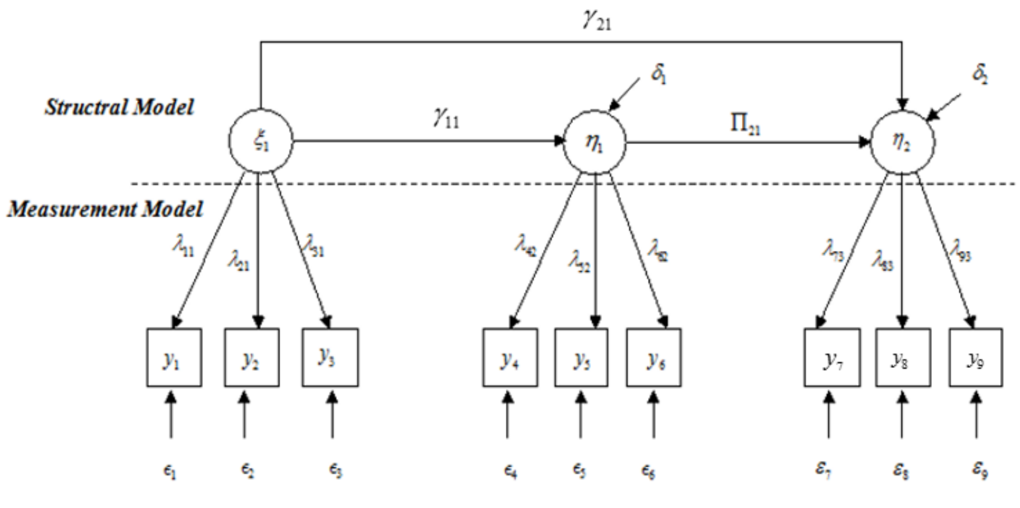

Figure 1: Path diagram associated with the linear SEM

4. Nonlinear Structural Equation Models

Let $y, \Lambda, \varepsilon, \omega, \eta$ and ξ random vectors and parameters with the same definitions as in Eqs. (2) and (3). There is no difference in the measurement equation and the structural equation is formulated as follows.

$$\eta = \Pi \eta + \Gamma \Gamma(\xi) + \delta \tag{4}$$

where Π, Γ and δ are given Eq. (3), and $F(\xi) = \left(f_1(\xi), \ldots, f_t(\xi)\right)^T$ is a $t \times 1$ vector-valued function with nonzero, known, and linearly independent differentiable functions $f_1, \ldots, f_t$ and $t \geq q_2$. Let $y = \{y_i, i = 1, \ldots, n\}$ be the observed data set with a sample size *n*, where the y_i are modeled through nonlinear SEM with variables η_i and ξ_i, and measurement errors ε_i and δ_i [49].

5. Bayesian Structural Equation Modeling Approach

In this chapter, we further explain Bayesian SEM, taking into account a standard linear SEM defined by (1) and (2). Let $Y = (y_1, \ldots, y_n)$ and $\omega = (\omega_1, \ldots, \omega_n)$, and θ let be the vector of unknown parameters in $\Lambda, \Psi_\varepsilon, \Pi, \Gamma, \Phi$ and Ψ_ε. In the posterior analysis, the observed data Y are augmented with the matrix of latent variable Ω, and consider the joint posterior distribution $[\theta, \Omega / Y]$. In order to achieve the aim, a sequence of random observations from the joint posterior distribution $[\theta, \Omega / Y]$, will be generated through the Gibbs sampler. The Gibbs sampler is a Markov chain Monte Carlo (MCMC) algorithm that generates a sequence of random observations from the full conditional posterior distribution of unknown model parameters, when direct sampling is difficult [55].

The Gibbs sampler operation starts with the initial start values set to $\left(\theta^{(0)}, \Omega^{(0)}, Y^{(0)}\right)$ and then performs the simulation for $\left(\theta^{(1)}, \Omega^{(1)}, Y^{(1)}\right)$.

