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## **Effect of Akaike Information Criterion on Model Selection in Analyzing Auto-crash Variables**

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### **Abstract**

Count data has become widely available in many disciplines. The mostly used distribution for modeling count data is the Poisson distribution (Horim and Levy; 1981) which assume equidispersion (Variance is equal to the mean). Since observed count data often exhibit over or under dispersion, the Poisson model becomes less ideal for modeling. To deal with a wide range of dispersion levels, Generalized Poisson regression, Poisson regression, and lately Conway-Maxwell-Poisson (COM-Poisson) regression can be used as alternative regression models. We compared the Generalized Poisson regression, Poisson Regression Model and Conway-Maxwell- Poisson. Data on road traffic crashes from the Anambra State Command of the Federal Road Safety Commission (FRSC), Nigeria were analyzed using these three methods, the results from the three methods are compared using the Akaike Information Criterion (AIC) with Poisson showing an AIC value of 2325.8 and GPR having an AIC value of 896.0278 and COM-Poisson showing an AIC value of 951.01. The GPR was considered a better model when analyzing road traffic crashes in Anambra State, Nigeria.

**Keywords:** Over-dispersion; Road Traffic; Crashes; Discrete; Akaike Information Criterion; equidispersion.

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## 1. Introduction

Count data arise in many fields which includes; biology, healthcare, psychology, marketing and many more. When response variable is a count and the researcher is interested in how this count changes as the explanatory variable increases. Classical Poisson regression is the most well-known methods for modeling count data, but its underlying assumption of equidispersion limits its use in many real-world applications with over-or under dispersed data. This excess variation may result to incorrect inference about parameter estimates, standard errors, tests and confidence intervals. Over-dispersion mostly arises for various reasons including mechanisms that generate excessive zero counts or censoring. As a result over-dispersed count data are common in many areas which in turn, have led to the development of statistical methodology for modeling over-dispersed data [1]. For over-dispersed data, the Negative Binomial model is a popular choice [2]. Other over-dispersion models include Poisson mixtures [3] and Conway-Maxwell-Poisson. A flexible alternative that captures both over- and under-dispersion is the Conway-Maxwell-Poisson (COM-Poisson) distribution. The COM-Poisson is a two-parameter generalization of the Poisson distribution which also includes the Bernoulli and Geometric distributions as special cases [4]. The COM-Poisson distribution has been used in so many count data application and has been extended methodologically in various directions [1]. Therefore in this work, because of the problem of model selection and the appropriate method to apply in the analysis of auto-crash data bearing in mind their underlying assumptions, we wish to find the model that is most adequate.

## 2. Methodology

In this section we shall review the models that most widely used in the analysis of count data which include: the Poisson models, Conway- Maxwell- Poisson models, Generalized Poisson Regression model and the Akaike Information Criterion

### 2.1 Poisson Models

This is a special case of Generalized Linear Models (GLM) framework. The simplest distribution used for modeling count data is the Poisson distribution with probability density function

$$P(Y_i = y_i) = \frac{\lambda_i^{y_i} \exp(-\lambda_i)}{y_i!} \quad y_i = 0, 1, 2, \dots \quad (1)$$

The canonical link is  $(\mu) = \log(\mu)$  resulting in a log-linear relationship between mean and linear predictor. The variance in the Poisson model is identical to the mean, thus the dispersion is fixed at  $\phi=1$  and the variance function is  $V(\mu)=\mu$  [5]. The mean Poisson regression can be assumed to follow a log link,  $(Y_i)=\mu_i=\exp(x_i'\beta)$ , where  $x_i$  denotes the vector of explanatory variables and  $\beta$  the vector of regression parameters. The maximum likelihood estimates can be obtained by maximizing the log likelihood.

### 2.2 Conway-Maxwell-Poisson (COM-Poisson) Models

The COM-Poisson distribution is a two parameter generalization of the Poisson distribution that is flexible

enough to describe a wide range of count data distributions, since its revival, it has been further developed in several directions and applied in multiple fields.

