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Using AVMINS and RVSM Models in Studying the Isotopes

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Abstract

Taking the effect of variation of the moment of inertia up to first approximation, the an harmonic variable moment of inertia model "AVMI" is reformulated and denoted "AVMINS " Using AVMINS,RVSM, and VMI models. The ground rotational band for Th isotopes are calculated and compared with the corresponding experimental data. The calculations show that the predicted results are in close agreement with experimental data.

Key words: rotational bands; variable moment of inertia (VMI); angular momentum; softness parameter (σ).

1-Introduction

According Bohr Mottelson the ground state bands of deformed nuclei are written as [1].

$$E(I)=\frac{\hbar^2}{2\theta}(I+1) \qquad (1)$$

$where\ , \theta$ is the moment of inertia, and I is the angular momentum,

Which takes $I^{\pi}=2^{+},4^{+},6^{+},...$

Take into consideration the rotation –vibration effect, equation(1) is modified as

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$$E(I)=\frac{\hbar^2}{2\theta_{\vdots}} I(I+1) \quad +B[I(I+1]^2 \qquad (2)$$

There are many efforts are made to established a formula to fit the experimental data of the yrast band of deformed nuclei. e.g. R.K .Gupta [2] introduced the softness concept of nuclear matter,. Harmonic vibrator model VMI, An harmonic vibrator model AVMI model and etc [3,4,5].

In this article we used the concept of softness of nuclear mater in modifying equation(2) , which is denoted RVSM model and in reformulation of AVMI Model which is denoted AVMINS model. We used the RVSM, AVMINS and VMI models to calculate the ground state energies for Th isotopes. We find that the predicting results of RVSM, AVMINS and VMI models are in close agreement compared with the experimental data.

2. Methods of Calculations:

We carried out the calculations on the isotopes which are Th^{224},Th^{226} ,Th^{228} ,Th^{230},Th^{232} and Th^{234} by using the following two methods :

2.1 AVMINS Model

Consider the nucleus as rigid rotator, the rotational band of deformed nuclei can be written as [1]:

$$E(I) = \frac{\hbar^2}{2\theta} I(I+1) \dots \dots \qquad (2.1)$$

Reference [4] proposed the AVMI model as follow:

$$E(I) = AI + \frac{J(J-2)}{\theta} + \frac{1}{2} C(\theta(I) - \theta_0)^2 \dots \dots \qquad (3)$$

Take the moment of inertia θ , depends on the angular momentum I, and using the Taylor expansion the moment of inertia can be written as:

$$\theta(I) = \theta_0(1 + \sigma_1 I + \sigma_2 I^2 + \sigma_3 I^3 + \cdots) \dots \dots \qquad (4)$$

Where σ_n is the softness parameter,

$$\sigma_n = \frac{1}{n!} \frac{\delta^n \theta(I)}{\delta I^n}\Big|_{J=0} \dots \dots \qquad (5)$$

$$n = 1, 2, 3, \dots$$

Takes only $n = 1$, " the first approximation ",

Then

$$\theta(I) = \theta_0(1 + \sigma_1 I) \dots \dots \qquad (6)$$

Substitute in equation (3), one gets

$$E(I) = AI + \frac{I(I-2)}{\theta_0(1+\sigma_1 I)} + \frac{1}{2}C(\theta_0\sigma_1 I)^2 \dots\dots (7)$$

Equation (7) is denoted **AVMINS** model

Using the experimental excitations energies $E(2)$, $E(4), E(6)$ & $E(8)$ and the equation (7) one can find the parameters σ_1, θ_0, A and C as follows:

$$\sigma_1 = \frac{\left[\frac{9E(2) - 9E(4) + 3E(6)}{3E(4) - E(2) + E(8) - 3E(6)} - 3\right]}{24} \qquad (8)$$

$$\theta_0 = \frac{-48\sigma_1}{[3E(2) - 3E(4) + E(6)][1 + 4\sigma_1][1 + 6\sigma_1]} (9)$$

$$A = \frac{1}{24}\left[\frac{48}{\theta_0(1 + 48\sigma_1)} + 16E(2) - E(8)\right] \qquad (10)$$

$$C = \frac{1}{2}\left[\frac{E(2) - 2A}{(\theta_0\sigma_1)^2}\right] \dots \qquad (11)$$

2.2 RVSM Model

Take $\theta(I)$ up to first approximation" Equation (6)" and substituting in equation (2). One gets:

$$E(I) = AI(I+1) + BI^2[I(I+!] + C\lfloor I(I+1)\rfloor^2 \quad (12)$$

Where $A = \frac{\hbar^2}{2\theta_0}$, $B = Ao_1$ *and C are fitting paramters*

VMI model

The variable moment of inertia [5] is written as

$$E(I) = \frac{\hbar^2}{2\theta_0} I(I+1) + \frac{1}{2}C(\theta(I) - \theta_0)^2 \quad . \qquad (13)$$

Where, $\frac{\hbar^2}{2\theta_0}$ and C are fitting parameters and the variable moment of inertia $\theta(I)$ is determined through use of the variation condition.

$$\frac{\partial E(I)}{\partial \theta}\Big|_I = 0$$

3. Results and Discussion

Using the experimental excitation energies E (2), E (4), E (6) &E (8) and equations (8, 9,10, &11), the parameters σ_1, θ_0, A and C are calculated and are listed in table (1-a) Using equation (7) "**AVMINS** model "and the given parameters σ_1, θ_0, A & C , we calculated the ground energies for the isotopes Th^{224},Th^{226},Th^{228},Th^{230},Th^{232} and Th^{234}.The predicted results are tabulated in table(2)

The parameters A,B and C of RVSM model as equation (12) are given by using the least square fitting method for the isotopes Th^{224},Th^{226},Th^{228},Th^{230},Th^{232} and Th^{234}. and are tabulated in table (1-b).The predicted energies of the ground band for these nuclei are listed in table (2).