At the rth iteration, by making use of the current values $\left(\theta^{(r)}, \Omega^{(r)}, Y^{(r)}\right)$, the Gibbs sampler is performed as follows:

(i) Generate $\Omega^{(r+1)}$ from the conditional distribution $\left(\Omega \setminus \theta^{(r)}, Y^{(r)}\right)$

(ii) Generate $\theta^{(r+1)}$ from the conditional distribution $\left(\theta \setminus \Omega^{(r+1)}, Y^{(r)}\right)$

(ii) Generate $\left(Y^{(r+1)}\right)$ from the conditional distribution $p\left(Y \setminus \Omega^{(r+1)}, \theta^{(r+1)}\right)$

Under mild regularity conditions, the samples are close to the desired posterior distribution. When determining the posterior distribution, the prior distribution selection should be made for $(\Lambda, \Psi_\varepsilon)$ and Φ. In this study, the prior distribution for these three parameters is taken via the following conjugate type distribution [55].

Let $\psi_{\varepsilon k}$ be the *k*th diagonal element of ψ_ε and Λ_k be the *k*th row of Λ. Then suppose that:

$$\psi_{\varepsilon k}^{-1} \sim Gamma(\alpha_{0\varepsilon k}, \beta_{0\varepsilon k}) \tag{5}$$

$$(\Lambda_k \setminus \psi_{\varepsilon k}) \sim N(\Lambda_{0k}, \psi_{\varepsilon k} H_{0yk}) \tag{6}$$

$$\Phi^{-1} \sim W_q(R_0, \rho_0) \tag{7}$$

where $Gamma(,)$ is the gamma distribution, W_q is an q dimensional Wishart distribution, parameters $\alpha_{0\varepsilon k}, \beta_{0\varepsilon k}, \Lambda_{0\varepsilon k}, \rho_0$, positive definite matrix H_{0yk} and R_0 are hyperparameters which are assumed to be described by an uninformative prior distribution [33,55].

5.1. Bayesian model comparison

Comparing the models and testing various hypotheses in the SEM is an important statistical inference. In a structural equation modeling, a classical approach to hypothesis testing is to use significance tests based on p values determined by asymptotic distributions of test statistics. Nevertheless, as pointed out in the statistical literature [10,11], the p-value of a significance test related to only the type-I error in the hypothesis test is a measure of evidence against the null model, but not a means of supporting the null model. For complex SEMs, it is often difficult to obtain asymptotic distributions of these test statistics. Furthermore, significance tests cannot be applied to test nested hypotheses or to compare nonnested models [50].

In this section, we describe two well-known model comparison statistics, Deviance Information Criterion (DIC) and Bayesian Information Criteria, which do not have the above mentioned problems [43,51].

Bayesian Information Criterion (BIC)

In this section, the Bayesian information criterion (BIC) or the Schwarz criterion (also SBC, SBIC) of the model selection criteria are illustrated. The model with the lowest BIC value is preferred.

Suppose that the observed data Y with a sample size of n appears under one of the M_1 and M_0 competing models according to the probability densities $p(Y \mid M_1)$ and $p(Y \mid M_0)$, respectively.

For $k = 0,1$. A simple approximation of $2\log B_{10}$ which does not depend on the prior density is the following Schwarz criterion S^*:

$$2\log B_{10} \cong 2S^* = 2\left[\log p\left(Y \mid \tilde{\theta}_1, M_1\right) - \log p\left(Y \mid M_0\right)\right] - \left(d_1 - d_0\right)\log n \quad (8)$$

where $\tilde{\theta}_1$ and $\tilde{\theta}_0$ are ML estimates of θ_1 and θ_0 under M_1 and M_0, respectively, d_1 and d_0 are the dimensions of θ_1, θ_0 and *n* is the sample size [43]. Minus S^* is the following well-known Bayesian information criterion (BIC) which used to compare M_1 and M_0

$$BIC_{10} = -2S^* \cong -2\log B_{10} = 2\log B_{01} \quad (9)$$

For each $M_k, k = 0,1,$ Lee (2007) defined

$$BIC_k = -2\log p\left(Y \mid \tilde{\theta}_k, M_k\right) + d_k \log n. \quad (10)$$

then $2\log B_{10} = BIC_0 - BIC_1$. Hence, the model M_k with the smaller BIC_k value is selected [29].