The COM-Poisson probability distribution function is given by the equation:

$$P(y; \lambda, \nu) = \frac{\lambda^y}{(y!)^\nu Z(\lambda, \nu)} \tag{2}$$

for a random variable Y, where  $Z(\lambda, \nu) = \sum_{s=0}^{\infty} \frac{\lambda^s}{(s!)^\nu}$ , and  $\nu \geq 0$  is a normalizing constant;  $\nu$  is considered the dispersion parameter such that  $\nu > 1$  represents under-dispersion, and  $\nu < 1$  over-dispersion. The COM-Poisson distribution includes three well-known distribution as special cases: Poisson ( $\nu=1$ ), Geometric ( $\nu=0, \lambda < 1$ ), and Bernoulli ( $\nu \rightarrow \infty$  with  $\frac{\lambda}{1+\lambda}$ ) [6].

Taking a GLM approach, Sellers and Shmueli (2010) [1] proposed a COM-Poisson regression model using the link function,

$$E(Y) = \log \lambda = X' \beta = \beta_0 + \sum_{j=1}^p \beta_j X_j \tag{3}$$

### 2.3 The Generalized Poisson Regression Model

The advantage of using the generalized Poisson regression model is that it can be fitted for both over-dispersion,  $\text{Var}(y_i) > E(y_i)$ , as well as under-dispersion,  $\text{Var}(y_i) < E(y_i)$  (Wang and Famoye (1997)). Suppose is a count response variable that follows a generalized Poisson distribution, the probability density function of  $y_i, i = 1, 2, \dots, n$  is given as (Famoye (1993), Wang and Famoye (1997)) [7].

$$f(y_i) = P(Y_i = y_i) = \left[ \frac{\mu_i}{1 + \alpha \mu_i} \right]^{y_i} \frac{(1 + \alpha y_i)^{y_i - 1}}{y_i!} \exp \left[ \frac{-\mu_i (1 + \alpha y_i)}{(1 + \alpha y_i)} \right], y = 0, 1 \tag{4}$$

### 2.4 Multicollinearity Test

One formal way of detecting multicollinearity is by the use of the variance inflation factors (VIF). The VIF is used to test for the presence of multicollinearity, and is given by

$$\text{VIF} = \frac{1}{1 - R_j^2} \tag{6}$$

Where  $R_j^2$  is the coefficient of determination of a regression of an explanatory variable j on all the other explanators. A VIF value of 10 and above indicates a multicollinearity problem (Wikipedia.org) [8].

Table 1 shows that all the variables have VIF values  $< 10$ . Thus all the variables can be included in the subsequent analyses and modeling with the Poisson regression, Generalized Poisson regression, and Negative Binomial Regression.

**Table 1:** Multicollinearity Test

Model	Collinearity Statistics	
	Tolerance	VIF
1 (Constant)		
NUMBER OF CRASHES	.609	1.643
WEEK	.971	1.030
NUMBER OF CAUSES	.621	1.611

### 2.5 Akaike Information Criterion (AIC)

When several models are available, one can compare the models performance based on several likelihood measures which have been proposed in statistical literatures. One of the most popularly used measures is AIC. The AIC penalized a model with larger number of parameters, and is defined as

$$AIC = -2 \ln L + 2p \tag{7}$$

Where  $\ln L$  denotes the fitted log likelihood and  $p$  the number of parameters. A relatively small value of AIC is favorable for the fitted model [6].

### 3. Analysis and Results

The data were analyzed using R Software and the results obtained are given below. Before performing the analysis on the three methods used, testing the data for multicollinearity was conducted. The test results are shown in table II below:

### 5. Conclusion

Poisson regression model, GPR, and Conway-Maxwell- Poisson regression model were compared to determine a better model used in modeling auto-crashes in Anambra State, Nigeria. The criterion for selection of the best model used is AIC. Best model is the model that has the smallest AIC value.

