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Nonsingular Big Bang and Black Holes

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Abstract

Penrose–Hawking singularity theorems pre-assume asymptote flat spacetime. This false flat spacetime paradigm produces such singularities at the Big Bang and the center of the Black Holes. The Universe is globally hyperbolic as we did prove mathematically in [3]. We prove that the hyperbolic time evolution equation of the universe $R(t)=\sqrt{3/8\pi\rho_j}\sinh t\sqrt{8\pi\rho_j/3}$ characteristics the hyperbolic universe and traces its manifold dynamical geometry wouldn`t break down at the initial Big Bang moment. The Schwarzschild solution is the simplest possible black hole. In Einstein's theory, gravity is described through the curvature of spacetime, and in the center of the black hole, the curvature goes to infinity. That infamous singularity indicates a breakdown of physics. We modify both Schwarzschild metric and Kerr metric in the hyperbolic spacetime, where they possess nonsingularity. Hence Black Holes are nonsingular.

Keywords: Nonsingular; Big Bang; Black Holes; Schwarzschild metric; Kerr metric.

1. Introduction

An initial-value problem: Given the state of a system at some moment in time, what will be the state at some later time? Future events can be understood as consequences of initial conditions plus the laws of physics. Could the dynamical nature of the spacetime background break down an initial-value formulation in general relativity?

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In general relativity, a singularity is a place that objects or light rays can reach in a finite time where the *curvature becomes infinite, or spacetime stops being a manifold*. Singularities can be found in all cosmological solutions which don't have scalar field energy or a cosmological constant.Curvature is associated with gravity and hence curvature singularities correspond to "infinitely strong gravity." There are several possibilities of how such infinitely strong gravity can manifest itself. For instance, it could be that the energy density becomes infinitely large - this is called a "Ricci singularity", As an example of a Ricci singularity, the evolution of energy density in a universe described by a big bang model. As you go towards the left - corresponding to earlier and earlier instances of cosmic time zero - the density grows beyond all bounds and at cosmic time zero - at the big bang - it was infinitely high. A ***path*** [1] in spacetime is a ***continuous*** chain of events through space and time. While there are competing definitions of spacetime singularities, the most central, and widely accepted, criterion rests on the possibility that some spacetimes contain incomplete paths. Indeed, the rival definitions (in terms of missing points or curvature pathology) still make use of the notion of path incompleteness.While path incompleteness seems to capture an important aspect of the intuitive picture of singular structure, it completely ignores another seemingly integral aspect of it: curvature pathology. If there are incomplete paths in a spacetime, it seems that there should be a *reason* that the path cannot go farther. The most obvious candidate explanation of this sort is something going wrong with the dynamical structure of the spacetime, which is to say, with the curvature of the spacetime. This suggestion is bolstered by the fact that local measures of curvature do in fact blow up as one approaches the singularity of a standard black hole or the big bang singularity. However, there is one problem with this line of thought: no species of curvature pathology we know how to define is either necessary or sufficient for the existence of incomplete paths. At the heart of all of our conceptions of a spacetime singularity is the notion of some sort of failing: a path that disappears, points that are torn out, spacetime curvature that becomes pathological. However, *perhaps the failing lies not in the spacetime of the actual world, but rather in the* theoretical description *of the spacetime*. That is, perhaps we shouldn't think that general relativity is accurately describing the world when it posits singular structure! Indeed, in most scientific arenas, singular behavior is viewed as an indication that the theory being used is deficient. It is therefore common to claim that general relativity, in predicting that spacetime is singular, is predicting its own demise, and that classical descriptions of space and time break down at black hole singularities and at the Big Bang. Such a view seems to deny that singularities are real features of the actual world, and to assert that they are instead merely artifices of our current (flawed) physical theories. Many physicists and philosophers resist that singularities are *real*. Some argue that singularities are too repugnant to be real. Others argue that the singular behavior at the center of black holes and at the beginning of time points to the limit of the domain of applicability of general relativity. **Note** that the hyperbolic universe inflates exponentially produces an accelerated expansion of the universe without cosmological constant or scalar field. We have shown in [2] that general relativity doesn't break down at large cosmological scale since it predicts both the accelerated expansion of the universe (without invoking dark energy) and predicts the galaxy flat rotation curve (without invoking dark matter). General relativity didn't break down at Planck scale as we had shown in [3].In this research we shall prove that the time evolution equation of the universe characteristics the hyperbolic universe and traces its manifold dynamical geometry shouldn`t break down even at the initial Big Bang moment. Our task is to remove the singularity from the mathematical model, represented by the General Relativity Theory and the hyperbolic spacetime, underlying the Big Bang Theory and Black Holes. Our main point is to examine whether the state point:

$$\left(R_{B.B},\rho_{B.B},t_{B.B}\right)=\left(0,\infty,0\right)$$

constitutes a singular point in the manifold? Is it really a missing point of the manifold? Does the local measure of curvature blow up as one approach this point? Does the density grow beyond all bounds, infinitely high as one approach this point?

2. The Hyperbolic Spacetime

To derive the dynamical equation of cosmology, we should combine Einstein field equations

$$R\mu\nu-\frac{1}{2}g_{\mu\nu}\Re=8\pi GT_{\mu\nu}$$

with the isotropic homogeneous Robertson- Walker's line-element:

$$ds^2=dt^2-R^2(t)\left(\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right)$$

Where $x^0=t,x^1=r,x^2=\theta,x^3=\phi$

The corresponding components of the metric tensor are:

$$g_{00}=1=g^{00},g_{11}=-\frac{R^2}{1-kr^2}=(g^{11})^{-1}$$
$$g_{22}=-R^2r^2=(g^{22})^{-1}$$
$$g_{33}=\sin^2\theta g_{22}=(g_{33})^{-1}$$

Now according to the affine connection:

$$\Gamma^{\lambda}_{\mu\nu}=\frac{1}{2}g^{\lambda\sigma}\left(\frac{\partial g_{\alpha\nu}}{\partial x^{\mu}}+\frac{\partial g_{\sigma\mu}}{\partial x^{\nu}}-\frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right)$$

