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On Returns to Scale Assumption in Endogenous Growth

Merter Mert*

Gazi University, Department of Economics, Ankara, 06500, Turkey
Email: mertermert@gazi.edu.tr

Abstract

This study points out to a contradiction between increasing returns to scale assumption and steady-state or long-run equilibrium or balanced growth analysis in models of endogenous growth. The paper reminds the necessity of the constant returns to scale assumption in models of economic growth, with special reference to models of endogenous growth, which are based on steady-state or long-run equilibrium or balanced growth. While analyzing balanced growth, some models of endogenous growth are based on increasing returns to scale but some others are based on constant returns to scale. The arguments in the present paper are simply explained by the some of the original texts on the debate. This study points out that assumption of constant returns to scale is compatible with the steady-state or long-run equilibrium or balanced growth, rather than increasing returns to scale. The paper emphasizes an important and neglected problem in endogenous growth.

Keywords: returns to scale; endogenous growth; steady-state; balanced growth; long-run equilibrium.

1. Introduction

Models of endogenous growth explain long-run rate of growth and technological progress endogenously. Some models of endogenous growth explain endogenous growth based on increasing returns to scale (for instance; [14,18]) while some of them assume constant returns to scale (for instance; [2,16]). However, this study is based on the idea that the assumption of constant returns to scale is compatible with the long-run equilibrium, rather than increasing returns to scale.

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* Corresponding author.

This study attempts to put this contradiction briefly, based on [14,18,2]. Note that our analysis is constrained with the analysis of [14,18,2].

The study is organized as follows. Section 2 discusses the main problem of the study. Specifically, Section 2 discusses returns to scale and steady-state in models of endogenous growth with special reference to [14,18] and [2]. Section 3 concludes the study.

2. Main Problem

Constant returns to scale occur if doubling of the all inputs leads to the same proportional increase in the output. Increasing returns to scale occur if doubling of the all inputs leads to a more than proportional increase in the output. Thus, as the first result, if there are constant returns to scale then all variables will grow at same rate. If all variables are growing, then, by definition, economy cannot be in short-run, economy should be in long-run. Thus, as the second result, if there are constant returns to scale there is long-run equilibrium.

As a consequence, the two results simply show that if there are constant returns to scale there will be steady-state, if there are increasing returns to scale there will be off-steady-state. It should be emphasized that, returns to scale definition above is the 'proportional definition'. There is another definition, namely, 'marginal definition'. One may refer to [12] for a discussion on the contradiction between proportional and marginal definitions.

[10: 14-15] points out relationship between constant returns to scale and steady-state: "Now relative prices can remain constant over time, even though quantities are varying, provided, first, that there is CRS [constant returns to scale] and, second, that the ratios between quantities produced and quantities absorbed in producing them remain constant over time; so the economy is in what had been called a steady-state."

In contrast, according to [9], steady-state is possible if there are increasing returns to scale: "So long as the tendency to increasing returns to scale is not so powerful as to cause increasing social returns to capital by itself, steady growth is possible, with *Y* growing at the same rate as *K*..." [9: 833]. [9] define a production function such as $Y = A\left(e^{mt}K^{\alpha}L^{1-\alpha}\right)^{\mu}$ where *Y, A, K* and *L* are output, level of technology, capital and labor, respectively. α and μ are parameters where $0<\alpha<1$ and $\mu>1$. Then "if *g*, the rate of growth of *Y*, equals the rate of growth of *K* we have (*n* rate of population growth)

$g = \frac{m\mu}{1-\alpha\mu} + n\left(1+\frac{\mu-1}{1-\alpha\mu}\right)$" [9: 833]. Note that, simply, by definition, if inputs (labor and capital) grow at same rate (say *n*), rate of growth of *Y* will be $g = \frac{m\mu}{1-\alpha\mu} + n\left(1+\frac{\mu-1}{1-\alpha\mu}\right)$ but rate of growth of *K* can not be the same, since, again by definition, labor and capital grow at same rate, *n*. Thus, capital-output ratio will not be constant, so, there will be off-steady-state.