### 6. Recommendation

Based on AIC values in Table 1, the smallest AIC value is a Generalized Poisson Regression model. Thus, the best model for analyzing traffic crash data or over and under-dispersed data is the Generalized Poisson Regression model.

**Table 2:** Parameter estimates, standard error and AIC value for models

	POISSON		QUASI-POISSON		GPR		COM-POISSON	
	Estimated coefficient	Std Error	Estimated coefficient	Std Error	Estimated coefficient	Std Error	Estimated coefficient	Std Error
intercept	2.907810	0.052759	2.907810	0.228054	2.255499	0.124604	16.9459	6.2129
Number of crashes	0.072249	0.006912	0.072249	0.029879	0.079212	0.027320	2.1853	0.9560
Season (wk of the year)	-0.005722	0.001229	-0.005722	-0.005314	-0.004059	0.004649	-0.1597	0.1509
Number of causes								
AIC	2325.8		-		896.0278		951.01	

**References**

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## Appendix

>y=

```
c(47,23,19,11,9,23,30,71,65,7,55,8,46,76,3,67,10,18,10,30,0,6,61,47,67,35,81,67,46,12,32,11,41,22,44,16,0,12,29,25,31,15,4,5,84,45,61,27,0,35,20,72,44,35,10,33,85,10,42,0,14,11,95,82,45,61,47,70,32,7,18,42,19,20,9,11,9,23,30,71,53,12,0,7,31,0,33,8,7,50,33,5,2,32,3,6,4,4,1,0,85,44,3,16)
```

> x1 =

```
c(5,4,10,6,3,6,1,1,7,7,7,6,2,1,3,3,1,8,3,2,0,10,11,5,1,4,6,4,7,2,5,4,4,2,4,2,0,3,2,5,2,6,1,1,5,6,8,7,0,3,10,6,4,2,2,2,6,2,1,0,3,2,11,10,11,5,10,7,8,8,7,4,5,4,10,6,6,1,1,3,9,4,0,2,4,0,7,2,2,4,2,2,1,7,1,1,1,2,1,0,8,4,1,3)
```

>

x2

=

```
c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52)
```

>

x3

=

```
c(5,4,3,2,3,3,1,1,7,7,7,6,2,1,3,3,1,8,3,2,0,3,3,5,1,4,6,4,7,2,5,4,4,2,4,2,0,3,2,5,2,6,1,1,5,6,8,7,0,3,3,6,4,2,2,2,6,2,1,0,3,2,3,4,5,1,4,2,3,2,3,4,5,4,3,2,2,1,1,3,3,4,0,2,4,0,3,2,2,4,2,2,1,2,1,1,1,2,1,0,2,1,1,3)
```

```
> local({pkg <- select.list(sort(.packages(all.available = TRUE)),graphics=TRUE)
```

```
+ if(nchar(pkg)) library(pkg, character.only=TRUE)})
```

Loading required package: stats4

Loading required package: splines

```
> model = vglm(y~x1+x2+x3, family = genpoisson)
```

Warning message:

In log(theta + x \* lambda) : NaNs produced

```
> summary(model)
```

Call:

```
vglm(formula = y ~ x1 + x2 + x3, family = genpoisson)
```

Pearson residuals:

	Min	1Q	Median	3Q	Max
rhobit(lambda)	-0.6557	-0.5331	-0.2506	0.4613	2.7072
loge(theta)	-4.4189	-0.3801	0.3893	0.8479	0.9771

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept):1	2.255499	0.124604	18.101	< 2e-16 ***
(Intercept):2	0.996075	0.200239	4.974	6.54e-07 ***
x1	0.079212	0.027320	2.899	0.003739 **
x2	-0.004059	0.004649	-0.873	0.382701
x3	0.137679	0.040374	3.410	0.000649 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Number of linear predictors: 2

Names of linear predictors: rhobit(lambda), loge(theta)