We compute:

$$\Gamma^3_{03} = \frac{\dot{R}}{R}, \Gamma^0_{11} = \frac{R\dot{R}}{1-kr^2}$$
$$\Gamma^0_{22} = R\dot{R}r^2, \Gamma^0_{33} = R\dot{R}r^2 \sin^2\theta$$
$$\Gamma^1_{11} = \frac{kr}{1-kr^2}, \Gamma^1_{22} = -r(1-kr^2), \Gamma^1_{33} = \sin^2\theta.\Gamma^1_{22}$$
$$\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r}, \Gamma^2_{33} = -\frac{1}{2}\sin 2\theta, \Gamma^3_{23} = \cot\theta$$

All other components of Γ either vanish or follow from the symmetry

$$\Gamma^\lambda_{\nu\mu} = \Gamma^\lambda_{\mu\nu}$$

A dot denotes differentiation with respect to time. Latin indices run over the values 1, 2 and 3. Note the Ricci tensor:

$$R_{\mu\nu} = \frac{\partial\Gamma^\lambda_{\mu\nu}}{\partial x^\lambda} - \frac{\partial\Gamma^\lambda_{\mu\lambda}}{\partial x^\nu} + \Gamma^\sigma_{\mu\nu}\Gamma^\lambda_{\sigma\lambda} - \Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\sigma\nu}$$

We calculate the nonzero components of the Ricci tensor, the non-vanishing components of which are easily found to be:

$$R^i_j = -\frac{1}{R^2}(R\ddot{R} + 2\dot{R}^2 + 2k)\delta^i_j$$
$$R^0_0 = -3\frac{\ddot{R}}{R}$$

$$R_{11} = \frac{R\ddot{R} + 2\dot{R}^2 + 2k}{1-kr^2}$$
$$R_{22} = r^2(R\ddot{R} + 2\dot{R}^2 + 2k)$$
$$R_{33} = r^2(R\ddot{R} + 2\dot{R}^2 + 2k)\sin^2\theta$$

And the Ricci scalar is then

$$\Re = g^{\mu\nu} R_{\mu\nu} = g^{00} R_{00}$$
$$+ g^{11} R_{11} + g^{22} R_{22} + g^{33} R_{33}$$
$$\Re = 6\left(\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2}\right)$$

Turning our attention now to the cosmological fluid, we assume that it is described by an ideal fluid. The fluid will be at rest in co-moving coordinates. The four-velocity is then $U^{\mu} = (1,0,0,0)$.

According to the energy momentum tensor

$$T_{\mu\nu} = -p g_{\mu\nu} + (p+\rho) U_{\mu} U_{\nu}, \qquad g_{\mu\nu} U^{\mu} U^{\nu} = 1$$

Where p is the pressure and ρ is the energy density of the cosmological fluid. The energy-momentum tensor becomes:

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & g_{11} p & 0 & 0 \\ 0 & 0 & g_{22} p & 0 \\ 0 & 0 & 0 & g_{33} p \end{pmatrix}$$
$$T^{\mu}_{\nu} = diag(-\rho, p, p, p)$$
$$T = T^{\mu}_{\mu} = -\rho + 3p$$

Consider the zero component of the conservation of energy equation:

$$0 = \nabla_{\mu} T^{\mu}_{0} =$$
$$= \partial_{\mu} T^{\mu}_{0} + \Gamma^{\mu}_{\mu\lambda} T^{\lambda}_{0} - \Gamma^{\lambda}_{\mu 0} T^{\mu}_{\lambda}$$
$$= -\partial_0 \rho - 3\frac{\dot{R}}{R}(\rho + p)$$
$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + p) = 0$$

Rewrite Einstein field equations

$$R\mu\nu - \frac{1}{2} g_{\mu\nu}\Re = 8\pi G T_{\mu\nu}$$

The $\mu\nu = 00$ equation is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\Re = 8\pi G T_{\mu\nu}$$
$$R_{00} - \frac{1}{2} g_{00}\Re = 8\pi G T_{00}$$

$$-3\frac{\ddot{R}}{R} + \frac{1}{2} \times 6\left(\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2}\right) = 8\pi G\rho$$
$$\dot{R}^2 + k = \frac{8\pi}{3} G\rho R^2$$

There is only one distinct equation from $\mu\nu = ij$, due to isotropy

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\Re = 8\pi G T_{\mu\nu}$$
$$R_{11} - \frac{1}{2} g_{11}\Re = 8\pi G T_{11}$$
$$r^2\left(R\ddot{R} + 2\dot{R}^2 + 2k\right) - \frac{1}{2} R^2 r^2 \times$$
$$6\left(\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2}\right) = 8\pi G p R^2 r^2$$
$$\therefore -2R\ddot{R}r^2 - \dot{R}^2 r^2 - r^2 k = 8\pi G p R^2 r^2$$
$$\therefore 2R\ddot{R} + \dot{R}^2 + k = -8\pi G p R^2$$

We get the following two dynamical equations of cosmology (Friedmann`s equations):

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$$\dot{R}^2 + k = (8\pi/3)\rho R^2 \qquad (1)$$

$$2R\ddot{R} + \dot{R}^2 + k = -8\pi p \qquad (2)$$

Where p is the pressure and ρ is the energy density of the cosmological fluid and k is the curvature. Now we shall solve the differential equation (1) by separating the variables. We assume the Big Bang Model as an initial condition (i.e. R=0 when t=0). .

$$\dot{R}^2 + k = (8\pi/3)\rho R^2$$

$$\dot{R}^2 = (8\pi/3)\rho R^2 - k$$

$$\dot{R} = \sqrt{(8\pi\rho/3)R^2 - k}$$

$$dR/\sqrt{(8\pi\rho/3)R^2 - k} = dt$$

$$dR/\sqrt{R^2 - 3k/8\pi\rho} = \sqrt{8\pi\rho/3}\,dt$$

Differential equation (1) allows one to deal with ρ_j as a parameter since it's not an explicit function of t , so Eq. (1) can be solved for any chosen fixed value, ρ_j from the stream of the various values of the parameter ρ :

$$\rho_1, \rho_2, \ldots, \rho_{planck}, \ldots, \rho_j, \ldots, \rho_{now}$$

By means of the mean value theorem, we assume approximately that ρ_j evolves to the fixed physical value ρ_j exactly simultaneously associated to the state (t_j, R_j) since ρ_j is not defined and not continuous at the point of singularity t = 0, put

$$\rho(c) = \rho_j \ :0<c \le t$$

$$\int_0^R dr/\sqrt{r^2 - 3k/8\pi\rho_j} = \sqrt{8\pi\rho_j/3}\int_0^t d\tau$$

$$\cosh^{-1}(R/\sqrt{3k/8\pi\rho_j}) - \cosh^{-1}0 = t\sqrt{8\pi\rho_j/3} \qquad (3)$$

