Hall Effect and Temperature Distribution on Unsteady Micropolar Fluid Flow in a Moving Walls

Mekonnen Shiferaw Ayano[a]*, Negussie Tadege Demeke[b]

[a]Department of Mathematics, University of Swaziland, Swaziland

[b]Mechanical Engineering Department, Arba Minch university, Ethiopia

[a]Email: mekk_aya@yahoo.com or msayano@uniswa.sz

[b]Email: negtag57@yahoo.com

Abstract

Unsteady incompressible electrically conducting micropolar fluid between parallel plates where one plate is moving with constant velocity in the presence of transverse magnetic field studied. The effect of hall is taken in to consideration and it is assumed that the flow is also due to decaying pressure gradient with walls of the channel have different temperatures. The profile of velocities, microrotation and temperature studied numerically for various parameters Hartmann number, hall parameter and coupling number. The flow is simulated and the results are discussed graphically for various parameters influencing the flow.

Keywords: unsteady; micropolar fluid; Hall effect; migenatohydrodynamic.

1. Introduction

In the last several years considerable attention has been given to the study of the hydrodynamic thermal convection due to its important practical applications such as artificial fibers, the aerodynamic extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the boundary layer along a liquid film in condensation processes paper production etc.

----

* Corresponding author.

Several authors including [1-6] have investigated analytically and numerically the effects of a transverse uniform magnetic field on the flow of aviscous incompressible electrically conducting fluid between two infinite parallel plates.

The studies extended for Newtonian and diverse non-Newtonian fluids when the case one of the plates or both moving with uniform velocity have studied by may authors to mention few, Hassanien [7] studied two-dimensional unsteady viscous flow through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field when one of the plates is oscillating in its own plane. unsteady unidirectional flows of a fluid of second grade with sudden application of a constant pressure gradient or by the impulsive motion of one or two boundaries examined by Erdogan [8]. Further, Erdogan [9] studied unsteady flow over a plane walls due to one rigid boundary moved suddenly and one being free. Gupta [10] studied analytically hydromagnetic flow between two parallel non-conducting infinite planes one oscillating and the other fixed. Attia [11] studied effect of hall current on the velocity and temperature distributions of Couette flow with variable properties and uniform suction and injection when the fluid is acted upon by an exponential decaying pressure gradient and an external uniform magnetic field, also Attia [12] studied the problem of the unsteady state with heat transfer for fixed boundaries. Reference [13] numerically studied magnetohydrodynamic flow past an infinite continuous moving surface embedded in porous medium.

The theory of micropolar fluid formulated by Eringen[14] can be used to analyze the behavior of many of the fluids involved in technical processes and engineering applications, polymers, flows of exotic lubricants, animal bloods and real fluids with suspensions. An excellent review of micropolar fluids and their applications was given by [15]. Heat transfer in micropolar fluids is also important in various applications.

In this paper it is considered magnetohydrodynamic unsteady heat transfer flow of microplar fluid when one of the plates moving. The resulting governing equations are solved numerically to obtain the local similarity solutions. Graphical results for non-dimensional velocity, microrotation and temperature profiles presented for values of the parameters characterizing the flow.

2. Mathematical formulation and non-dimensionalization

2.1. Formulation of the problem

Consider an incompressible micropolar fluid flowing between two parallel plates distance h apart. Let x and y axes be chosen along and perpendicular to the walls respectively. Let the two plates are kept at two constant temperatures, T_1 for the lower plate and T_2 for the upper plate, with $T_2 > T_1$ .The motion is assume due to the constant velocity, U_0, in the lower plate and an exponential decaying pressure gradient in the x-direction where initially both the fluid and plates are at rest. At time t = 0 a uniform suction from above and injection from below, with velocity v_0 , are applied. Also magnetic field of uniform strength B_0 is applied perpendicular to the plate in the positive y-direction. The magnetic Reynolds number is taken to be very small enough so that the induced magnetic field can be neglected.

The current density given by the generalized equation of Ohm's law when the hall term is retained will have the

form:

$$\vec{J} = \sigma \left[\vec{q} X \vec{B} + \eta(\vec{J} X \vec{B}) \right] \tag{1}$$

Within the framework of the above-noted assumptions, the appropriate conservation equations can be described by the following equations:

$$\rho \frac{\partial u}{\partial t} + \rho \, v_0 \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + (\mu + \kappa) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial v_2}{\partial y} - \frac{\sigma B_0^2}{1 + B_h}(u + B_h w) \tag{2}$$

$$\rho \frac{\partial w}{\partial t} + \rho \, v_0 \frac{\partial w}{\partial y} = (\mu + \kappa) \frac{\partial^2 w}{\partial y^2} + \kappa \frac{\partial v_1}{\partial y} - \frac{\sigma B_0^2}{1 + B_h}(w - B_h u) \tag{3}$$

$$\rho \, j \frac{\partial v_1}{\partial t} + \rho \, j \, v_0 \frac{\partial v_1}{\partial y} = -2\kappa v_1 - \kappa \frac{\partial w}{\partial y} + \gamma \frac{\partial^2 v_1}{\partial y^2} \tag{4}$$

$$\rho \, j \frac{\partial v_2}{\partial t} + \rho \, j \, v_0 \frac{\partial v_2}{\partial y} = -2\kappa v_2 - \kappa \frac{\partial u}{\partial y} + \gamma \frac{\partial^2 v_2}{\partial y^2} \tag{5}$$

$$\rho \, C_p \frac{\partial T}{\partial t} + \rho \, v_0 \frac{\partial T}{\partial y} = k_f \frac{\partial^2 T}{\partial y^2} + (2\mu + \kappa) \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] + \gamma \left[\left(\frac{\partial v_1}{\partial y} \right)^2 + \left(\frac{\partial v_2}{\partial y} \right)^2 \right]$$

$$+ \frac{\kappa}{2} \left[\left(\frac{\partial w}{\partial y} - v_1 \right)^2 + \left(\frac{\partial u}{\partial y} + v_2 \right)^2 \right] + \frac{\sigma B_0^2}{1 + B_h}(u^2 + w^2) \tag{6}$$

Initial conditions:

$$u(y, 0) = w(y, 0) = v_1(y, 0) = v_2(y, 0) = T(y, 0) = 0 \tag{7}$$

the boundary conditions:

$$u(0, t) = w(0, t) = v_1(0, y) = v_2(0, t) = 0, \quad T(0, t) = T_1, \qquad t > 0 \tag{8}$$

$$u(h, t) = w(h, t) = v_1(h, y) = v_2(h, t) = 0, \quad T(h, t) = T_2, \qquad t > 0 \tag{9}$$

where ρ is density, u and w are fluid velocity components of $\vec{q}$, v_1 and v_2 microrotation components, μ the dynamic viscosity, κ the gyroviscosity, σ the electrical conductivity of the fluid, $\vec{J}$ is current density, η is hall factor, B_0 magnetic induction, $B_h = \sigma \eta B_0$ is a hall parameter, j the microinertia, γ is material constant, C_p the specific heat, κ_f is thermal conductivity of the fluid.

The exponentially decaying pressure gradient is assumed of the form, $\frac{dP}{dx} = \Gamma e^{-\alpha t}$, where Γ and α are constants.