[11] defines Arrow type models and points out [13] which assumes increasing returns to scale. According to [11: 99], [13] shows the existence of a steady growth path. However, [13] does not explicitly analyze steady-state or balanced or equilibrium growth. [13], following Arrow, assumes only 'exponential growth'. [13] assumes an Arrow type of technical progress for a general type of production function and shows capital and output grow at common rate: $\frac{\sigma}{1-n}$

where σ is growth rate of labor, n is an elasticity parameter for 'cumulative gross investment' or capital, and $0<n<1$. Although capital and output grow at common rate, according to [13] there are also increasing returns to scale: "If we multiply our cumulated investment by λ, and our labor force by λ^{1-n} we multiply production by λ. If we also multiply the labor force by λ, production will therefore be multiplied by more than λ." [13: 120] Note that, simply, if production is multiplied by more than λ while capital grows by rate of λ, capital-output ratio will not be constant, so there will be off-steady-state.

On the other hand, there is also a concept called 'balanced growth'. [20: 412] define balanced growth as "a state of affairs in which the output of each commodity increases (or decreases) by a constant percentage per unit of time, the mutual proportions in which commodities are produced remaining constant." [21], which explains endogenous growth using a multi-sector nonlinear dynamic input–output model, defines balanced growth path as "prices – but not quantities – are assumed constant over time" (223). It should be emphasized that balanced growth definition given by [21] is almost same with the quotation of [10] about constant returns to scale. Note that [21: 235] also assumes constant returns to scale.

Besides, there are also some studies on the existence of equilibrium in an economy with linear constant and also nonlinear constant returns to scale. As an example, [6: 79-96] makes an analysis in order to find equilibrium in an economy with linear constant returns to scale production activities. Apart from linear constant returns to scale production, [8] generalize approach of [6] and show that there is an equilibrium in an economy with nonlinear constant returns to scale production.

Moreover, there are studies which attempt to relax assumption of constant returns to scale. As an example, [4: 369] defines "an asymptotic equilibrium in which neither relative factor share goes to zero under marginal product factor pricing", namely, 'well-behaved equilibrium'. [4: 369] claims that "when returns to scale are non-constant (but still homogeneous), unit elasticity of substitution as well as Harrod neutrality is necessary for a well-behaved equilibrium". [5] shows that how the 'technical change frontier' may be used to relax assumption of constant returns to scale. Another study, [15: 171] states that property of homogeneity "may be interpreted economically as the assumption of constant returns to scale, implies that there is a constant growth rate on the balanced growth path." However [15: 171] does not assume "homogeneity, so that the growth rate along the balanced growth path may fluctuate". The idea in [15: 171] is based on the fact that "national economies move on or near a balanced growth path". Thus according to [15] balanced growth is somewhat different from steady-state.

Interestingly, according to [19: 48] neoclassical "model can get along perfectly well without constant returns to scale" and "it is not essential to the working of the model nor even overwhelmingly useful in an age of cheap computer simulation." [19: 48] states that "the assumption of constant returns to scale is a considerable simplification, both because it saves a dimension by allowing the whole analysis to be conducted in terms of ratios and because it permits the further simplification that the basic market-form is competitive."

[14: 9] constructs his "system's balanced growth path: the particular solution ... such that the rates of growth of each of these variables is constant". Interestingly, [14: 9] says that, in parentheses, "I have never been sure exactly what it is that is 'balanced' along such a path, but we need a term for solutions with this constant growth rate property and this is as good as any".