Now we use complex analysis as follows:

$$\cosh^{-1} x = \ln(x \pm \sqrt{x^2 - 1})$$
$$\cosh^{-1} 0 = \ln \pm \sqrt{-1} = \ln \sqrt{-1}, or, \cosh^{-1} 0 = \ln - \sqrt{-1}$$
$$\cosh^{-1} 0 = (1/2)\ln(-1), or, \cosh^{-1} 0 = \ln(-1) + (1/2)\ln(-1)$$
$$\because e^{i\pi} = -1$$
$$\therefore \ln(-1) = i\pi$$
$$\therefore \cosh^{-1} 0 = i\pi/2, or$$
$$\cosh^{-1} 0 = 3i\pi/2$$

Substitute the first value $\cosh^{-1} 0 = i\pi/2$ in equation (3), we get:

$$R(t) = \sqrt{3k/8\pi\rho_j}.\cosh(t\sqrt{8\pi\rho_j/3} + \pi i/2)$$
$$R(t) = \sqrt{3k/8\pi\rho_j}.(\cosh t\sqrt{8\pi\rho_j/3}.\cosh(\pi i/2) + \sinh(\pi i/2).\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t) = \sqrt{3k/8\pi\rho_j}.(\cosh t\sqrt{8\pi\rho_j/3}.\cos(\pi/2) + i\sin(\pi/2).\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t) = i\sqrt{3k/8\pi\rho_j}.\sinh t\sqrt{8\pi\rho_j/3}$$

Since the function $\rho(t)$ is always positive, so is any chosen fixed value ρ_j. A simple analysis shows that the R(t) scale solution represented in the last equation is complex if k is positive, negative if k is negative and vanishes if k is zero. So the first value $\cosh^{-1} 0 = i\pi/2$ is rejected. Substitute the other value $\cosh^{-1} 0 = 3i\pi/2$ in equation (3), we get:

$$R(t) = \sqrt{3k/8\pi\rho_j}.\cosh(t\sqrt{8\pi\rho_j/3} + 3\pi i/2)$$
$$R(t) = \sqrt{3k/8\pi\rho_j}(\cosh t\sqrt{8\pi\rho_j/3}.\cosh(3\pi i/2) + \sinh(3\pi i/2)\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t) = \sqrt{3k/8\pi\rho_j}(\cosh t\sqrt{8\pi\rho_j/3}.\cos(3\pi/2) + i\sin(3\pi/2)\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t) = -i\sqrt{3k/8\pi\rho_j}\sinh t\sqrt{8\pi\rho_j/3}$$

The R(t) scale solution in the last equation is real, positive and non-vanishing if and only if k is negative. Since k is normalized, substitute k=-1, in the last equation, we get: .

$$R(t) = -i\sqrt{3k/8\pi\rho_{\mathrm{j}}}\,\sinh t\sqrt{8\pi\rho_{\mathrm{j}}/3}$$
$$R(t) = -i\sqrt{-3/8\pi\rho_{\mathrm{j}}}\,\sinh t\sqrt{8\pi\rho_{\mathrm{j}}/3}$$
$$R(t) = -i.i\sqrt{3/8\pi\rho_{\mathrm{j}}}\,\sinh t\sqrt{8\pi\rho_{\mathrm{j}}/3}$$
$$R(t) = \sqrt{3/8\pi\rho_{\mathrm{j}}}\,\sinh t\sqrt{8\pi\rho_{\mathrm{j}}/3} \qquad (4)$$

Which mean that $R(t)$ either vanishes if $k = 0$ or complex if $k = 1$. Thus, the curvature k must be negative and consequently the universe must be hyperbolic and open. *Note* that the solution represented by Eq. (4) is evaluated only for the values simultaneously associated with ρ_j, namely (R_j, t_j)

$$R_j = \sqrt{3/8\pi\rho_j}\,\sinh t_j\sqrt{8\pi\rho_j/3} \qquad (5)$$

Our hyperbolic universe is a manifold weaved by the time evolution equation of the universe since the Big Bang

$$R_j = \sqrt{3/8\pi\rho_j}\,\sinh\left[t_j\sqrt{8\pi\rho_j/3}\right]$$

Which reflects the structure of the manifold whether it possesses a singular point or not?

3. Verification of the time evolution equation of the universe

(i) Planck scale**[4]**

It is well known that the time evolution equation of the universe successfully predicts the Planck length at micro-cosmos scale as well as it predicts the current observed large structure at macro-cosmos scale.

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1 sec = 2.997×10^{10}cm

Planck length= $L_p = \sqrt{Gh/c^3} = 1.6\times10^{-33}\,cm$

Planck time = $t_p = L_p/c = \sqrt{Gh/c^5} = 5.4\times10^{-44}\,s$

Planck density= =3.8789×10^{62}cm

Substitute the above data in the time evolution equation of the universe at Planck scale

$$R_p = \sqrt{3/8\pi\rho_p}\,\sinh\sqrt{8\pi\rho_p/3}\,t_p$$

$$R_p = \sqrt{3/8\pi\times 3.8789\times 10^{62}}\,\sinh\sqrt{8\pi\times 3.8789\times 10^{62}/3}\times 5.4\times 10^{-44}\times 2.997\times 10^{10} =$$

$0.175423 \times 10^{-31} \times \sinh 0.092255888$

$= 0.175423\times10^{-31} \times 0.092386811$

$= 1.62 \times 10^{-33}$ cm = L_p = Planck length. Hence

$$R_p = \sqrt{3/8\pi\rho_p}\,\sinh\sqrt{8\pi\rho_p/3}\,t_p = L_p = \sqrt{Gh/c^3}$$

(ii) current scale

The energy density now $\rho_{now}=10^{-31}$ g/cm^3 $=7.425\times10^{-60}$cm^{-2}

The age of the Universe (approximately) $t_{now}=13.7\times10^9$ yr$=1.2974585\times10^{28}\,cm$

Substitute the above data in the hyperbolic time evolution equation of the Universe, yields

$$R_j = \sqrt{3/8\pi\rho_j}\,\sinh\left[t_j\sqrt{8\pi\rho_j/3}\right]$$
$$R_{now} = \sqrt{3/8\pi\rho_{now}}\,\sinh\left[t_{now}\sqrt{8\pi\rho_{now}/3}\right]$$

$$R_{now} = \sqrt{3/\left(8\pi\times 7.425\times 10^{-60}\right)}\times$$
$$\sinh\left[1.2974585\times 10^{28}\times\sqrt{8\pi\times 7.425\times 10^{-60}/3}\right]$$
$$R_{now} = 1.6\times 10^{29}\times \sinh 0.08287$$
$$R_{now} = 1.3\times 10^{28}\,cm$$