2.2. Non-dimensionalization

Introduce the non- dimensional variables through

$$\hat{u} = \frac{u}{U_0}, \qquad \hat{w} = \frac{w}{U_0}, \qquad \widehat{v_1} = \frac{v_1 h}{U_0}, \qquad \widehat{v_1} = \frac{v_2 h}{U_0},$$

$$\theta = \frac{T - T_1}{T_2 - T_1}, \qquad \hat{t} = \frac{tU_0}{h}, \hat{y} = \frac{y}{h}, \qquad \hat{\Gamma} = \frac{h\Gamma}{\rho U_0^2} \qquad (10)$$

Substituting equation (10) into equations (2-6) after dropping the hat we get the following non-dimensional equations

$$\frac{\partial u}{\partial t} + \frac{R}{Re}\frac{\partial u}{\partial y} = \Gamma e^{-\alpha t} + \frac{1}{Re\,(1-N)}\frac{\partial^2 u}{\partial y^2} + \frac{N}{Re(1-N)}\frac{\partial v_2}{\partial y} - \frac{H_a^2}{Re(1+B_h)}(u + B_h w) \qquad (11)$$

$$\frac{\partial w}{\partial t} + \frac{R}{Re}\frac{\partial w}{\partial y} = \frac{1}{Re\,(1-N)}\frac{\partial^2 u}{\partial y^2} + \frac{N}{Re(1-N)}\frac{\partial v_1}{\partial y} - \frac{H_a^2}{Re(1+B_h)}(w - B_h u) \qquad (12)$$

$$\frac{\partial v_1}{\partial t} + \frac{R}{Re}\frac{\partial v_1}{\partial y} = \frac{N}{Rea_j(1-N)}\left(-2v_1 - \frac{\partial w}{\partial y}\right) + \frac{2-N}{M^2\,Rea_j}\frac{\partial^2 v_1}{\partial y^2} \qquad (13)$$

$$\frac{\partial v_2}{\partial t} + \frac{R}{Re}\frac{\partial v_2}{\partial y} = \frac{N}{Rea_j(1-N)}\left(-2v_2 - \frac{\partial u}{\partial y}\right) + \frac{2-N}{M^2\,Rea_j}\frac{\partial\,v_2}{\partial y^2} \qquad (14)$$

$$\frac{\partial \theta}{\partial t} + \frac{R}{Re}\frac{\partial T}{\partial y} = \frac{1}{Pr\,Re}\frac{\partial^2 \theta}{\partial y^2} + \frac{2-N}{1\,N}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{(2-N)ReEc}{M^2}\left[\left(\frac{\partial v_1}{\partial y}\right)^2 + \left(\frac{\partial v_2}{\partial y}\right)^2\right]$$

$$+ \frac{NReEc}{1-N}\left[\left(\frac{\partial w}{\partial y} - v_1\right)^2 + \left(\frac{\partial u}{\partial y} + v_2\right)^2\right] + \frac{H_a^2}{Re(1+B_h)}(u^2 + w^2) \qquad (15)$$

Where $R = \frac{\rho\,v_0\,h}{\mu}$(suction parameter), $Re = \frac{\rho\,U_0\,h}{\mu}$ (Reynolds number), $N = \frac{\kappa}{\mu}$ (coupling number), $H_a^2 = \frac{\sigma\,B_0^2 h^2}{\mu}$ (Hartmann number), $a_j = \frac{j}{h^2}$ (micro-inertia density parameter), $M = \frac{h^2\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}$ (micropolarparameter), $pr = \frac{\mu\,C_p}{k_f}$ (Prandtl number), $Ec = \frac{\mu^2}{\rho^2 C_p h^2(T_2-T_1)}$ (Eckert number).

Initial conditions:

$$u(y, 0) = w(y, 0) = v_1(y, 0) = v_2(y, 0) = \theta(y, 0) = 0 \qquad (16)$$

the boundary conditions:

$$u(0, t) = w(0, t) = v_1(0, y) = v_2(0, t) = 0, \quad \theta(0, t) = 0, \;\; t > 0 \qquad (17)$$

$$u(1, t) = 1, \quad w(1, t) = v_1(1, y) = v_2(1, t) = 0, \quad \theta(1, t) = 1, \;\; t > 0 \qquad (18)$$

3. Results and discussion

To study the velocity, microrotation and the temperature we fix the parameters pr = 0.72, Re=1, Ec = 0.4, Γ = 2, α = 2, R= -2. Figure 1 and figure 2 shows the profile of velocities components, microrotations and temperature

for N=0.5, H_a = 2 and B_h= 2.

It can be clearly observed from figure 1 the velocity *u* increases towards the lower plate as it is expected. The velocity increase due to the influence of the decaying pressure gradient with time up till a maximum value and then constant due to the motion of the lower plate. Figure2 shows the velocity component *w* this component velocity is due to the assumption $B_h \neq 0$ otherwise vanishes as it can be seen from figure 8. The microrotation component v_1 shows the nature of the micropolar fluid, that is the symmetric effect with the center Eringen [13].