It should be emphasized that the author is aware of the concepts namely private and social factors of production which are important for this debate. Indeed, there are studies on endogenous growth based on private returns to scale and social returns to scale conceptualization, investigate steady-state under increasing returns to scale (see, for example; [1,3]). Note that, there are also studies which assume decreasing returns to scale while analyzing endogenous growth. As an example, [7: 293] extends his basic model with contractual costs of research and development under Stackelberg competition which assumes "each entrant is investing under (local) increasing returns to scale, while the leader does it at a larger scale but under decreasing returns to scale." In another extension, he assumes that there are two types of investments and one of them is characterized by decreasing returns to scale [7: 294]. After extensions, [7] makes applications based on steady-state solutions.

Apart from other studies, one should refer [17] in order to explain the problem. [17] makes an assumption on the production function $F(k, K, \mathrm{x})$ where k is firm-specific input, K is the sum of k for all firms (i.e. aggregate level of knowledge in the economy) and x is the set of additional factors such as physical capital, labor, and etc. According to [17: 1015], if all inputs grow at same rate which is equal to $\psi > 1$, then output represented by $F(\psi k, \psi K, \psi \mathrm{x})$ will be greater than $F(\psi k, K, \psi \mathrm{x}) = \psi F(k, K, \mathrm{x})$, so this explains increasing returns to scale. However, one can see that, by definition, this explanation is not valid since K is assumed to be constant by [17: 1015]. Note that, by definition, returns to scale are defined if all inputs grow at same rate. Thus, if one does not assume that all inputs grow at a common rate, returns to scale cannot be defined. At this point [17] gives an explanation about **concavity** of F. According to him, "once concavity is granted, … assuming that F is homogeneous of degree one as a function of k_i and x_i when K is held constant; any concave function can be extended to be homogeneous of degree one by adding an additional factor to the vector x if necessary." Thus, explanation of [17] can be mathematically convincing but from the point of view of economics, it is not compatible with the definition of returns to scale. On the other hand, [17] mainly deals with long-run equilibrium rather than steady-state. [17: 1003] states that "even if all other inputs are held constant, it will not be optimal to stop at some steady state where knowledge is constant and no new research is undertaken." As a consequence, [17] shows the existence of a competitive equilibrium in the long-run in the presence of externalities, increasing returns in the production of output and decreasing returns in the production of new knowledge.

The present study is based on the claim that steady-state occurs at balanced growth. However, as it is referred above, for technical reasons, some studies prefer balanced growth or long-run equilibrium rather than steady-state. Besides, in the light of the discussion above, the present study attempts to emphasize that if there are constant returns to scale there will be steady-state, if there are increasing returns to scale there will be off-steady-state.

Let us now discuss the [14] and [18], respectively.

2.1. The Lucas model

[14] introduces the following production function:

$$Y(t) = AK(t)^{\alpha}[u(t)h(t)L(t)]^{1-\alpha}h(t)^{\gamma} \tag{1}$$

where $Y(t)$, $K(t)$, A, $u(t)$, $h(t)$ and $L(t)$ represent output, physical capital stock, level of technology, time devoted to production, human capital stock and labor, respectively. $u(t)h(t)L(t)$ is effective labor. α, $1-\alpha$ and γ are output elasticity parameters with respect to physical capital, effective labor and human capital, respectively.

According to Lucas (1988), the variable $h(t)$ within effective labor shows internal effect of human capital stock. The variable $h(t)^{\gamma}$ points out external effect of human capital stock. Equation (1) is rewritten by rate of growth:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)} = \alpha\frac{dK(t)}{dt}\frac{1}{K(t)} + (1-\alpha)\left(\frac{dh(t)}{dt}\frac{1}{h(t)} + \frac{dL(t)}{dt}\frac{1}{L(t)}\right) + \gamma\frac{dh(t)}{dt}\frac{1}{h(t)} \tag{2}$$

Note that $\frac{dA}{dt}$ and $\frac{du(t)}{dt}$ are assumed to be equal to zero.

According [14] physical capital stock and output grow at same rate at balanced growth path $\left(\frac{dK(t)}{dt}\frac{1}{K(t)} = \frac{dY(t)}{dt}\frac{1}{Y(t)}\right)$.