4. Nonsingular Big Bang .

If we assume the density ρ_j and the time t_j runs independently from each other, we may evaluate the limit at the Big Bang:

$$\left(\rho_j, t_j\right) = \left(\rho_{BB}, t_{BB}\right) \to (\infty, 0)$$

$$R_{B.B} = \sqrt{3/8\pi\rho_{B.B}} \sinh\left[t_{B.B}\sqrt{8\pi\rho_{B.B}/3}\right]$$

$$R_{B.B} = \lim_{(\rho_{B.B},t_{B.B})\to(\infty,0)} \sqrt{3/8\pi\rho_{B.B}} \sinh\left[t_{B.B}\sqrt{8\pi\rho_{B.B}/3}\right]$$

$$R_{B.B} = \lim_{(\rho_{B.B},t_{B.B})\to(\infty,0)} \left(t_{B.B}\right) \frac{\sinh\left[t_{B.B}\sqrt{8\pi\rho_{B.B}/3}\right]}{t_{B.B}\sqrt{8\pi\rho_{B.B}/3}}$$

$$R_{B.B} = \lim_{(\rho_{B.B})\to(\infty)} \left[\lim_{(t_{B.B})\to(0)} \left(t_{B.B}\right) \frac{\sinh\left[t_{B.B}\sqrt{8\pi\rho_{B.B}/3}\right]}{t_{B.B}\sqrt{8\pi\rho_{B.B}/3}} \right]$$

$$R_{B.B} = \lim_{(\rho_{B.B})\to(\infty)} [0\times 1] = 0$$

$$R_{B.B} = \lim_{(t_{B.B})\to(0)} \left[\lim_{(\rho_{B.B})\to(\infty)} \left(t_{B.B}\right) \frac{\sinh\left[t_{B.B}\sqrt{8\pi\rho_{B.B}/3}\right]}{t_{B.B}\sqrt{8\pi\rho_{B.B}/3}} \right]$$

$$R_{B.B} = \lim_{(t_{B.B})\to(0)} [\infty] = \infty$$

The limit does not exist since it is not unique. Let us treat the limit from a different point of view, namely the dependent evolution for both the density ρ_j and the time t_j.

Now we are interesting to explore how both the density ρ_j and the time t_j are dependently evolved? Consider the factor $t_j\sqrt{\rho_j}$ appears in the time evolution equation of the Universe. Calculate the value of $t_j\sqrt{\rho_j}$ at the given two well known sets of data, namely the Planck scale and the current scale:

$$t_p\sqrt{\rho_p} = 5.4\times 10^{-44} \times 2.997\times 10^{10}\sqrt{3.8789\times 10^{62}} = 0.032$$

$$t_{now}\sqrt{\rho_{now}} = 1.2974585\times 10^{28} \times \sqrt{7.425\times 10^{-60}} = 0.034$$

The two values are approximately equal no matter how large the difference between the two states, which is of order 10^{61}. Hence it is very reasonable that $t_j\sqrt{\rho_j}$ remains approximately constant through the whole evolution of the cosmos, even at the Big Bang. *The infinitely large density is struggled by the infinitesimally small time and vice versa, in our mathematical model*. This process prevents the scale factor from blows up by the infinitely large density. Since the data at Planck scale is accurate, we assume $t_j\sqrt{\rho_j} = 0.032$. Hence,

$$R_{B.B} = \lim_{\left[\left(t_{B.B}\sqrt{\rho_{B.B}}\right), t_{B.B}\right]\to(0.032,0)} \left[\left(t_{B.B}\right)\right]\frac{\sinh\left[t_{B.B}\sqrt{\rho_{B.B}}\sqrt{8\pi/3}\right]}{t_{B.B}\sqrt{\rho_{B.B}}\sqrt{8\pi/3}}$$

$$R_{B.B} = \lim_{t_{B.B}\to 0}\left[\left(t_{B.B}\right)\frac{\sinh\left[0.032\sqrt{8\pi/3}\right]}{0.032\sqrt{8\pi/3}}\right]$$

$$R_{B.B} = \lim_{t_{B.B}\to 0}\left[\left(t_{B.B}\right)\times 1.00187\right] = 0$$

The limit exists. The manifold consists of its limiting point and hence it is a complete space. Thus there exists a ***continuou**s **path*** governs the time evolution of the Universe since the Big Bang. Hence the Big Bang is nonsingular.

5. Black Holes

The black hole is an object whose escape velocity exceeds the speed of light *c*. The idea goes back to natural scientist and Anglican rector John Michell, who in 1783 observed that an object with the density of the Sun but 500 times its radius would be a black hole. The nonrelativistic Newtonian equations he used give the correct relativistic formula for the black hole radius. His argument can be summarized beginning with the formula for the energy *E* of a particle of mass *m* moving in the gravitational potential of a spherical mass *M*

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r}$$

where *v* is velocity, *r* is radial distance, and *G* is Newton constant. A particle just barely escapes to infinity if *E*=0; if it has an initial velocity $v = c$, the condition of marginal escape determines an initial radius $R(M) = 2GM/c^2$, now called the Schwarzschild radius for mass *M*. For an Earth- sized mass, *R* is approximately 1 cm; for a solar mass, it is about 3 km[31]. Our current classical understanding of gravity is via general relativity and Einstein's equations. 1916, mere weeks after their final formulation, Karl Schwarzschild presented a solution giving the gravitational field of a spherically symmetric mass. That Schwarzschild solution is the simplest possible black hole. In Einstein's theory, gravity is described through the curvature of spacetime, and in the center of the black hole, the curvature goes to infinity. That infamous singularity indicates a breakdown of physics, but one far removed from scrutiny. In particular, since in classical physics nothing escapes the region within the so-called event horizon located at *R*, the singularity has no effect outside the black hole. The need for physics to smooth out the singularity is nonetheless one of the motivators for pursuing a quantum theory of gravity. Many feel that a correct quantum description will resolve such singularities. One of the most remarkable features of relativistic black holes is that they are purely gravitational entities. A pure black hole spacetime contains no matter whatsoever. It is a "vacuum" solution to the Einstein field equations, which just means that it is a solution of Einstein's gravitational field equations in which the matter density is everywhere zero. (Of course, one can also consider a black hole with matter present.) In pre-relativistic physics we think of gravity as a force produced by the mass contained in some matter. In the context of general relativity, however, we do away with gravitational force, and instead postulate a curved spacetime geometry that produces all the effects we standardly attribute to gravity. Thus a black hole is not a "thing" *in* spacetime; it is instead a feature of spacetime itself.