Figure 1: The profile of velocity(*u*) for   N =0.5,Ha = 2 and B_h = 20.5

Figure 2: The profile of velocity (*w*) for N =0.5,Ha = 2 and B_h = 2
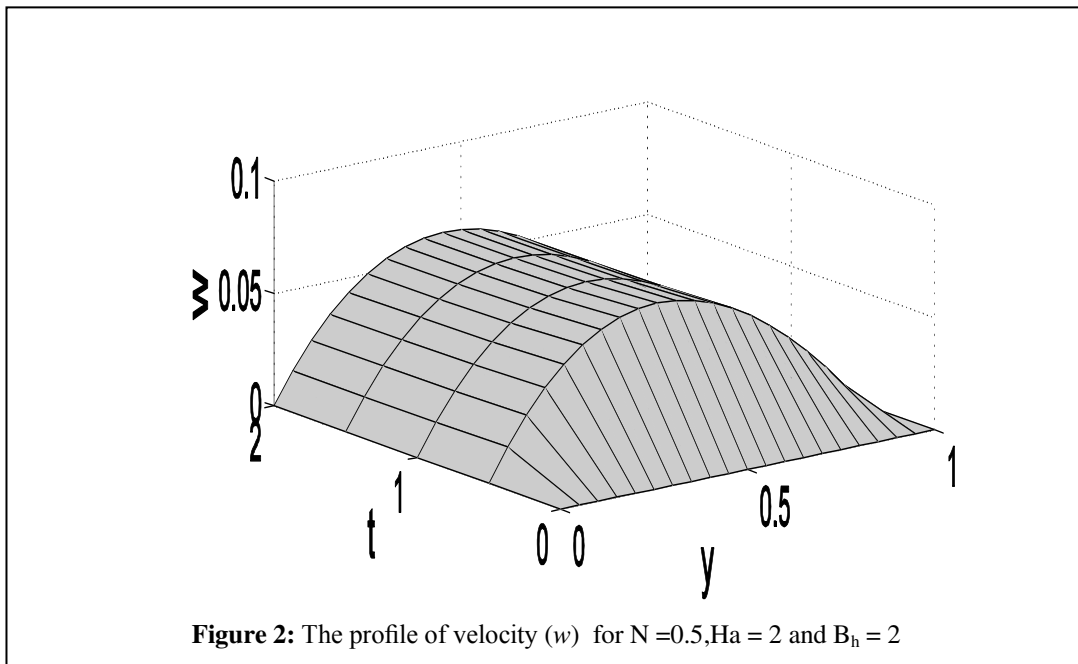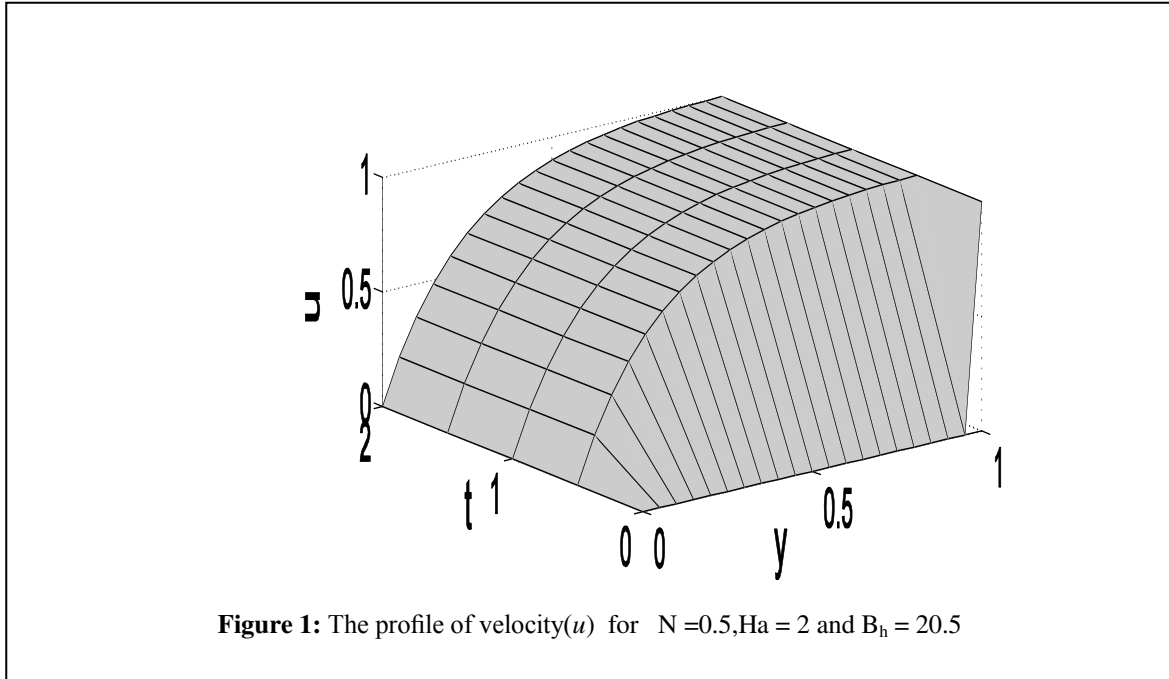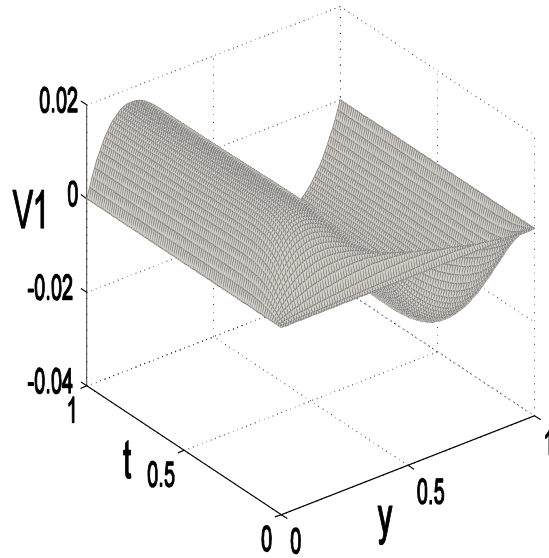
Figure 3: The profile of microrotation (v_1) for N =0.5,Ha = 2 and B_h = 2

In figure 4 it can be seen the profile of microrotation component v_2 through time. Figure 5 shows the distribution of temperature from its maximum at the lower boundary. Figure 6 shows the distribution of temperature at different time levels and after sufficiently long time the whole domain will be filled homogeneously.
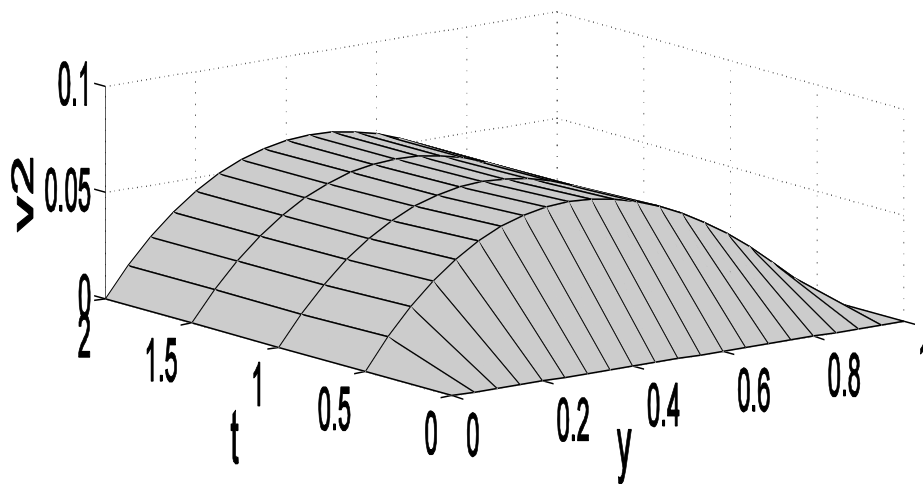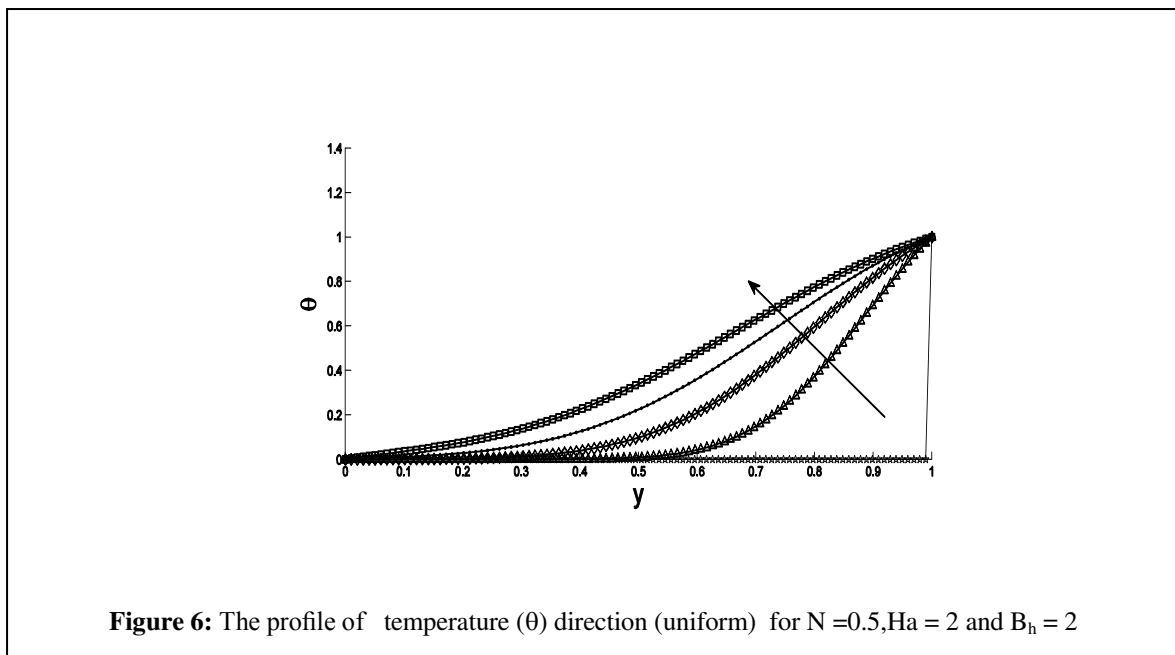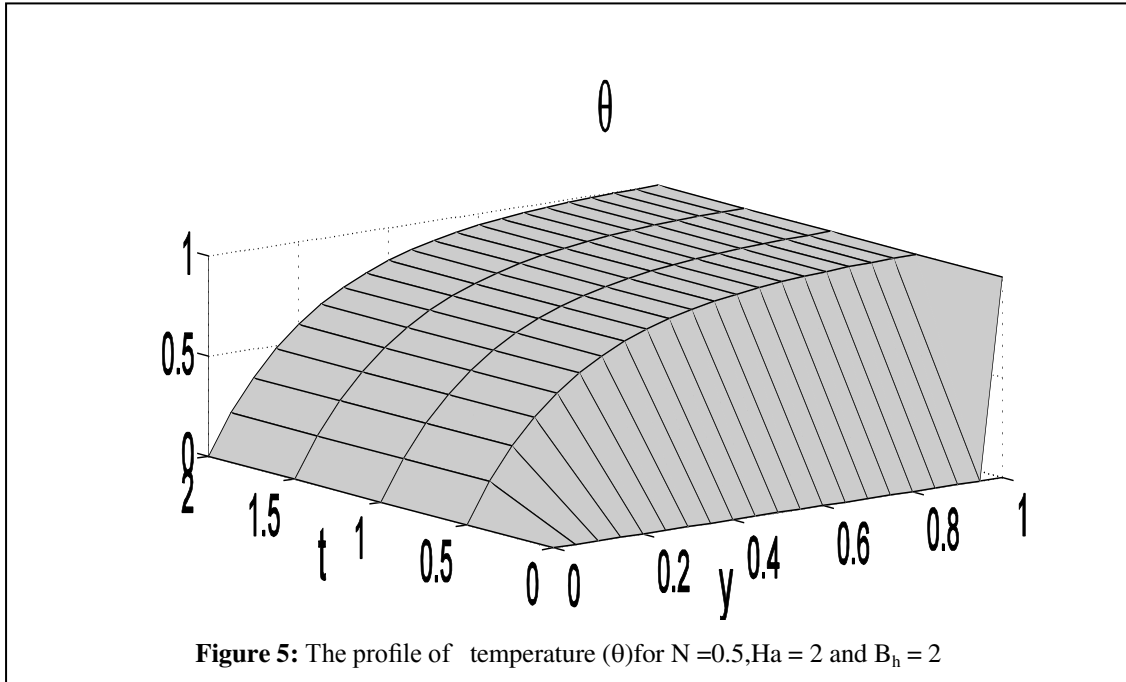