Akaike Information Criterion (AIC)

The Akaike information criterion used to compare the model M_k is given as follows:

$$AIC_k = -2\log p\left(Y \mid \tilde{\theta}_k, M_k\right) + 2d_k \quad (11)$$

which does not depend on the sample size *n*. The interpretation of AIC_k is similar to BIC_k. Therefore, M_k is selected if its AIC_k is smaller. When comparing Equations (10) and (11), it appears that the BIC tends to prefer simpler models than those selected by the AIC [1,29].

Deviance Information Criterion (DIC)

An alternative goodness of fit and model comparison test statistic is the deviance information criterion (DIC).

This statistic is considered as a generalization of the AIC. Under a competing model M_k with a vector of unknown parameter θ_k, the DIC is defined as

$$DIC_k = \overline{D\left(\theta_k\right)} + d_k, \quad (12)$$

where $\overline{D\left(\theta_k\right)}$ measures the goodness of fit of the model, and is defined as

$$\overline{D\left(\theta_k\right)} = E\left\{-2\log p\left(Y \mid \theta_k, M_k\right) \mid Y\right\}. \quad (13)$$

where d_k is the effective number of parameters in M_k, and is defined as

$$d_k = E_{\theta_k}\left\{-2\log p\left(Y \mid \theta_k, M_k\right) \mid Y\right\} + 2\log p\left(Y \mid \tilde{\theta}_k\right) \tag{14}$$

in which $\tilde{\theta}_k$ is the Bayesian estimate of θ_k. Let $\left\{\theta_k^{(j)}, j = 1, \ldots, J\right\}$ be a sample of observations simulated from the posterior distribution. The expectations in (13) and (14) can be estimated as follows:

$$E_{\theta_k}\left\{-2\log p\left(Y \mid \theta_k, M_k\right) \mid Y\right\} = \frac{2}{J}\sum_{j=1}^{J}\log p\left(Y \mid \theta_k^{(j)}, M_k\right). \tag{15}$$

In the application areas, a model with a smaller DIC value is selected [29,51].

Posterior Predictive p-value

The Bayes factor can be used to evaluate the goodness of fit of the hypothesized model by taking M_1 or M_0 to be the saturated model. Although, it is difficult to define a saturated model when analyzing some complex SEMs. For example, in the analysis of non-linear SEMs, the distribution associated with the hypothesized model is not normal. Therefore, a model with normal distribution with a general unstructured covariance matrix cannot be regarded as a saturated model. A simple and more convenient alternative without involving a basic saturated model is the PP *p*-values (posterior predictive p-values) [35,40].

Let $D\left(\boldsymbol{Y} \middle| \theta, \Omega\right)$ be a discrepancy measure that is used capture the discrepancy between hypothesized model M_0 and the data, and let $\boldsymbol{Y}^{rep}$ be the generated hypothetical replicate data the PP *p*-value defined

$$p_B\left(\boldsymbol{Y}\right) = P\left\{D\left(\boldsymbol{Y}^{rep} \middle| \theta, \Omega\right) \geq D\left(\boldsymbol{Y} \middle| \theta, \Omega \middle| \boldsymbol{Y}, M_0\right)\right\} \tag{16}$$

Which is the upper-tail probability of the discrepancy measure under its posterior predictive distribution. PP *p*-values not far from 0.5 indicate that realized discrepancies are near the center of the posterior predictive distribution of the discrepancy measure. Hence the hypothesized model may be considered as plausible when its PP *p*-value is reasonably close to 0.5. This approach is conceptually and computationally simple and useful for model checking of a wide variety of complex situations [15,22,50].

6. An Application of Bayesian Sem

We illustrate the Bayesian approach with WinBUGS by an analysis of Residence Satisfaction for Turkish Statistical Institute's LSS data. The analysis was performed this survey to a random sample of 2124 Turkish citizens living in the 81 provinces.