6. The Schwarzschild metric

In General Relativity, the unique spherically symmetric vacuum solution is the Schwarzschild metric. In spherical coordinates (t,r,θ,ϕ), the metric is given by:

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$where$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$$

We are interested in the solution outside a spherical body, Einstein`s equation in vacuum

$$R_{\mu\nu} = 0$$

The Minkowski metric (flat spacetime metric) in polar coordinates

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

The Schwarzschild metric in the spherically symmetric

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + e^{2\lambda(r)}r^2 d\Omega^2$$

Put $\bar{r} = e^{\lambda(r)}r$,then

$$d\bar{r} = e^{\lambda(r)}dr + e^{\lambda(r)}r d\lambda(r) = \left(1 + r\frac{d\lambda}{dr}\right)e^{\lambda(r)}dr$$

$$ds^2 = -e^{2\alpha(r)}dt^2 + \left(1 + r\frac{d\lambda}{dr}\right)^{-2} e^{2\beta(r)-2\lambda(r)}d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

Relabeling

$$\bar{r} \rightarrow r$$

$$\left(1 + r\frac{d\lambda}{dr}\right)^{-2} e^{2\beta(r)-2\lambda(r)} \rightarrow e^{2\beta(r)}$$

We get

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2 d\Omega^2$$

Let us now use Einstein`s equation to solve for the functions

$$\alpha(r) \;\; and \, \beta(r)$$

The Christoffel symbols are given by

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}\left(\frac{\partial g_{\alpha\nu}}{\partial x^{\mu}} + \frac{\partial g_{\sigma\mu}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right)$$

$$g_{tt} = -e^{2\alpha}, g_{rr} = e^{2\beta}, g_{\theta\theta} = r^2, g_{\phi\phi} = r^2 \sin^2\theta$$

$$g^{tt} = -e^{-2\alpha}, g^{rr} = e^{-2\beta}, g^{\theta\theta} = r^{-2}, g^{\phi\phi} = r^{-2}\cos ec^2\theta$$

$$\Gamma^{t}_{tr} = \partial_r \alpha \qquad \Gamma^{r}_{tt} = e^{2(\alpha-\beta)}\partial_r \alpha \qquad \Gamma^{r}_{rr} == \partial_r \beta$$

$$\Gamma^{\theta}_{r\theta} = \frac{1}{r} \qquad \Gamma^{r}_{\theta\theta} = -re^{-2\beta} \qquad \Gamma^{\phi}_{r\phi} = \frac{1}{r}$$

$$\Gamma^{r}_{\phi\phi} = -re^{-2\beta}\sin^2\theta \quad \Gamma^{\theta}_{\phi\phi} = -\sin\theta\cos\theta \quad \Gamma^{\phi}_{\theta\phi} = \cot\theta$$

From these we get the following components of the Riemann tensor **[5]**

$$R^t_{rtr} = \partial_r \alpha \partial_r \beta - \partial_r^2 - (\partial_r \alpha)^2$$
$$R^t_{\theta t \theta} = -re^{-2\beta} \partial_r \alpha$$
$$R^t_{\phi t \phi} = -re^{-2\beta} \sin^2 \theta \partial_r \alpha$$
$$R^r_{\theta r \theta} = re^{-2\beta} \partial_r \beta$$
$$R^r_{\phi r \phi} = re^{-2\beta} \sin^2 \theta \partial_r \beta$$
$$R^\theta_{\phi \theta \phi} = (1 - e^{-2\beta}) \sin^2 \theta$$

Taking the contraction yields the Ricci tensor:

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$$
$$R_{tt} = R^\lambda_{t\lambda t} = R^r_{trt} + R^\theta_{t\theta t} + R^\phi_{t\phi t}$$
$$= g^{rr} g_{tt} R^t_{rtr} + g^{\theta\theta} g_{tt} R^t_{\theta t\theta} + g^{\phi\phi} g_{tt} R^t_{\phi t\phi}$$
$$= e^{2(\alpha-\beta)} \left[\partial_r^2 \alpha + (\partial_r \alpha)^2 - \partial_r \alpha \partial_r \beta + \frac{2}{r} \partial_r \alpha \right]$$

$$R_{rr} = -\partial_r^2 \alpha - (\partial_r \alpha)^2 + \partial_r \alpha \partial_r \beta + \frac{2}{r} \partial_r \beta$$
$$R_{\theta\theta} = e^{-2\beta} \left[r(\partial_r \beta - \partial_r \alpha) - 1 \right] + 1$$
$$R_{\phi\phi} = \sin^2 \theta R_{\theta\theta}$$

We would like to set the Ricci tensor equal to zero

$$0 = e^{2(\beta-\alpha)} R_{tt} + R_{rr}$$
$$= \frac{2}{r} (\partial_r \alpha + \partial_r \beta)$$
$$\therefore \alpha = -\beta + c$$

We can set this constant equal to zero by rescaling our time coordinate

$$t \longrightarrow e^{-c}t$$
$$\therefore \alpha = -\beta$$
$$\because R_{\theta\theta} = 0$$
$$\therefore e^{2\alpha}\left(2r\partial_r \alpha + 1\right) = 1$$

This is equivalent to

$$\partial_r\left(re^{2\alpha}\right) = 1$$
$$e^{2\alpha} = 1 - \frac{R_s}{r}$$
$$R_s \quad is \quad constant$$

$$ds^2 = -\left(1 - \frac{R_s}{r}\right)dt^2 + \left(1 - \frac{R_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

R_s is called Schwarzschild radius. In the weak field limit its found that

$$g_{tt} = -\left(1 - \frac{2GM}{r}\right)$$
$$\therefore R_s = 2GM$$
$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

7. Schwarzschild metric in the hyperbolic spacetime

We modify the Schwarzschild metric in the hyperbolic spacetime. The required modified Schwarzschild spherically symmetric metric will be,

$$d\tau^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2 d\Omega^2$$
$$d\tau^2 = \left(1 + \nu + (1/2)\nu^2 + ...\right)dt^2 - \left(1 + \lambda + (1/2)\lambda^2 + ...\right)dr^2 - r^2 d\Omega^2$$