Figure 4: The profile of microrotation (v_2)for N =0.5,Ha = 2 and B_h = 2

Figure 5: The profile of temperature (θ)for N =0.5,Ha = 2 and B$_h$ = 2

Figure 6: The profile of temperature (θ) direction (uniform) for N =0.5,Ha = 2 and B$_h$ = 2

It is seen from figures7 that with increase in Hartmann number the variation of distribution of velocity *u* decreases whereas the induced velocity component*w* increases and also showing that when the Hartmann number is zero this component of velocity vanishes as depicted in figure 8. Figure 9 shows the effect of Hartmann number on the micrortation component, increasing the value of *Ha* will increase in microrotationv_1 showing the reverse flow with symmetric effect about the center. The effect of Ha on microrotation $v_2$shows it pushed towards the upper plate as the value of *Ha* increases.The temperature has an increasing effect as it can be seen from figure 11 when Ha increased.
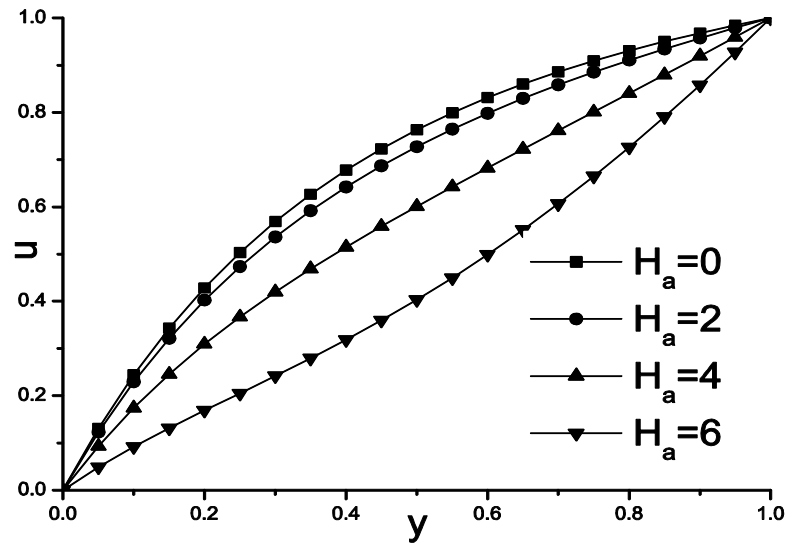
Figure 7: Effect of Hartman numbers (Ha) on velocity (u), for N = 0.5, t = 2 and B_h = 2
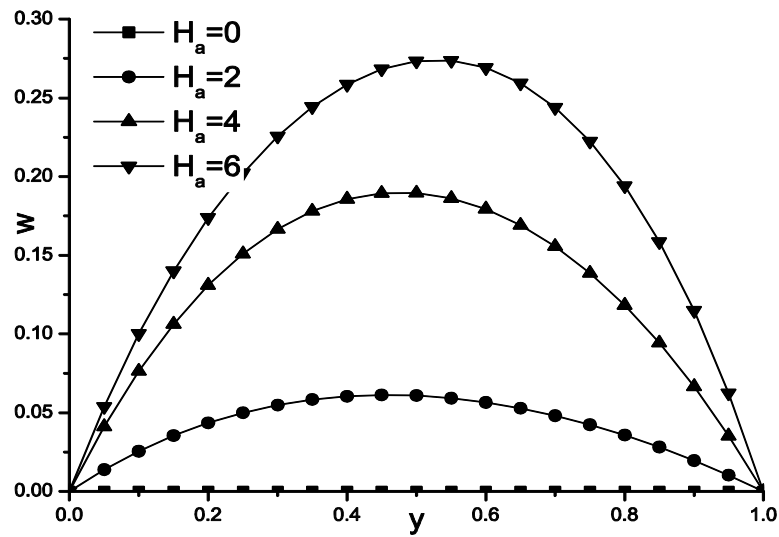