Then equation (3) is written:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)} = \alpha\frac{dY(t)}{dt}\frac{1}{Y(t)} + (1-\alpha)\left(\frac{dh(t)}{dt}\frac{1}{h(t)} + \frac{dL(t)}{dt}\frac{1}{L(t)}\right) + \gamma\frac{dh(t)}{dt}\frac{1}{h(t)} \tag{3}$$

Rearranging:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)} - \frac{dL(t)}{dt}\frac{1}{L(t)} = \alpha\left(\frac{dY(t)}{dt}\frac{1}{Y(t)} - \frac{dL(t)}{dt}\frac{1}{L(t)}\right) + (1-\alpha)\frac{dh(t)}{dt}\frac{1}{h(t)} + \gamma\frac{dh(t)}{dt}\frac{1}{h(t)} \tag{4}$$

$$\frac{dY(t)}{dt}\frac{1}{Y(t)} - \frac{dL(t)}{dt}\frac{1}{L(t)} = \left(\frac{1-\alpha+\gamma}{1-\alpha}\right)\left(\frac{dh(t)}{dt}\frac{1}{h(t)}\right) \tag{5}$$

Thus if $\gamma = 0$, balanced growth is compatible with constant returns to scale:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)}-\frac{dL(t)}{dt}\frac{1}{L(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}-\frac{dL(t)}{dt}\frac{1}{L(t)}=\frac{dh(t)}{dt}\frac{1}{h(t)} \tag{6}$$

On the contrary, if $\gamma > 0$, balanced growth is not compatible with constant returns to scale:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)}-\frac{dL(t)}{dt}\frac{1}{L(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}-\frac{dL(t)}{dt}\frac{1}{L(t)}>\frac{dh(t)}{dt}\frac{1}{h(t)} \tag{7}$$

More importantly, since there are no constant returns to scale, there is off-steady-state.

2.2. The Romer model

[18: 89] uses following function:

$$Y(H_A, L, x)=(H_Y A)^{\alpha}(LA)^{\beta}K^{1-\alpha-\beta}\eta^{\alpha+\beta-1} \tag{8}$$

According to equation (8), η is unit of forgone consumption to create one unit of any type of durable, K is physical capital stock, L is population or labor, A is level of technology or a count of the number of designs. H_Y is human capital devoted to final output, x is input used by a firm that produces final output, and finally H_A is marginal product of research sector. α, β and γ are output elasticity parameters.

Equation (8) is rewritten by rate of growth:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)}=\alpha\frac{dH_Y(t)}{dt}\frac{1}{H_Y(t)}+(\alpha+\beta)\frac{dA(t)}{dt}\frac{1}{A(t)}+\beta\frac{dL(t)}{dt}\frac{1}{L(t)}+(1-\alpha-\beta)\frac{dK(t)}{dt}\frac{1}{K(t)}+(\alpha+\beta-1)\frac{d\eta(t)}{dt}\frac{1}{\eta(t)} \tag{9}$$

[18: 89] defines physical capital stock as $K=\eta A\bar{x}$. If it is written by rate of growth:

$$\frac{dK(t)}{dt}\frac{1}{K(t)}-\frac{dA(t)}{dt}\frac{1}{A(t)}-\frac{d\bar{x}}{dt}\frac{1}{\bar{x}}=\frac{d\eta(t)}{dt}\frac{1}{\eta(t)} \tag{10}$$

Besides, according to [18: 79, 80] it is assumed that $\frac{dL(t)}{dt}\frac{1}{L(t)}=0$ and $\frac{dH_Y(t)}{dt}\frac{1}{H_Y(t)}=0$.