For which the Schwarzschild metric is just an approximation

$$d\tau^2 = \left(e^{\nu}\right)dt^2 - \left(e^{\lambda}\right)dr^2 - r^2 d\Omega^2$$
$$d\tau^2 \approx (1+\nu)dt^2 - (1+\lambda)dr^2 - r^2 d\Omega^2$$

The Ricci tensor

$$0 = R_{tt} = -(1/2)e^{\nu-\lambda}\left(\nu'' + \nu'^2/2 - \nu'\lambda'/2 + 2\nu'/r\right), ...(i)$$
$$0 = R_{rr} = (1/2)\left(\nu'' + \nu'^2/2 - \nu'\lambda'/2 + 2\lambda'/r\right), ...(ii)$$
$$0 = R_{\theta\theta} = -\left\{1 - \left(e^{-\lambda}r\right)' + e^{-\lambda}r\left(\frac{\nu' + \lambda'}{2}\right)\right\}, ...(iii)$$

From $R_{tt} = R_{\theta\theta} = 0$ web have $\nu' + \lambda' = 0$, so $\nu + \lambda = konstant$

Write simply $\lambda = -\nu + \log k$

Equation (i) is now just

$$\left(e^{\nu}r\right)'' = 0$$
$$e^{\nu}r = -\alpha + \beta r$$

Equation (iii) is

$$\left(e^{-\lambda}r\right)' = 1$$
$$\left(e^{\nu}r\right)' = k$$
$$\therefore \beta = k$$

We have now the complete solution

$$e^{\lambda} = \left(1 - 2\mu/kr\right)^{-1} \approx \left(e^{-2\mu/kr}\right)^{-1} = e^{2\mu/kr}$$
$$e^{\nu} = k\left(1 - \alpha/kr\right) = k\left(1 - 2\mu/kr\right) = \left(k - 2\mu/r\right)$$

For radial motion, $d\Omega^2 = 0$ .The modified Schwarzschild metric in the hyperbolic spacetime, will be

$$d\tau^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2$$

$$d\tau^2 = \left(k - 2\mu/r\right) dt^2 - e^{2\mu/kr} dr^2 - r^2 d\Omega^2 \qquad (6)$$

8. Equation of motion in the hyperbolic spacetime

Assuming flat space and circular orbit, the *Virial* theorem $M = V^2 R/G$ fails to account for the observed flat rotation curve of the Galaxy. To develop an equation describes the speed up motion in the hyperbolic spacetime and predicts the flat rotation curve, we seek for an equation whose limit is the Newtonian hyperbolic trajectory – in flat space- as *Vallado* theorem: $V = \sqrt{\mu(2/r - 1/a)}$,where a is the negative semi-major axis of orbit's hyperbola, with constant excess velocity $V_\infty = \sqrt{-\mu/a}$.

To do this, I will follow the following strategy

1- Seek for an equation $v = f(r)$ such that $v = \lim_{r\to 0} f(r) = 0$

2- $v = f(r) \xrightarrow{large.r} \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} \xrightarrow{r\to\infty} \sqrt{-\frac{\mu}{a}}$

3- I guess the required equation , that fits the data, should be

$$v = f(r) = e^{-\frac{1}{r}} \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)},$$

4- The final step in the mathematical problem solving method is to prove the conjecture

$$v = f(r) = e^{-\frac{1}{r}} \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

To find such an equation of the radial motion in the galaxy`s hyperbolic space-time, we proceed as follows, The free fall from rest of a star (of mass m and energy E) far from the center possesses [6] .

$$\frac{E}{m} = \left(1 - \frac{2\mu}{r}\right)\frac{dt}{d\tau} = 1$$

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{2\mu}{r}\right)^2$$

$$\left(\frac{d\tau}{dt}\right)^2 = \left(k - 2\mu/r\right) - e^{2\mu/kr}\left(\frac{dr}{dt}\right)^2$$

$$\left(1 - 2\mu/r\right)^2 = \left(k - 2\mu/r\right) - e^{2\mu/kr}\left(\frac{dr}{dt}\right)^2$$

To our purpose for the hyperbolic space- time, the velocity far away from the center would be

$V_\infty = \sqrt{-\mu/a}$ and consequently $k = 1 - \mu/a$

$$d\tau^2 = \left(1 - 2\mu/r - \mu/a\right)dt^2 - e^{2\mu/r}dr^2 - r^2 d\Omega^2$$

$$\left(1 - 4\mu/r + \left(2\mu/r\right)^2\right) = \left(1 - 2\mu/r - \mu/a\right) - e^{2\mu/\left[(1-\mu/a)r\right]}V^2$$

neglect the term $\left(2\mu/r\right)^2$ and rearrange

$$\left(1 - 4\mu/r\right) = \left(1 - 2\mu/r - \mu/a\right) - e^{2\mu/\left[(1-\mu/a)r\right]}V^2$$

$$e^{2\mu/\left[(1-\mu/a)r\right]}V^2 = \left(2\mu/r - \mu/a\right)$$

$$V = e^{-\mu/\left[(1-\mu/a)r\right]}\sqrt{2\mu/r - \mu/a}$$

$$V = e^{-a\mu/\left[(a-\mu)r\right]}\sqrt{2\mu/r - \mu/a}$$

$$\because -a >> \mu$$

$$\therefore a - \mu \approx a$$

$$V = e^{-\mu/r}\sqrt{2\mu/r - \mu/a} \qquad (7)$$

Example

A typical galaxy of ordinary enclosed mass (Milky Way or Andromeda) [7]

$$M = 10^{11} M_\odot = 10^{11} \times 2 \times 10^{30}\,kg$$

$$\mu = 10^{11} \times 2 \times 10^{30} \times 7.4 \times 10^{-31}\,km$$

$$\mu = 1.5 \times 10^{11}\,km$$

$$\mu = 1.5 \times 10^{11}\,km \times \left(s/s\right)$$

$$\mu = 1.5 \times 10^{11}\,km \times \left(3 \times 10^{5}\,km/s\right)$$

$$\mu = 4.5 \times 10^{16}\left(km^2/s\right)$$

$$210 = \sqrt{\mu/-a}$$
$$(210)^2 = \mu/-a$$

$$V = e^{\mu/r(kpc)} \sqrt{2\mu/r(kpc) - \mu/a}$$
$$V = e^{-4.5\times10^{16}(km^2/s)/(1(km/s)\times r(3.1\times10^{16}\,km))} \times$$
$$\sqrt{9\times10^{16}(km^2/s)/(1(km/s)\times r(3.1\times10^{16}\,km))+210^2}$$
$$V = e^{-1.45}\sqrt{3/r+210^2}$$