Figure 8: Effect of Hartman numbers (Ha) on induced velocity (w), for N = 0.5, t = 2 and B_h = 2
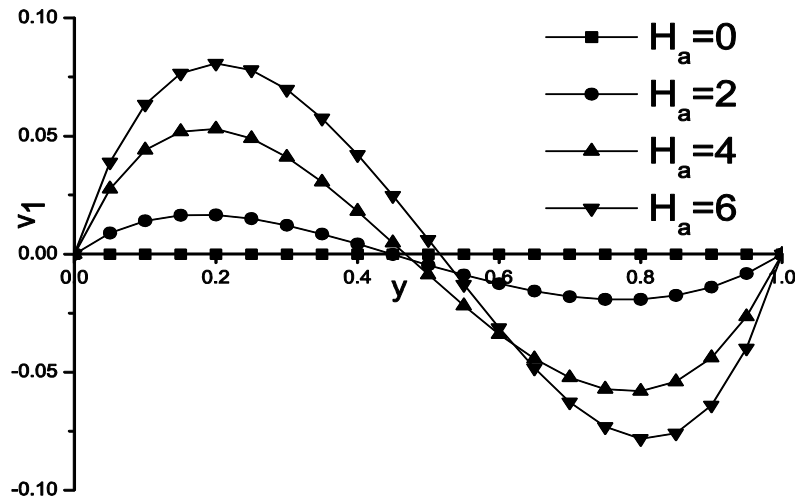
Figure 9: Effect of Hartman numbers (*Ha*)on microrotation (v_1), for N = 0.5, t = 2 and B_h = 2
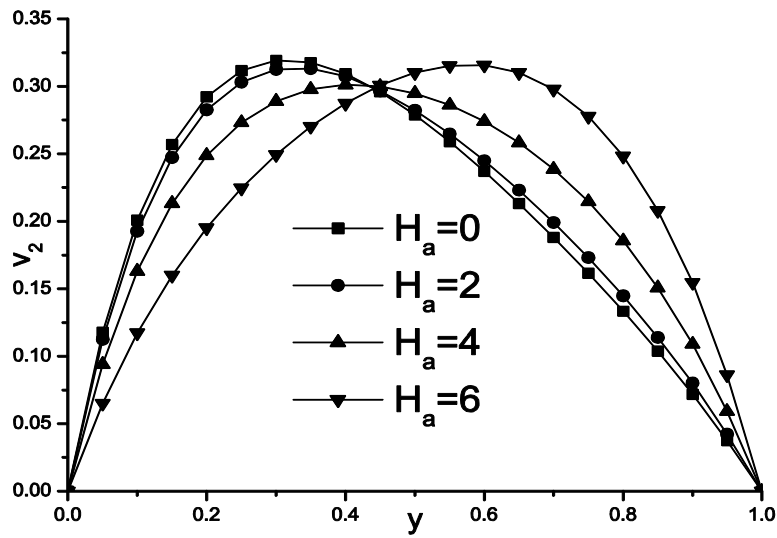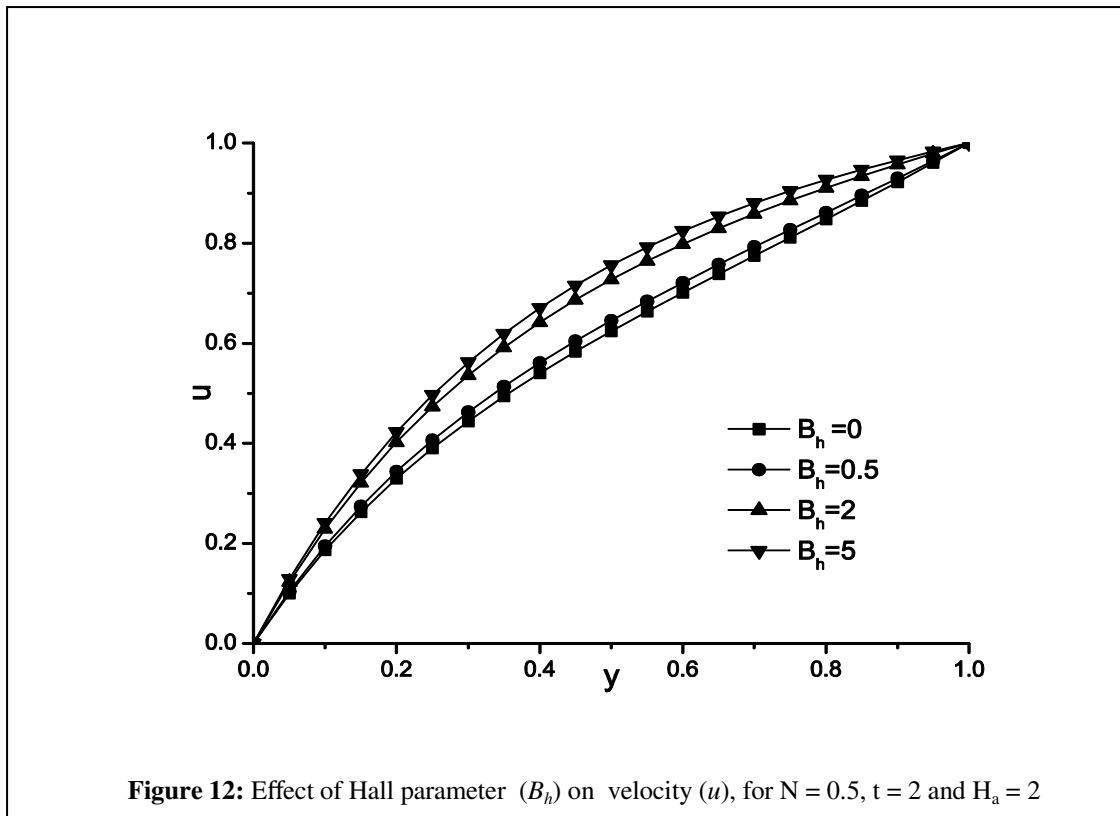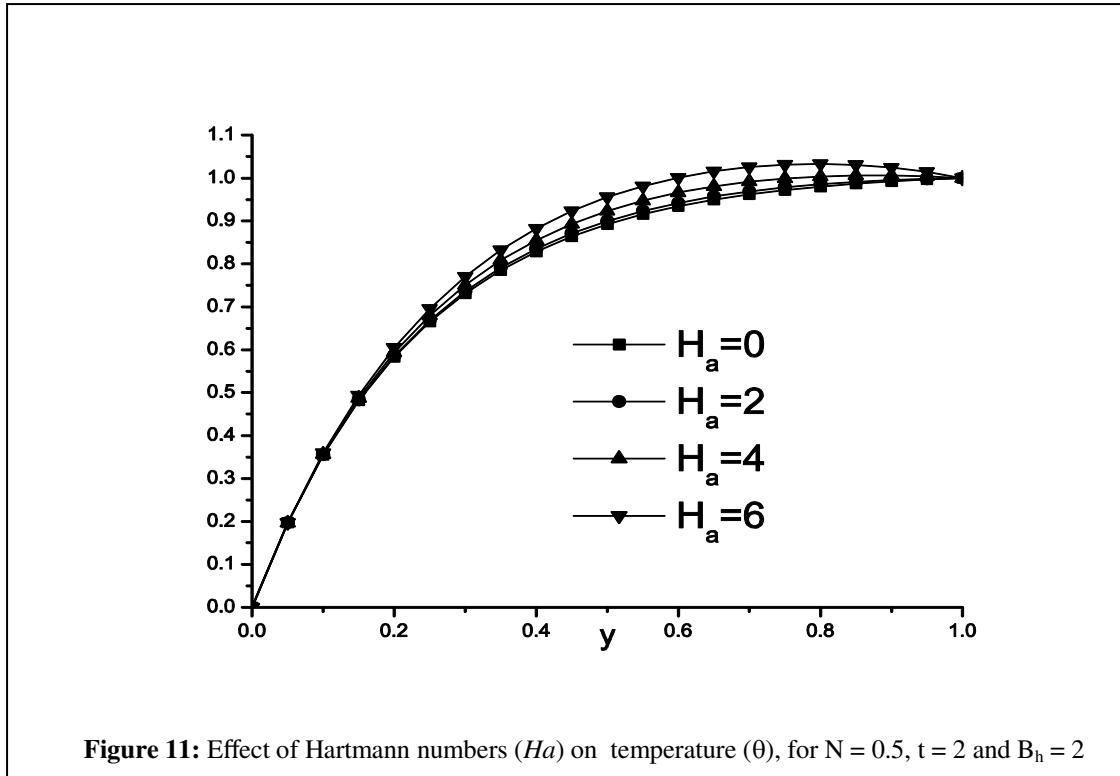
Figure 10: Effect of Hartman numbers (*Ha*) on induced velocity (*w*), for N = 0.5, t = 2 and B_h = 2