As the objective is to investigate the effects of other latent variables on Residence Satisfaction. In this study, Residence Satisfaction and Public Utility Satisfaction variables are the endogenous latent variables, η_2 and η_1 respectively; Work Life and Gain Satisfaction and Personal Relationship Satisfaction variables are the exogenous latent variables, ξ_1 and ξ_2 respectively. In order to give an illustration for model comparison procedure via BIC, AIC and DIC value, we consider the following competing models:

Model 1:

$$PUS = \gamma_1 \times WLGS + \delta_1$$

$$RS = \gamma_2 \times WLGS + \gamma_3 \times PRS + \gamma_4 \times PUS + \gamma_5 \times (WLGS)^2 + \gamma_6 \times (PRS)^2 + \gamma_7 \times (WLGS \times PRS) + \delta_2 \tag{17}$$

$$Var(\delta_1) = \Psi_{\delta_1}, Var(\delta_2) = \Psi_{\delta_2}, Cov(WLGS, PRS) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$$

Model 2:

$$\begin{aligned} &PUS = \gamma_1 \times WLGS + \delta_1 \\ &RS = \gamma_2 \times WLGS + \gamma_3 \times PRS + \gamma_4 \times PUS + \gamma_5 \times (WLGS \times PRS) + \delta_2 \\ &Var(\delta_1) = \Psi_{\delta_1}, Var(\delta_2) = \Psi_{\delta_2}, Cov(WLGS, PRS) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \end{aligned} \tag{18}$$

Model 3:

$$\begin{aligned} &PUS = \gamma_1 \times WLGS + \delta_1 \\ &RS = \gamma_2 \times WLGS + \gamma_3 \times PRS + \gamma_4 \times PUS + \gamma_5 \times (PRS)^2 + \delta_2 \\ &Var(\delta_1) = \Psi_{\delta_1}, Var(\delta_2) = \Psi_{\delta_2}, Cov(WLGS, PRS) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \end{aligned} \tag{19}$$

Model 4:

$$\begin{aligned} &PUS = \gamma_1 \times WLGS + \delta_1 \\ &RS = \gamma_2 \times WLGS + \gamma_3 \times PRS + \gamma_4 \times PUS + \gamma_5 \times (WLGS)^2 + \delta_2 \\ &Var(\delta_1) = \Psi_{\delta_1}, Var(\delta_2) = \Psi_{\delta_2}, Cov(WLGS, PRS) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \end{aligned} \tag{20}$$

Model 5:

$$\begin{aligned} &PUS = \gamma_1 \times WLGS + \delta_1 \\ &RS = \gamma_2 \times WLGS + \gamma_3 \times PRS + \gamma_4 \times PUS + \delta_2 \\ &Var(\delta_1) = \Psi_{\delta_1}, Var(\delta_2) = \Psi_{\delta_2}, Cov(WLGS, PRS) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \end{aligned} \tag{21}$$

Applying this to five models, we estimate the four models by Bayesian SEM method. In the solution process, we used WinBUGS program. We show the BIC, AIC and DIC value of four models in Table 1.

Table 1: Comparison of BIC, DIC and PP value for four models

MODEL	DIC	BIC	AIC	PP p-value
Model 1	74691.2	62986.670	62161.7	0.4555
Model 2	74715.4	62948.827	62149.5	0.4546
Model 3	74686.8	62941.327	62142.8	0.4565
Model 4	74710.1	62959.827	62160.5	0.4547
Model 5	74722.8	62944.205	62158.2	0.4545

We observe from Table 1 that Model 3 has a lowest BIC, AIC and DIC value in our models; there is enough evidence to consider Model 3 is the best model in our models. Moreover, Model 3's PP *p*-value estimate is not far 0.5 (0.4565), so it shows good fit.