The curve of the above equation is plot by visual mathematics program

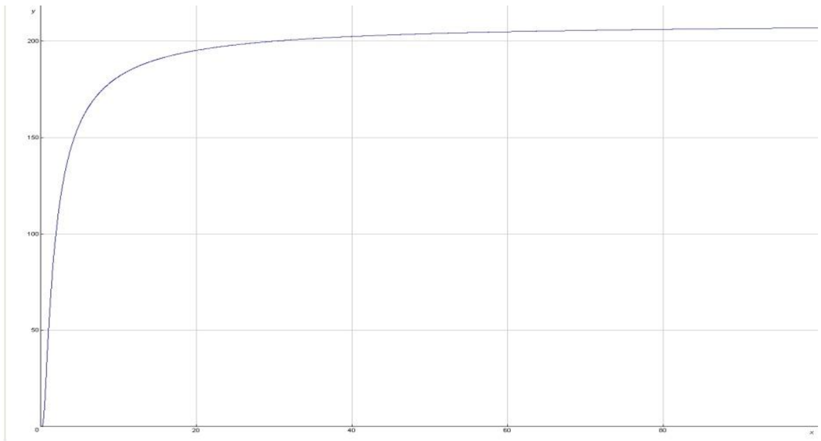

FIG. 1. The curve describes the motion of a star in the Milky way (or Andromeda) galaxy. The vertical axis represents the velocity, while the horizontal axis represents the distance from the center of the galaxy.

A typical cluster of galaxies of ordinary enclosed mass

$$M = 10^{14} M_{\odot} = 10^{14} \times 2 \times 10^{30} kg$$
$$\mu = 10^{14} \times 2 \times 10^{30} \times 7.4 \times 10^{-31} km$$
$$\mu = 1.5 \times 10^{14} km$$
$$\mu = 1.5 \times 10^{14} km \times (3 \times 10^{5} km/s)$$
$$\mu = 4.5 \times 10^{19} (km^2/s)$$

$$1000 = \sqrt{\mu/-a}$$
$$(1000)^2 = \mu/-a$$

$$V = e^{-\mu/(r\mathrm{M}pc)}\sqrt{2\mu/r(10\mathrm{M}pc) - \mu/a}$$
$$V = e^{-4.5\times10^{19}/\left(3.1\times10^{19}\times r\right)}\sqrt{2\times4.5\times10^{19}/\left(3.1\times10^{19}\times r\right)+(1000)^2}\,(km/s)$$ The curve of
$$V = e^{-1.45/r}\sqrt{3/r+(1000)^2}$$

the above equation is plot by visual mathematics program

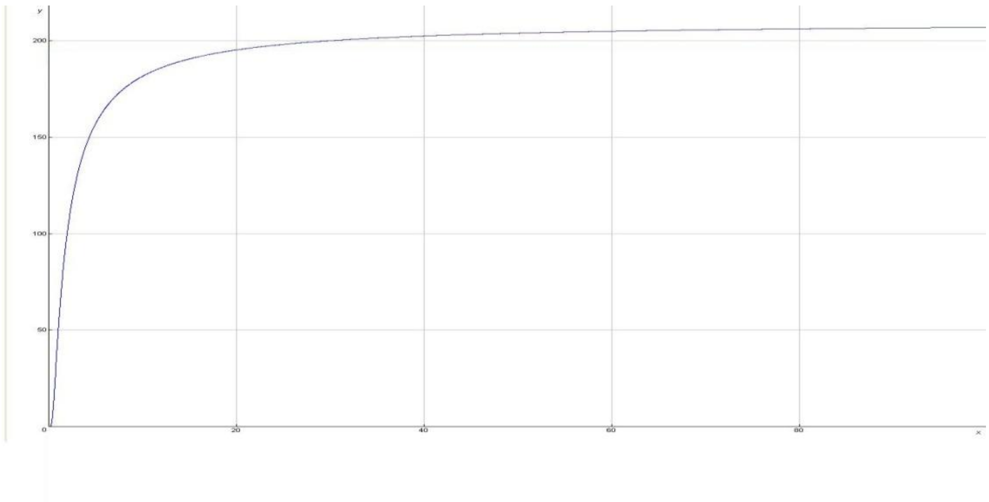

FIG. 2. The curve describes the motion of a cluster of galaxies. The vertical axis represents the velocity, while the horizontal axis represents the distance from the center of the cluster.

The dark matter halo is nothing but instead of it we have a cell of hyperbolic negative curvature. Virial theorem $\left(M = V^2R/G\right)$ does no longer hold for Non-Euclidian space. We developed the equation of motion in the hyperbolic space-time : $V = e^{-\mu/r}\sqrt{\mu(2/r - 1/a)}$, that describes the speed up motion in the hyperbolic space-time and predicts the flat curve. Farther away from the center the exponential factor $e^{-1/r}$ drops to one. Galaxies furthest away from the center are moving fastest until they reached large distance from the center the space-time turns flat and they possessed hyperbolic trajectory: $V = \sqrt{\mu(2/r - 1/a)}$, according to Vallado **[8]** theorem, with constant speed called hyperbolic excess velocity: $V_\infty = \sqrt{-\mu/a}$ that can explain the galaxy flat rotation curve problem, a is the negative semi-major axis of orbit's hyperbola.