Figure 11: Effect of Hartmann numbers (*Ha*) on temperature (θ), for N = 0.5, t = 2 and B_h = 2

Figure 12: Effect of Hall parameter (*B_h*) on velocity (*u*), for N = 0.5, t = 2 and H_a = 2
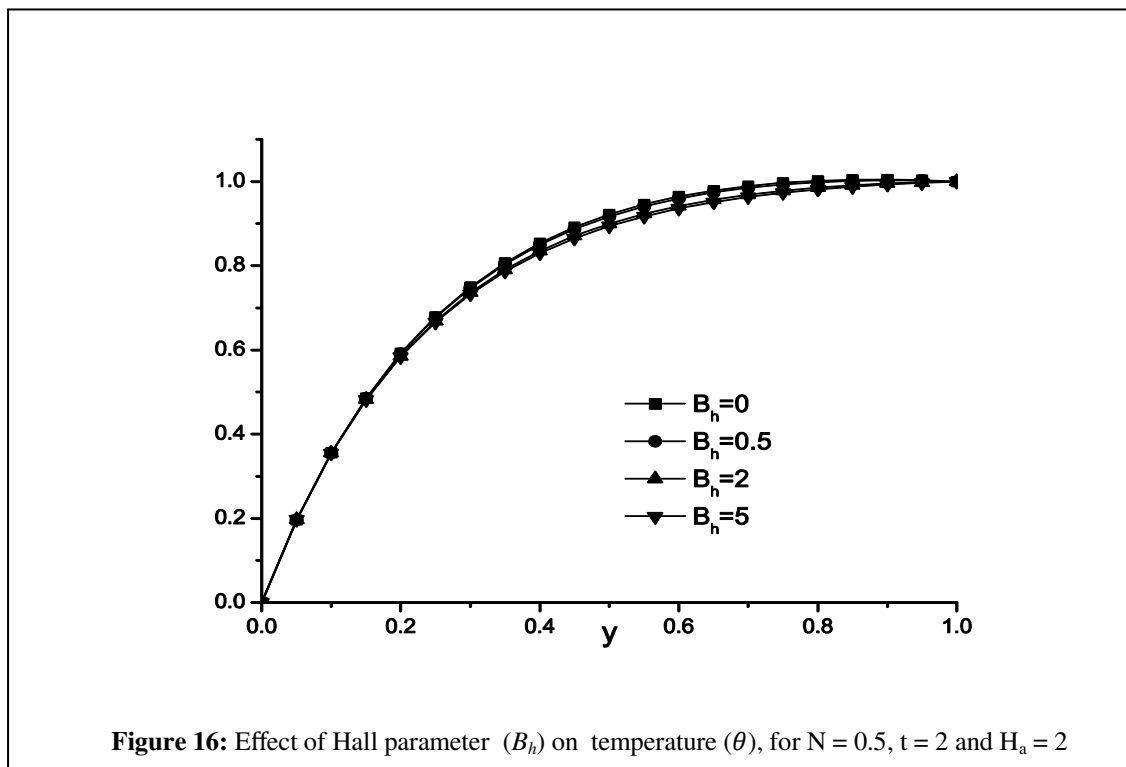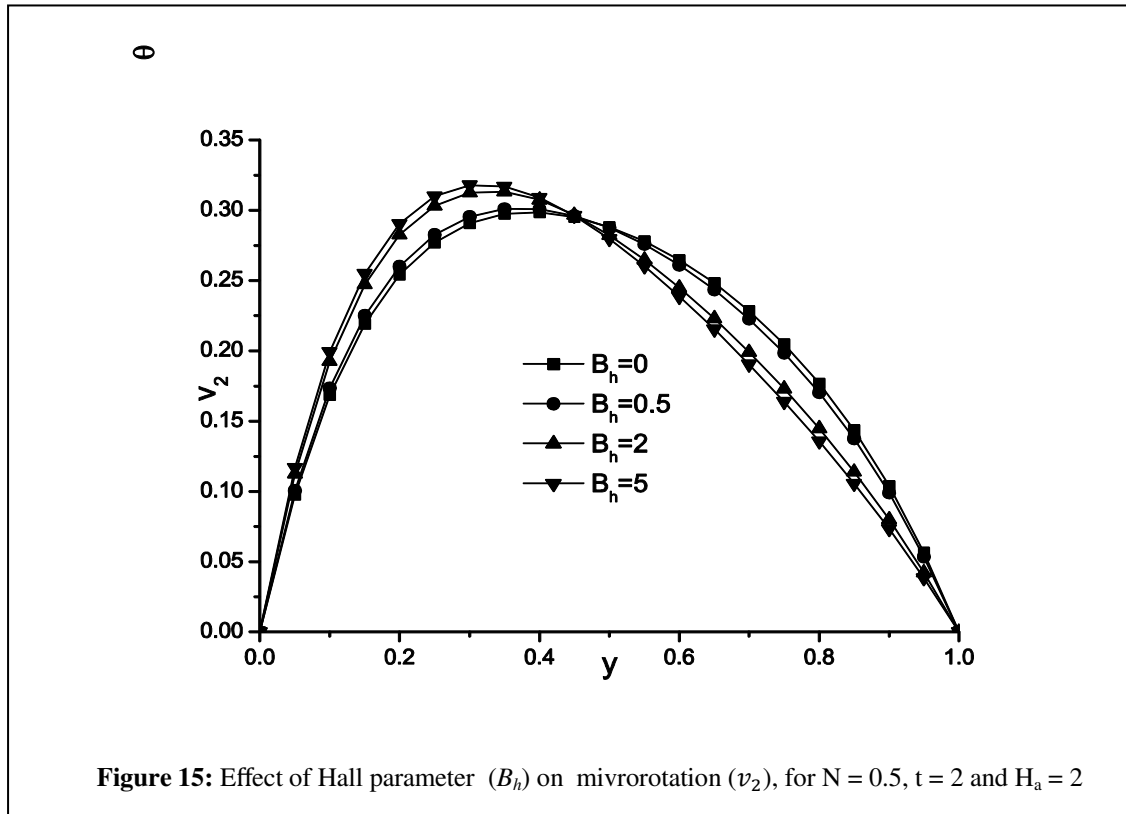
The hall parameter, *B_h*, effect on velocity components shown on figure 12 and figure 13, it is seen that by increasing B_h the flow can be enhanced. However, the effect of B_h on velocity component *w* shown in figure 13