Then equation (9) becomes:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)}=(\alpha+\beta)\frac{dA(t)}{dt}\frac{1}{A(t)}+(1-\alpha-\beta)\frac{dK(t)}{dt}\frac{1}{K(t)}+(\alpha+\beta-1)\left(\frac{dK(t)}{dt}\frac{1}{K(t)}-\frac{dA(t)}{dt}\frac{1}{A(t)}-\frac{d\bar{x}}{dt}\frac{1}{\bar{x}}\right) \tag{11}$$

Similar to [14], [18: 90] defines a balanced growth path, thus, along the balanced growth path $\bar{x}$ is constant, so,

$\frac{d\bar{x}}{dt}\frac{1}{\bar{x}}=0$.

Note that, K/A should also be constant along the balanced growth path [18: 90]. Thus,

$$\frac{d\eta(t)}{dt}\frac{1}{\eta(t)}=0 \tag{12}$$

$$\frac{dK(t)}{dt}\frac{1}{K(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)} \tag{13}$$

Equation (11) can be rewritten by:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}. \tag{14}$$

As a result, [18] shows steady-state equilibrium along the balanced growth path as follows:

$$\frac{dK(t)}{dt}\frac{1}{K(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)}=\frac{dY(t)}{dt}\frac{1}{Y(t)} \tag{15}$$

Thus balanced growth seems to be compatible with constant returns to scale.

According to [18] growth rate of technology (or a count of the number of designs) is written by:

$$\frac{dA(t)}{dt}\frac{1}{A(t)}=\delta H_A \tag{16}$$

where δ is a productivity parameter. [18] assumes H_A as a constant. Thus, economy grows at a constant rate at balanced growth. [18: 95] states that "if the research technology exhibited constant returns to scale, a doubling of both [human capital and the stock of knowledge] the human capital and the stock of knowledge would leave the marginal product of human capital in research unchanged."

Note that, human capital is the scale variable in [18]. Thus, if scale variable is doubled, then capital, output and level of technology are doubled and it will be $\frac{dH(t)}{dt}\frac{1}{H(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)}=\frac{dY(t)}{dt}\frac{1}{Y(t)}$ at **balanced growth**, so there are constant returns to scale. However, [18: 95] also states that "Under the specification used here, a doubling of both leads to an increase in the marginal product of human capital in research. As a result, a permanent increase in the total stock of human capital in the population leads to an increase in the ratio of *A* to *K* and a more than proportional increase in the amount of human capital that is devoted to the research sector…"

Let's rewrite equation (9) where H_A and the ratio of *A* to *K* are not a constant $\left(\frac{dH_A(t)}{dt}\frac{1}{H_A(t)}>0\right)$ and $\left(\frac{dK(t)}{dt}\frac{1}{K(t)}\neq\frac{dA(t)}{dt}\frac{1}{A(t)}\right)$ and under other assumptions:

$$\frac{dY(t)}{dt}\frac{1}{Y(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)} \tag{17}$$

By definition, if there are increasing returns to scale, doubling of the all inputs; human capital, level of technology and capital (labor is excluded as in [18: 79, 80]) should lead to a more than proportional increase in the output:

$$\frac{dH(t)}{dt}\frac{1}{H(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}<\frac{dY(t)}{dt}\frac{1}{Y(t)} \tag{18}$$

However, according to [18: 95] a doubling of human capital and the stock of knowledge

$$\frac{dH(t)}{dt}\frac{1}{H(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)} \tag{19}$$

leads to an increase in the ratio of *A* to *K*

$$\frac{dK(t)}{dt}\frac{1}{K(t)}<\frac{dA(t)}{dt}\frac{1}{A(t)}. \tag{20}$$

Thus, equations (17) and (19) and statement (20) show a contradiction as in statement (21):

$$\frac{dH(t)}{dt}\frac{1}{H(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}<\frac{dY(t)}{dt}\frac{1}{Y(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)} \tag{21}$$