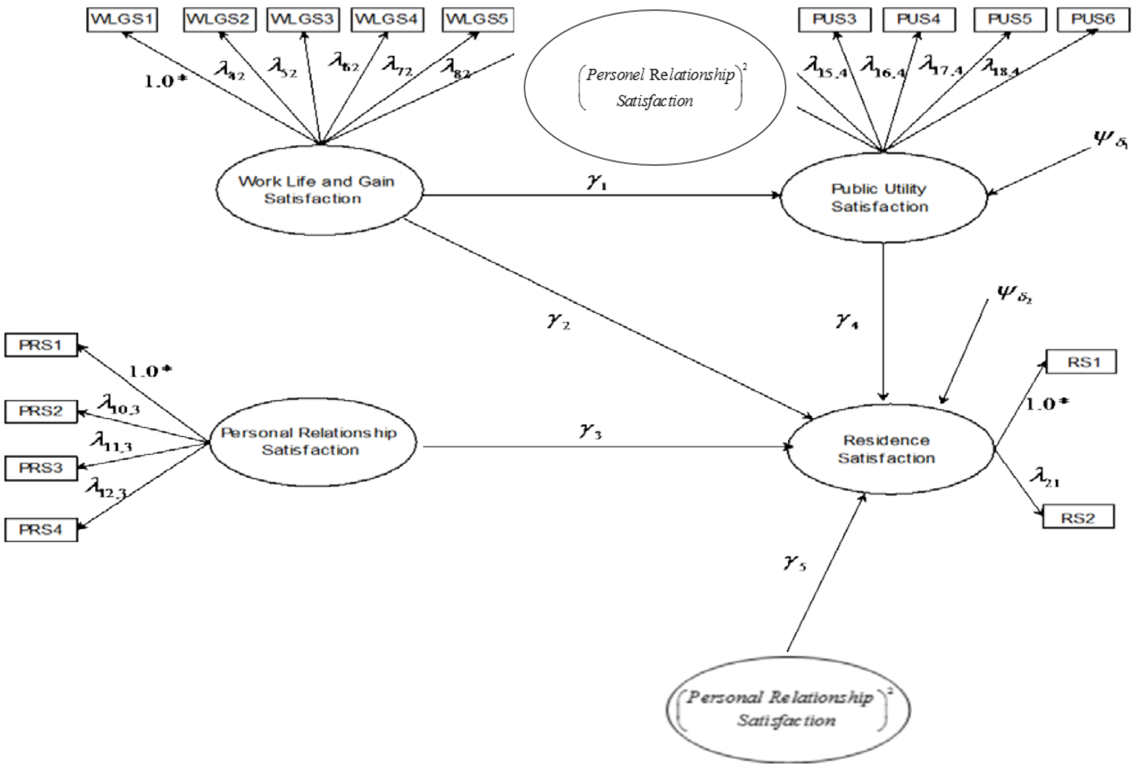

Figure 2: The path diagram of the proposed Model 3 in analyzing the LSS data

Since the best model is Model 3 among the five models in this study, we give only this model for our results. Bayesian estimates of the unknown parameters and standard error estimates are obtained. Estimation results and path diagram are given in Table 2 and Figure 3, respectively. It is observed that for Turkish citizens, Work Life and Gain Satisfaction has impact on Public Utility Satisfaction; Work Life and Gain Satisfaction, Public Utility

Satisfaction and Personal Relationship Satisfaction have significant impact on Residence Satisfaction. Also quadratic effect of Personal Relationship Satisfaction $\left(\xi_1^2\right)$, has significant impact on Residence Satisfaction.