with increase in B_h the velocity w decreases when $0 < B_h < 1$ and further showing that if $B_h = 0$ no flow in this direction i.e $w = 0$.
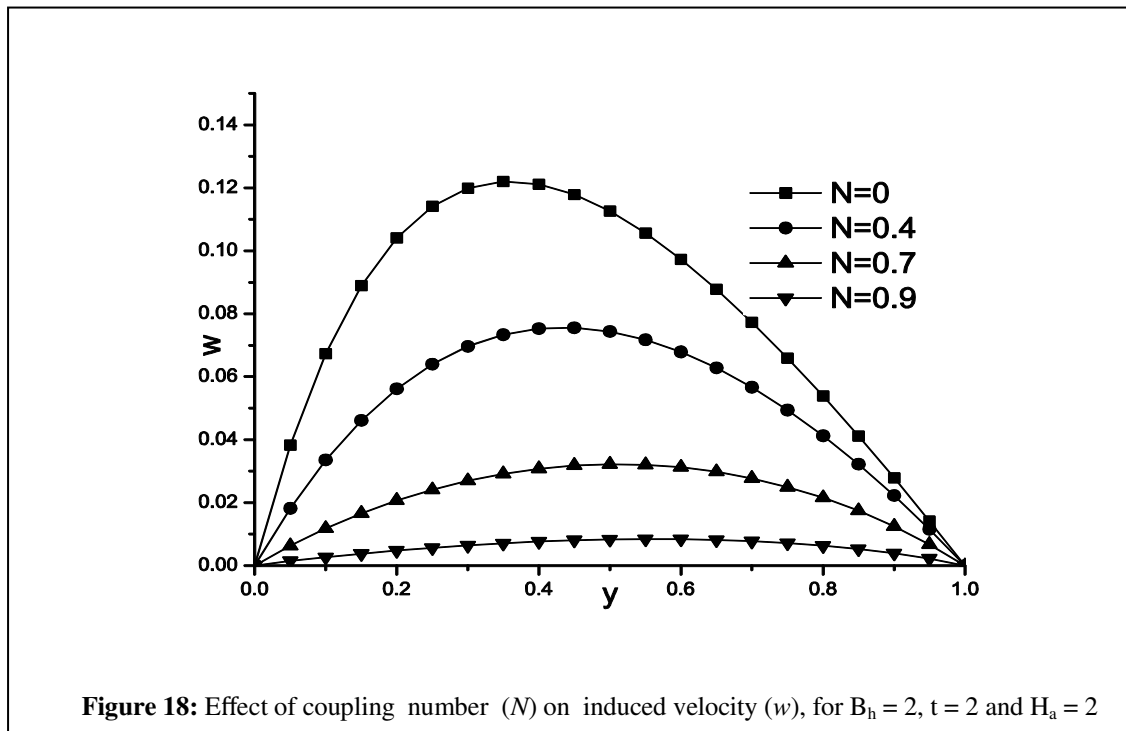

Figure 13: Effect of Hall parameter (B_h) on velocity (w), for N = 0.5, t = 2 and H_a = 2

Figure 14: Effect of Hall parameter (B_h) on mivrorotation (v_1) for N = 0.5, t = 2 and H_a = 2
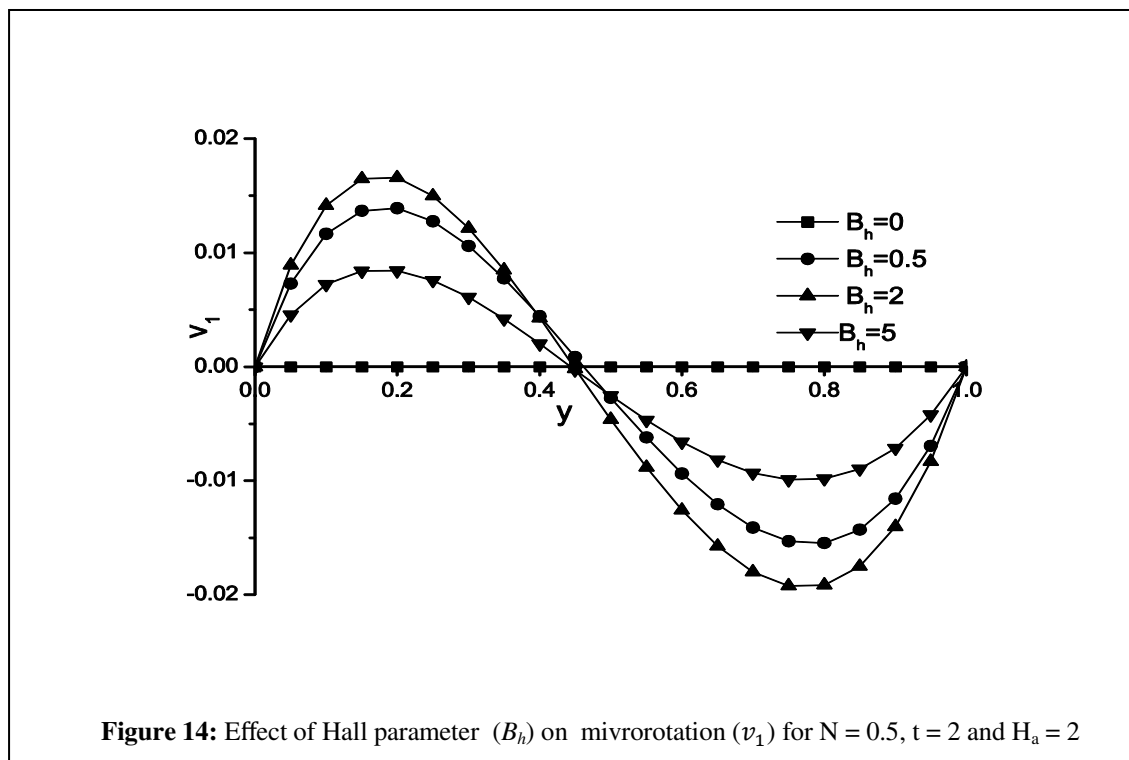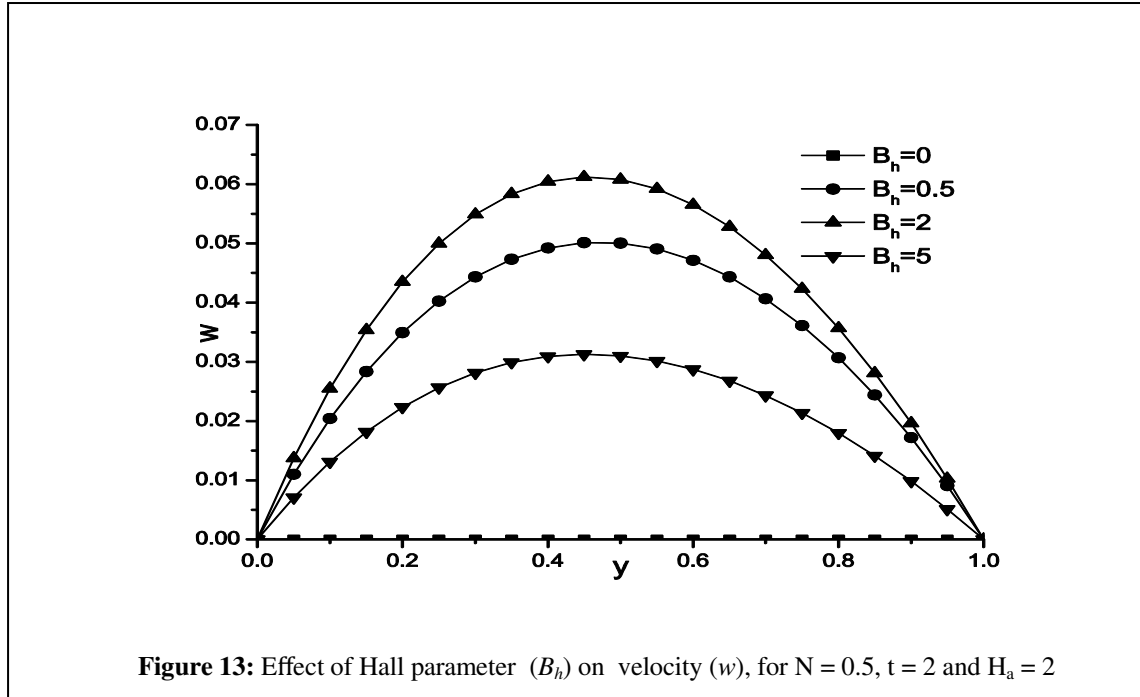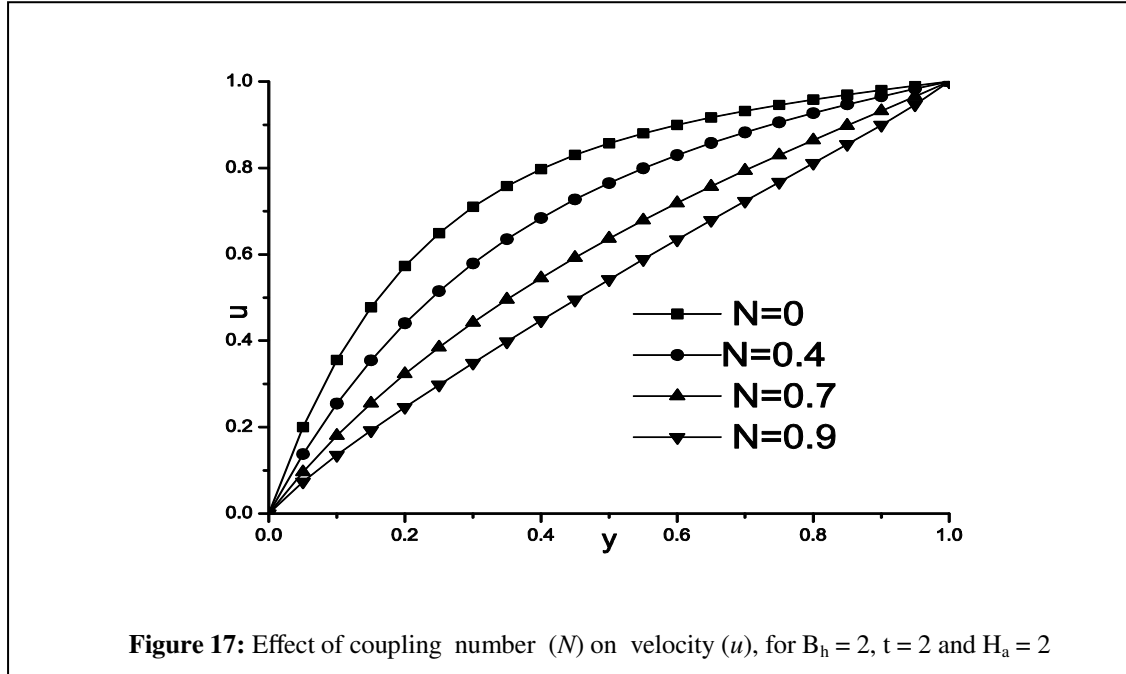
From figure 14 it can be seen microrotaion velocity v_1 vanishes when $B_h = 0$. The effect of B_h on microrotation v_2 studied in figure 15 shows increase in B_h increase v_2, it is also noted that the flow tilted