9. Nonsingular Schwarzschild metric in the hyperbolic spacetime

The modified Schwarzschild metric in the hyperbolic spacetime for radial null trajectory is

$$d\tau^2 = (1 - 2\mu/r - \mu/a)dt^2 - e^{2\mu/r}dr^2 - r^2 d\Omega^2$$
$$0 = (1 - 2\mu/r - \mu/a)dt^2 - e^{2\mu/r}dr^2$$
$$\frac{dr}{dt} = e^{-\mu/r}\sqrt{1 - 2\mu/r - \mu/a}$$

doesn`t possess singularity at $r = 0$, since

$$\lim_{r\to 0}\frac{dr}{dt} = \lim_{r\to 0} e^{-\mu/r}\sqrt{1 - 2\mu/r - \mu/a}$$

$$\lim_{r\to 0} V = \lim_{r\to 0}\frac{\sqrt{1 - 2\mu/r - \mu/a}}{e^{\mu/r}} = \lim_{r\to 0}\frac{\left(2\mu/r^2\right)\Big/2\sqrt{1 - 2\mu/r - \mu/a}}{\left(-\mu/r^2\right)e^{\mu/r}}$$

$$\lim_{r\to 0} V = \lim_{r\to 0}\frac{-1}{\sqrt{1 - 2\mu/r - \mu/a}.e^{\mu/r}} = 0$$

Note that Schwarzschild metric in the flat spacetime possesses singularity at $r = 0$, since

$$d\tau^2 = (1 - 2\mu/r)dt^2 - (1 - 2\mu/r)^{-1}dr^2 - r^2 d\Omega^2$$

for radial null trajectory is

$$d\tau^2 = 0 = (1 - 2\mu/r)dt^2 - (1 - 2\mu/r)^{-1}dr^2$$
$$\left(\frac{dr}{dt}\right)^2 = (1 - 2\mu/r)^2$$
$$V = \frac{dr}{dt} = (1 - 2\mu/r)\xrightarrow{r\to o}\infty$$

10. Nonsingular Kerr Black Hole in the hyperbolic spacetime

Kerr metric of a rotating black hole is given by **[9].**

$$d\tau^2 = \left(1 - \frac{2\mu r}{\rho^2}\right) dt^2 + \frac{4\mu\alpha r \sin^2\theta}{\rho^2} d\phi dt - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2$$

$$-\left(r^2 + \alpha^2 + \frac{2\mu r \alpha^2 \sin^2\theta}{\rho^2}\right) \sin^2\theta d\phi^2$$

$$\alpha \equiv \frac{J}{\mu}$$

$$\rho^2 = r^2 + \alpha^2 \cos^2\theta$$

$$\Delta = r^2 + \alpha^2 - 2\mu r$$

The singularity arises when

$$\rho^2 = r^2 + \alpha^2 \cos^2\theta = 0$$

Kerr metric reduces to Schwarzschild metric when $\alpha = 0$.

By **analogy** we can rewrite Kerr metric in the hyperbolic spacetime that can be reduced to the modified Schwarzschild metric in the hyperbolic spacetime, as follows:

$$\rho^2 = r^2 + \alpha^2 \cos^2\theta$$

$$\Delta = r^2 + \alpha^2 - 2\mu r$$

$$\frac{\rho^2}{\Delta} = \left(\frac{r^2 + \alpha^2 - 2\mu r}{r^2 + \alpha^2 \cos^2\theta}\right)^{-1}$$

$$\frac{\rho^2}{\Delta} = \left(\frac{r^2 + \alpha^2 \cos^2\theta - \alpha^2 \cos^2\theta + \alpha^2 - 2\mu r}{r^2 + \alpha^2 \cos^2\theta}\right)^{-1}$$

$$\frac{\rho^2}{\Delta} = \left(1 - \frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}\right)^{-1}$$

$$\frac{\rho^2}{\Delta} = \left(1 + \frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}\right)$$

$$\frac{\rho^2}{\Delta} = e^{\frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}}$$

$$\frac{\rho^2}{\Delta} = e^{\frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}} \xrightarrow{\alpha \to 0} e^{\frac{2\mu}{r}}$$

The Hyperbolic spacetime Kerr metric can be rewritten as

$$d\tau^2 = \left(1 - \frac{2\mu r}{\rho^2} - \frac{\mu}{a}\right)dt^2 + \frac{4\mu\alpha r\sin^2\theta}{\rho^2}d\phi dt - e^{\frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}}dr^2 - \rho^2 d\theta^2$$
$$-\left(r^2 + \alpha^2 + \frac{2\mu r\alpha^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2$$

whichreduces to our modified Schwarzschild metric in the Hyperbolic spacetime when $\alpha = 0$

$$d\tau^2 = (1 - 2\mu/r - \mu/a)dt^2 - e^{2\mu/r}dr^2 - r^2 d\Omega^2$$

The Kerr metric in the hyperbolic spacetime for radial null trajectory doesn`t possess singularity at $r = 0$, since

$$d\tau^2 = \left(1 - \frac{2\mu r}{r^2 + \alpha^2\cos^2\theta} - \frac{\mu}{a}\right)dt^2 - e^{\frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}}dr^2$$

$$0 = \left(1 - \frac{2\mu r}{r^2 + \alpha^2\cos^2\theta} - \frac{\mu}{a}\right)dt^2 - e^{\frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}}dr^2$$

$$\frac{dr}{dt} = e^{-\frac{1}{2}\frac{\alpha^2\cos^2\theta - \alpha^2 + 2\mu r}{r^2 + \alpha^2\cos^2\theta}}\sqrt{1 - \frac{2\mu r}{r^2 + \alpha^2\cos^2\theta} - \frac{\mu}{a}}$$

$$\rho \to 0 \Rightarrow \theta = \pi/2, and, r \to 0$$

$$\frac{dr}{dt} = e^{-\frac{1}{2}\frac{1-\alpha^2+2\mu r}{r^2}} \sqrt{1 - \frac{2\mu r}{r^2} - \frac{\mu}{a}}$$

$$\lim_{r \to 0} \frac{dr}{dt} = 0$$

The limit is taken by L`Hospital`s rule where θ = π/2.

11. Conclusion

The singularities are not real features of the actual world, they are instead merely artifices of our current (flawed) physical theories based on flat spacetime.The failing lies not in the spacetime of the actual world, but rather in the theoretical description of the flat spacetime. Penrose [10]–Hawking [11] singularity theorems pre-assume asymptote flat spacetime. This false flat spacetime paradigm produces such singularities at the Big Bang and the center of the Black Holes. The Universe is globally hyperbolic as we did prove mathematically. We prove that the hyperbolic time evolution equation of the universe $R(t) = \sqrt{3/8\pi\rho_j}\,\sinh t\sqrt{8\pi\rho_j/3}$,traces its manifold ,didn`t break down at the initial Big Bang moment. General Relativity possesses nonsingular Big Bang Hyperbolic Universe. We modify both Schwarzschild metric and Kerr metric in the hyperbolic spacetime. Neither Schwarzschild Black Holes nor Kerr Black Holes possess singularity in the hyperbolic spacetime.

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