Table 2: Estimated values for the parameters in Model 3

Node	**Mean**	**Sstandard deviation**	**MC error**	**2.50%**	**97.50%**	**Node**	**Mean**	**Sstandard deviation**	**MC error**	**2.50%**	**97.50%**
μ_1	2.28	0.0184	0.0006	2.245	2.317	$\lambda_{17,4}$	0.9172	0.0400	0.0021	0.8415	0.997
μ_2	2.202	0.0168	0.0006	2.17	2.235	$\lambda_{18,4}$	0.8183	0.0420	0.0019	0.7399	0.9053
μ_3	2.273	0.0174	0.0008	2.239	2.307	γ_1	0.417	0.0376	0.0017	0.3462	0.4948
μ_4	2.886	0.0226	0.0017	2.841	2.929	γ_2	0.2535	0.0367	0.0014	0.1821	0.3252
μ_5	2.875	0.0223	0.0017	2.831	2.918	γ_3	0.350	0.0394	0.0020	0.2752	0.4288
μ_6	2.465	0.0199	0.0007	2.426	2.503	γ_4	0.1969	0.0302	0.0012	0.1388	0.2558
μ_7	2.825	0.0216	0.0009	2.783	2.868	γ_5	-0.132	0.0339	0.0015	-0.198	-0.064
μ_8	2.792	0.0217	0.0009	2.749	2.835	$\Psi_{\varepsilon 1}$	0.3617	0.0205	0.0011	0.3226	0.4021
μ_9	2.139	0.0142	0.0006	2.111	2.168	$\Psi_{\varepsilon 2}$	0.268	0.0168	0.0010	0.2348	0.3012
μ_{10}	2.01	0.0112	0.0005	1.989	2.032	$\Psi_{\varepsilon 3}$	0.4499	0.0144	0.0002	0.4228	0.4798
μ_{11}	2.096	0.013	0.0006	2.069	2.123	$\Psi_{\varepsilon 4}$	0.2569	0.0132	0.0006	0.231	0.2837
μ_{12}	2.05	0.0114	0.0003	2.028	2.073	$\Psi_{\varepsilon 5}$	0.2553	0.0127	0.0006	0.2311	0.2811
μ_{13}	2.381	0.0194	0.0006	2.344	2.42	$\Psi_{\varepsilon 6}$	0.701	0.0222	0.0003	0.6571	0.7441
μ_{14}	2.283	0.0169	0.0005	2.25	2.317	$\Psi_{\varepsilon 7}$	0.7262	0.0237	0.0005	0.6813	0.774
μ_{15}	2.507	0.0170	0.0005	2.473	2.54	$\Psi_{\varepsilon 8}$	0.7662	0.0248	0.0005	0.7192	0.8158
μ_{16}	2.501	0.0196	0.0006	2.462	2.541	$\Psi_{\varepsilon 9}$	0.2241	0.0089	0.0003	0.207	0.242
μ_{17}	2.35	0.0168	0.0006	2.317	2.384	$\Psi_{\varepsilon 10}$	0.1109	0.0053	0.0002	0.1007	0.1216
μ_{18}	2.36	0.0181	0.0006	2.323	2.395	$\Psi_{\varepsilon 11}$	0.1787	0.0080	0.0003	0.1635	0.1943
λ_{21}	1.036	0.0699	0.0051	0.9098	1.181	$\Psi_{\varepsilon 12}$	0.2171	0.0071	0.0001	0.2033	0.2313
λ_{42}	2.159	0.0833	0.0082	2.013	2.328	$\Psi_{\varepsilon 13}$	0.5113	0.0183	0.0004	0.4764	0.5479
λ_{52}	2.07	0.0793	0.0079	1.928	2.238	$\Psi_{\varepsilon 14}$	0.3591	0.0138	0.0003	0.3323	0.3873
λ_{62}	0.822	0.0540	0.0030	0.7187	0.9265	$\Psi_{\varepsilon 15}$	0.4093	0.0145	0.0003	0.3814	0.4385
λ_{72}	1.266	0.0649	0.0047	1.144	1.400	$\Psi_{\varepsilon 16}$	0.483	0.0185	0.0004	0.4478	0.5201
λ_{82}	1.066	0.0611	0.0039	0.9507	1.186	$\Psi_{\varepsilon 17}$	0.3409	0.0129	0.0002	0.3161	0.3672
$\lambda_{10,3}$	0.8912	0.0317	0.0023	0.8311	0.9538	$\Psi_{\varepsilon 18}$	0.5072	0.0176	0.0003	0.4737	0.5423
$\lambda_{11,3}$	1.057	0.0373	0.0026	0.9877	1.13	ψ_{δ_1}	0.2793	0.0196	0.0012	0.2422	0.3204
$\lambda_{12,3}$	0.5373	0.0301	0.0015	0.4787	0.5971	ψ_{δ_2}	0.1758	0.0147	0.0009	0.1473	0.206
$\lambda_{14,4}$	0.9258	0.0409	0.0022	0.8465	1.01	Φ_{11}	0.1782	0.0136	0.0013	0.1528	0.2049
$\lambda_{15,4}$	0.8075	0.039	0.0020	0.7323	0.8883	Φ_{12}	0.0356	0.0052	0.0002	0.0256	0.0463
$\lambda_{16,4}$	1.075	0.0462	0.0025	0.9857	1.168	Φ_{22}	0.1993	0.0121	0.0009	0.1765	0.2235