As a consequence, it should be emphasized that if there is a balanced growth path, this path exists in the long-run, so in the conditions of constant returns to scale. Thus, if the economy is stable or moving to the long-run equilibrium, then, there cannot be increasing returns to scale. The equilibrium solution in [18] is simply for the long-run and shows where $\frac{dH(t)}{dt}\frac{1}{H(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}=\frac{dY(t)}{dt}\frac{1}{Y(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)}$. However, increasing returns to scale occurs if

$\frac{dH(t)}{dt}\frac{1}{H(t)}=\frac{dK(t)}{dt}\frac{1}{K(t)}=\frac{dA(t)}{dt}\frac{1}{A(t)}<\frac{dY(t)}{dt}\frac{1}{Y(t)}$ and this is not compatible with balanced growth. Moreover, even if one

assumes increasing returns, statement (21) shows that there is a contradiction.

Let us now discuss the [2] which assumes constant returns to scale in order to explain endogenous growth. It should be emphasized that [16: 501] also assumes constant returns to scale while explaining endogenous growth but the present study does not explain it.

2.3. The Barro model

[2: 106-107], uses the following production function which assumes constant returns to scale:

$$y(t)=Ak(t)^{\alpha}g(t)^{1-\alpha} \tag{22}$$

where y is output per worker, A is constant net marginal product of capital, k is (private) capital per worker, g is per capita quantity of government purchases of goods and services or government expenditure per labor. α

is output elasticity parameter with respect to private capital stock.

[2] assumes balanced budget conditions. Government revenue $[T(t)]$ can be written as:

$$T(t) = \pi y(t) \tag{23}$$

where π is tax rate.

Since there is balanced budget, then government expenditures should be equal to government revenue:

$$g(t) = \pi y(t) \tag{24}$$

Assume that private investments are determined by the following equation:

$$\frac{dk(t)}{dt} = (1-\pi)y(t) - c(t) - (n+\delta)k(t) \tag{25}$$

where c is consumption per person, δ and n represent depreciation rate and growth rate of population, respectively.

Consumption per person is equal to:

$$c(t) = (1-\pi)y(t) - s(1-\pi)y(t) \tag{26}$$

where s is saving rate.

Using equations (26), (25) and (22), equation (27) can be written:

$$\frac{dk(t)}{dt} = s(1-\pi)Ak(t)^{\alpha} g(t)^{1-\alpha} - (n+\delta)k(t) \tag{27}$$

Since $g(t) = \pi y(t)$ and $y(t) = Ak(t)^{\alpha} g(t)^{1-\alpha}$, equation (28) will be as follows:

$$g(t) = \pi^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} k(t) \tag{28}$$

Using equations (28) and (27) the following can be written:

$$\frac{dk(t)}{dt} = s(1-\pi)k(t)\pi^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} - (n+\delta)k(t) \tag{29}$$

Hence if $\pi = 0$ then equation (30) shows the investment equation of the Solow model, so there will not be endogenous growth:

$$\frac{dk(t)}{dt} = sy(t) - (n+\delta)k(t) \tag{30}$$

However, if $0 < \pi < 1$, economy will grow at same and positive rate; i.e. there will be endogenous growth. Since

$s(1-\pi)\pi^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} > (n+\delta)$ at steady state economy will grow at same *positive* rate:

$$\frac{dg(t)}{dt}\frac{1}{g(t)} = \frac{dk(t)}{dt}\frac{1}{k(t)} = \frac{dy(t)}{dt}\frac{1}{y(t)} = s(1-\pi)\pi^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} - (n+\delta) \tag{31}$$

As a consequence, under constant returns to scale assumption, equation (31) shows that positive long-run rate of growth is endogenous. Thus, [2] explains endogenous growth assuming constant returns to scale in order to be compatible with steady-state conditions.

3. Conclusion

The aim of this study is to show that there is a contradiction in models of endogenous growth which assume increasing returns to scale and long-run equilibrium or balanced growth. It is shown that, although long-run is compatible with constant returns to scale rather than increasing returns to scale, [14] and [18] explain long-run growth based of increasing returns to scale. On the other hand, positive long-run rate of growth can also explained endogenously based on constant returns to scale as it is explained by [2] and [16]. This study suggests that one should take into account this contradiction while investigating long-run equilibrium or balanced growth. As a consequence, this paper recommends that making an economic analysis on endogenous growth theory should be consistent with concepts of economics.