Dispersion Parameter for genpoisson family: 1

Log-likelihood: -443.0139 on 203 degrees of freedom

Number of iterations: 13

```
> AIC(model)
```

```
[1] 896.0278
```

```
> model = glm(y~x+x2+x3, family = poisson)
```

```
Error in eval(expr, envir, enclos) : object 'x' not found
```

```
> model = glm(y~x1+x2+x3, family = poisson)
```

```
> summary(model)
```

Call:

```
glm(formula = y ~ x1 + x2 + x3, family = poisson)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-8.100	-3.884	-1.411	2.021	9.565

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	2.907810	0.052759	55.115	< 2e-16 ***
x1	0.072249	0.006912	10.452	< 2e-16 ***
x2	-0.005722	0.001229	-4.654	3.25e-06 ***
x3	0.090689	0.010355	8.758	< 2e-16 ***



---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 2276.6 on 103 degrees of freedom

Residual deviance: 1840.5 on 100 degrees of freedom

AIC: 2325.8

Number of Fisher Scoring iterations: 5

```
> local({pkg <- select.list(sort(.packages(all.available = TRUE)),graphics=TRUE)
```

```
+ if(nchar(pkg)) library(pkg, character.only=TRUE)})
```

```
> model.nb = glm(y~x1+x2+x3)
```

```
> summary(model.nb)
```

Call:

```
glm(formula = y ~ x1 + x2 + x3)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-42.656	-14.817	-7.673	11.592	56.389

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	16.9459	6.2129	2.728	0.00754 **
x1	2.1853	0.9560	2.286	0.02438 *
x2	-0.1597	0.1509	-1.059	0.29233
x3	2.7158	1.4705	1.847	0.06771 .

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 517.7724)

Null deviance: 64857 on 103 degrees of freedom

Residual deviance: 51777 on 100 degrees of freedom

AIC: 951.01

Number of Fisher Scoring iterations: 2

```
> model.compis = glm.compois(y~x1+x2+x3)
```

Error: could not find function "glm.compois"

```
> model.compois = glm.compois(y~x1+x2+x3)
```

Error: could not find function "glm.compois"

```
> model.compois =glm(y~x1+x2+x3)
```

```
> summary(model.compois)
```

Call:

```
glm(formula = y ~ x1 + x2 + x3)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-42.656	-14.817	-7.673	11.592	56.389

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	16.9459	6.2129	2.728	0.00754 **
x1	2.1853	0.9560	2.286	0.02438 *

x2 -0.1597 0.1509 -1.059 0.29233

x3 2.7158 1.4705 1.847 0.06771 .

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 517.7724)

Null deviance: 64857 on 103 degrees of freedom

Residual deviance: 51777 on 100 degrees of freedom

AIC: 951.01

Number of Fisher Scoring iterations: 2

```
> local({pkg <- select.list(sort(.packages(all.available = TRUE)),graphics=TRUE)
```

```
+ if(nchar(pkg)) library(pkg, character.only=TRUE)})
```

```
> local({pkg <- select.list(sort(.packages(all.available = TRUE)),graphics=TRUE)
```

```
+ if(nchar(pkg)) library(pkg, character.only=TRUE)})
```

```
> model.compois =glm(y~x1+x2+x3)
```

```
> summary(model.compois)
```

Call:

```
glm(formula = y ~ x1 + x2 + x3)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-----	----	--------	----	-----

-42.656	-14.817	-7.673	11.592	56.389
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Coefficients:

Estimate	Std. Error	t value	Pr(> t )
----------	------------	---------	----------

(Intercept) 16.9459 6.2129 2.728 0.00754 \*\*

x1 2.1853 0.9560 2.286 0.02438 \*

x2 -0.1597 0.1509 -1.059 0.29233

x3 2.7158 1.4705 1.847 0.06771 .

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 517.7724)

Null deviance: 64857 on 103 degrees of freedom

Residual deviance: 51777 on 100 degrees of freedom

AIC: 951.01

Number of Fisher Scoring iterations: 2