Based on Table 2, we could show the structural model diagram of Model 3 in Figure 3. We could also present this model's structural equation models in the following equation:

$$PUS = 0.417 \times WLGS$$
$$RS = 0.253WLGS + 0.350PRS + 0.196 \times PUS - 0.132(PRS)^2 \quad (22)$$

This equation indicate that the greatest effect to the Residence Satisfaction is Personal Relationship Satisfaction then followed by Work Life and Gain Satisfaction, Public Utility Satisfaction and Public Utility Satisfaction quadratic effect. All of the latent variables give the significant effect to the Residence Satisfaction. Furthermore Work Life and Gain Satisfaction has significant impact on Public Utility Satisfaction.

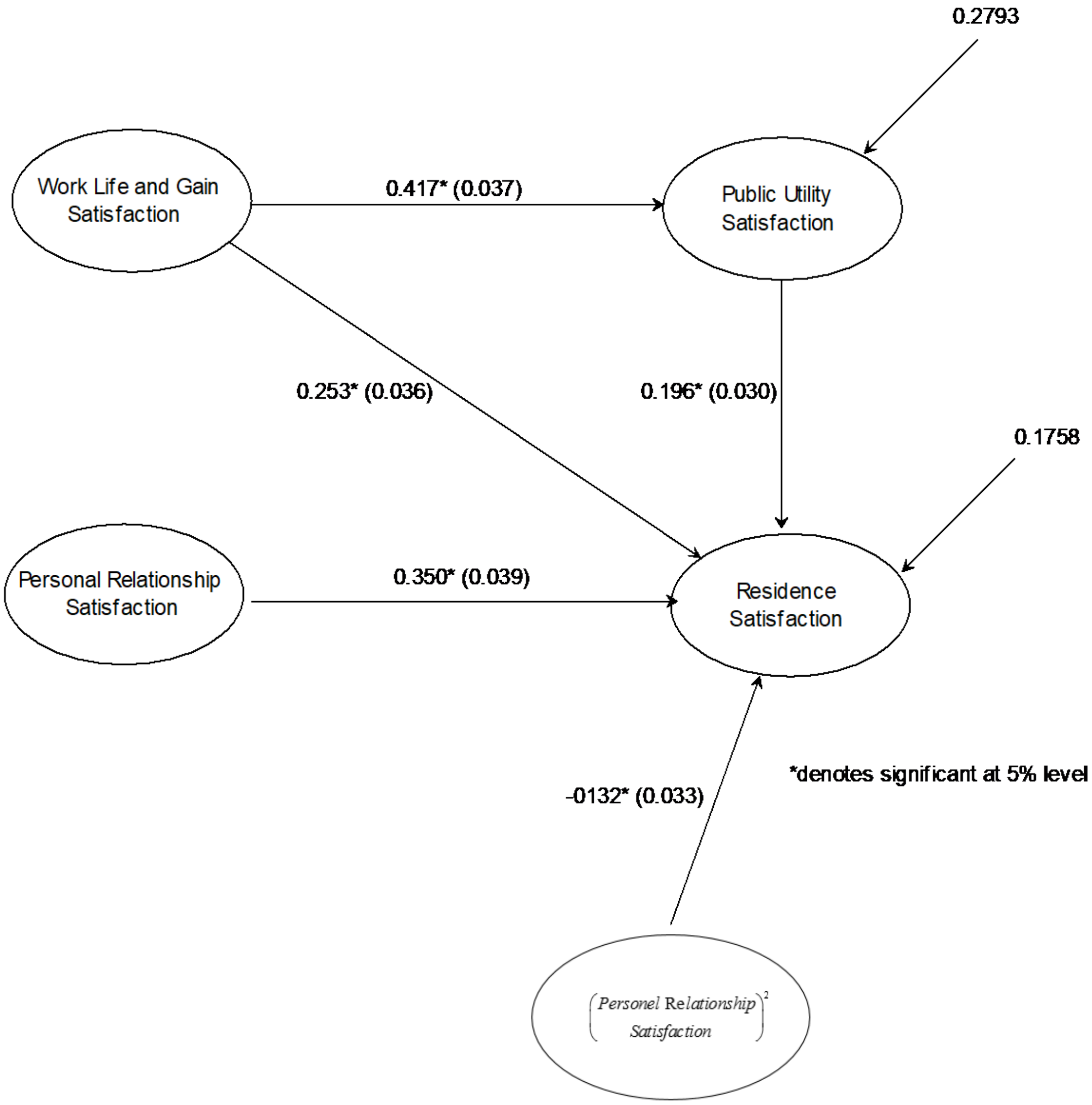

Figure 3: The path diagram and estimated results of the proposed model 3 in analyzing the LSS data

7. Discussion

SEMs are widely used to define the relationship between observed and latent variables in modern behavioral, social, and biomedical sciences. In the standard SEM, the computational algorithm is improved based on the sample covariance matrix and normal assumptions for observations. However, in many studies it is often seen that the data collected in a survey do not conform to the assumption of multivariate normality. Bayesian method in SEM is believed to be potential tools to overcome the normal assumption [4,33,41]. The basic objective of this study is to present Nonlinear Bayesian Structural Equation which applied Residence Satisfaction. In contrast to maximum likelihood method, in Bayesian method, parameters are considered as random with prior distribution and a prior density function [29]. There are many advantages of estimating multiple nonlinear effects. One of the advantages refers to the development of behavioral theories involving interaction and quadratic effects [21]. The methodology of Bayesian SEM then applied to a real data set. Linear and nonlinear five models are solved by Bayesian structural equation model. We compared these five models based on the BIC, AIC and DIC values of the comparison criteria and we determine the best model.