References

[1] C. Azariadis, A. Drazen. “Threshold Externalities in Economic Development.” The Quarterly Journal of Economics, vol. 105 no. 2, pp. 501-526, 1990.

[2] R. J. Barro. “Government Spending in a Simple Model of Endogeneous Growth.” Journal of Political Economy vol. 98 no. 5, pp. S103-S125, 1990.

[3] G. Cazzavillan, T. Lloyd-Braga, P. A. Pintus. “Multiple Steady States and Endogenous Fluctuations with Increasing Returns to Scale in Production.” Journal of Economic Theory, vol. 80, pp. 60-107, 1998.

[4] J. Conlisk. “Non-Constant Returns to Scale in a Neoclassical Growth Model.” International Economic Review, vol. 9 no. 3, pp. 369-373, 1968.

[5] J. Conlisk. “Nonconstant Returns to Scale and the Technical Change Frontier.” The Quarterly Journal of Economics, vol. 85 no. 3, pp. 483-493, 1971.

[6] A. van den Elzen. Lecture Notes in Economics and Mathematical Systems Volume 402. Berlin: Springer-Verlag, 1993.

[7] F. Etro. “Innovation by Leaders.” The Economic Journal, vol. 114, pp. 281-303, 2004.

[8] G. van der Laan, H. Kremers. “On the existence and computation of an equilibrium in an economy with constant returns to scale production.” Annals of Operations Research, vol. 44, pp. 143-160, 1993.

[9] F. H. Hahn, R. C. O. Matthews. “The Theory of Economic Growth: A Survey.” The Economic Journal, vol. 74 no. 296, pp. 779-90, 1964.

[10] J. Hicks. “The assumption of constant returns to scale.” Cambridge Journal of Economics, vol. 13 no. 1, pp. 9-17, 1989.

[11] K.-I. Inada. “Endogenous Technical Progress and Steady Growth.” The Review of Economic Studies, vol. 36 no. 1, pp. 99-107, 1969.

[12] R. E. Kohn. “Returns to Scale in Welfare Economics.” Atlantic Economic Journal, vol. 11 no. 1, pp. 54-57, 1983.

[13] D. Levhari. “Extensions of Arrow's "Learning by Doing".” The Review of Economic Studies, vol. 33 no. 2, pp. 117-131, 1966.

[14] R. E. Jr. Lucas. “On the Mechanics of Economic Development.” Journal of Monetary Economics, vol. 22, pp. 3-42, 1988.

[15] T. T. Read. “Balanced Growth Without Constant Returns to Scale.” Journal of Mathematical Economics, vol. 15, pp. 171-178, 1990.

[16] S. Rebelo. “Long-Run Policy Analysis and Long-Run Growth.” Journal of Political Economy, vol. 99 no. 3, pp. 500-521,1991.

[17] P. Romer, “Increasing Returns and Long-Run Growth.” Journal of Political Economy, vol. 94 no. 5, pp. 1002-103, 1986.

[18] P. Romer. “Endogenous Technological Change.” Journal of Political Economy, vol. 98 no. 5, pp. 71-102, 1990.

[19] R. M. Solow. “Perspectives on Growth Theory.” The Journal of Economic Perspectives, vol. 8 no. 1, pp. 45-54, 1994.

[20] R. M. Solow, P. A. Samuelson. “Balanced Growth under Constant Returns to Scale Source.” Econometrica, vol. 21 no. 3, pp. 412-424, 1953.

[21] J. S. Zhang. "A Multi-sector Nonlinear Dynamic Input–Output Model with Human Capital." Economic Systems Research, vol. 20 no. 2, pp. 223-237, 2008.