Also, the VMI model was used .The predicted results of VMI model are " taken from ref [6] " also. tabulated in Table (2).

The deviation from experimental data is given according to his equation

$Dev = \frac{1}{N}\sum_{i=1}^{N}\left(E_{Cal} - E_{exp}\right)$. and is written in the last row in table (2) .

4. Conclusion

From table (2) it is clear that the three models AVMINS, RVSM and VMI described well the ground band of the considered The isotopes. This means that the three models are nearly equivalent.

Table 1-a: The parameters of the AVMINS model as equation (6)

Nucleus	*A*	θ_o	σ	*C*
Th^{224}	*3.063211E-02*	10928.74	-.1936715	4.111198E-09
Th^{226}	1.804268E-02	22464.99	-.1823517	1.076023E-09
Th^{228}	1.252299E-02	44281.03	-.1731955	2.775862E-10
Th^{230}	1.071106E-02	193208	-.1702922	1.467761E-11
Th^{232}	9.662266E-03	1220916	-.167304	3.592128E-13
Th^{234}	9.466416E-03	1137149	-.1675604	4.209666E-13

Table 1-b: The parameters of the RVSM model as Equation (12)

Nucleus	*A*	*B*	*C*
Th^{224}	1.639815E-02	-7.341633E-04	1.521334E-05
Th^{226}	1.271513E-02	3.89517E-04	5.837017E-06
Th^{228}	1.029557E-02	-2.300387E-04	2.581105E-06
Th^{230}	9.294759E-03	-1.353166E-04	4.415862E-07
Th^{232}	8.483634E-03	-8.325296E-05	-1.2292E-06
Th^{234}	8.306866E-03	-2.2271E-05	-3.857183E-06

Table 2: Experimental (E) and predicted (Ep) energies in (Mev) using AVMINS ,RVSM and VMI models

Th^{224}				
I^+	E	Ep AVMINS	Ep RVSM	Ep VMI
2	0.0981	0.0981	0.0981	0.1065
4	0.2841	0.2841	0.2841	0.3109
6	0.5347	0.5347	0.5347	0.5807
8	0.8339	0.8339	0.8339	0.9000
10	1.1738	1.1738	1.1738	1.2598
12	1.5498	1.5498	1.5498	1.6540
14	1.9589	1.9589	1.9589	2.0783
16	2.398	2.398	2.398	2.3980
18	2.864	2.864	2.864	2.8640
Dev		-0.16623	1.494964E-03	0.0508
Th^{226}				
I^+	E	Ep **AVMINS**	Ep RVSM	Ep VMI
2	0.0722	0.0722	0.0722	0.0745
4	0.2264	0.2179453	0.22643	0.2347
6	0.4473	0.4219359	0.4473	0.4628
8	0.7219	0.7175187	0.7219	0.7455
10	1.0403	1.078968	1.0403	1.0736
12	1.3952	1.512143	1.3952	1.4406
14	1.7815	2.017399	1.7815	1.8419
16	2.1958	2.59482	2.1958	2.2740
18	2.6351	3.244437	2.6351	2.6351
20	3.0971	3.966263	3.0971	3.0971
Dev		0.22308	1.627676E-04	0.0318
Th^{228}				
I^+	E	Ep **AVMINS**	Ep RVSM	Ep VMI
2	0.0577	0.0577	0.057759	0.0585
4	0.1868	0.1812961	0.186823	0.1898
6	0.3717	0.3551883	0.378179	0.3844
8	0.6225	0.6198368	0.6225	0.6333
10	0.9118	0.9391121	0.9118	0.9291
12	1.2394	1.323307	1.2394	1.2659
14	1.5995	1.772706	1.5995	1.6392
16	1.9881	2.287369	1.9881	1.9880

18	2.4079	2.867318	2.4079	2.4079
Dev		-0.11316	-2.787622E-04	0.0158

Th^{230}				
I^+	E	Ep **AVMINS**	Ep RVSM	Ep VMI
2	0.0532	0.0532	0.0532	0.0522 a
4	0.1741	0.1700856	0.1741	0.1720
6	0.3566	0.3445569	0.3566	0.3547
8	0.5941	0.593449	0.5941	0.5943
10	0.8797	0.9009687	0.8797	0.8834
12	1.2078	1.271941	1.2078	1.2151
14	1.5729	1.706443	1.5729	1.5827
16	1.9715	2.204489	1.9715	1.9809
18	2.3978	2.766086	2.3978	2.4055
Dev		8.93E-02	1.785308E-04	0.00368
Th^{232}				
I^+	E	Ep **AVMINS**	Ep RVSM	Ep VMI
2	0.0493	0.0493	0.049369	0.0498
4	0.1621	0.1585708	0.16212	0.1633
6	0.3332	0.3226123	0.3332	0.3351
8	0.5569	0.5567895	0.5569	0.5592
10	0.827	0.845912	0.827	0.8299
12	1.1371	1.194967	1.1371	1.1421
14	1.4828	1.603967	1.4828	1.4920
Dev		2.62E-02	8.20245E-05	0.0033
Th^{234}				
I^+	E	Ep **AVMINS**	Ep RVSM	Ep VMI
2	0.0495	0.0495	0.04955	0.0498
4	0.163	0.1601557	0.163	0.1642
6	0.3365	0.327967	0.3365	0.3394
8	0.5648	0.5646821	0.5648	0.5709
10	0.843	0.8587392	0.843	0.8430
12	1.1602	1.213911	1.1602	1.1602
Dev		9.66E-03	2.716978E-05	0.0059

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