8. Conclusion

The main purpose of this paper is to demonstrate the Bayesian method for analyzing nonlinear SEM. The methodology of Bayesian SEM applied to a real data set obtained from Turkish Statistical Institute. The Bayesian approach allows researchers to apply nonlinear structural equation modeling and has various advantages. For instance, the first moment properties of raw observations are simpler than the second moment properties of the sample covariance matrix. Therefore the Bayesian approach can overcome more complex situations and provides a direct estimate of latent variables. This approach can directly model raw observations and latent variables through the familiar regression model. Hence it provides more direct interpretation [49]. In this study, we contribute to the actual knowledge by identifying significant relationships between Residence Satisfaction and Public Utility Satisfaction, Personal Relationship Satisfaction, Work Life & Gain Satisfaction using a SEM framework and determine the best model among the five different potential models according to BIC, AIC and DIC comparison criteria value. For this, we apply the Bayesian SEM using WINBUGS version 1.4.3.

9. Recommendation

In the future works, the proposed Bayesian SEM model in the study may be performed to different nonlinear SEM approaches such as Product Indicator (PI), Latent Moderated Structural Equations (LMS), Quasi-Maximum Likelihood (QML) and Extended Unconstrained (ExUC). Furthermore, in the future, the model obtained in this study can be re-analyzed using the new up-to-date LSS data set and the two models can be compared. In this regard, it can be tested whether the preferences of the individuals have changed or not in terms of non-linear SEM modeling on the residential satisfaction.

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