towards the lower plate. The temperature decrease with increase in the parameter B_h as it is depicted in figure 16.

Figure 15: Effect of Hall parameter (B_h) on mivrorotation (v_2), for N = 0.5, t = 2 and H_a = 2

Figure 16: Effect of Hall parameter (B_h) on temperature (θ), for N = 0.5, t = 2 and H_a = 2

In equation (2) when κ = 0 and hence in equation (11) when N=0 the equations (11) and (13) are decuple. It can

be seen from figure 19 and figure 20 when $N=0$ or (ie.,as $\kappa \rightarrow 0$) there is no microrotation and hence the fluid is Newtonian. Increasing the coupling number N both the main velocity u and the component velocity w decreases. The microrotation v_1 and v_2 both increases with coupling number increases. The temperature increases with increase in the value of N.
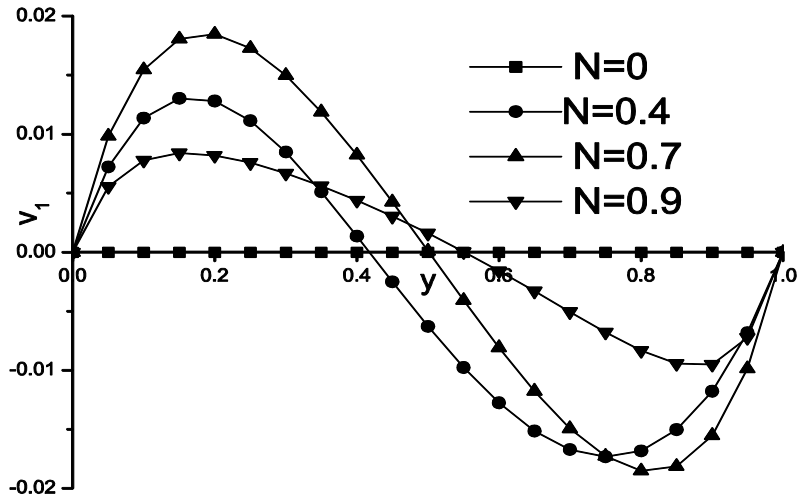

Figure 17: Effect of coupling number (N) on velocity (u), for $B_h = 2$, t = 2 and $H_a = 2$

Figure 18: Effect of coupling number (N) on induced velocity (w), for $B_h = 2$, t = 2 and $H_a = 2$

Figure 19: Effect of coupling number (N) on microrotation (v_1) for $B_h = 2$, $t = 2$ and $H_a = 2$

Figure 20: Effect of coupling number (N) on microrotation (v_2) for $B_h = 2$, $t = 2$ and $H_a = 2$
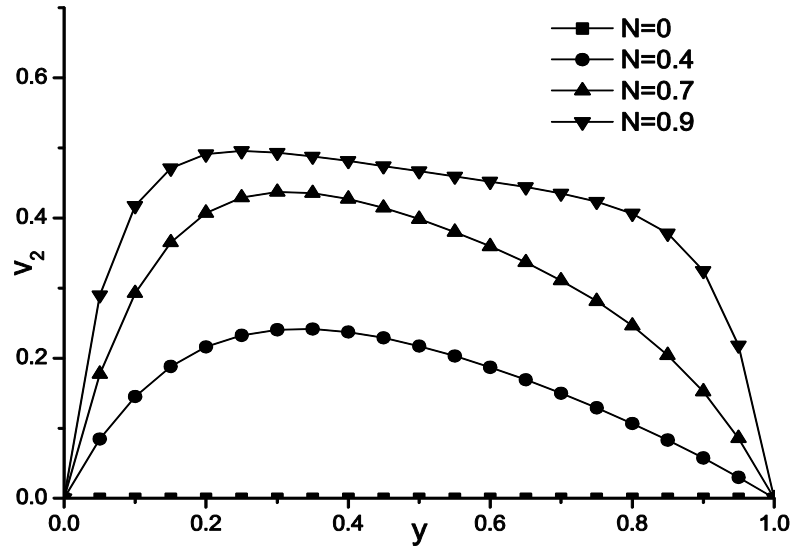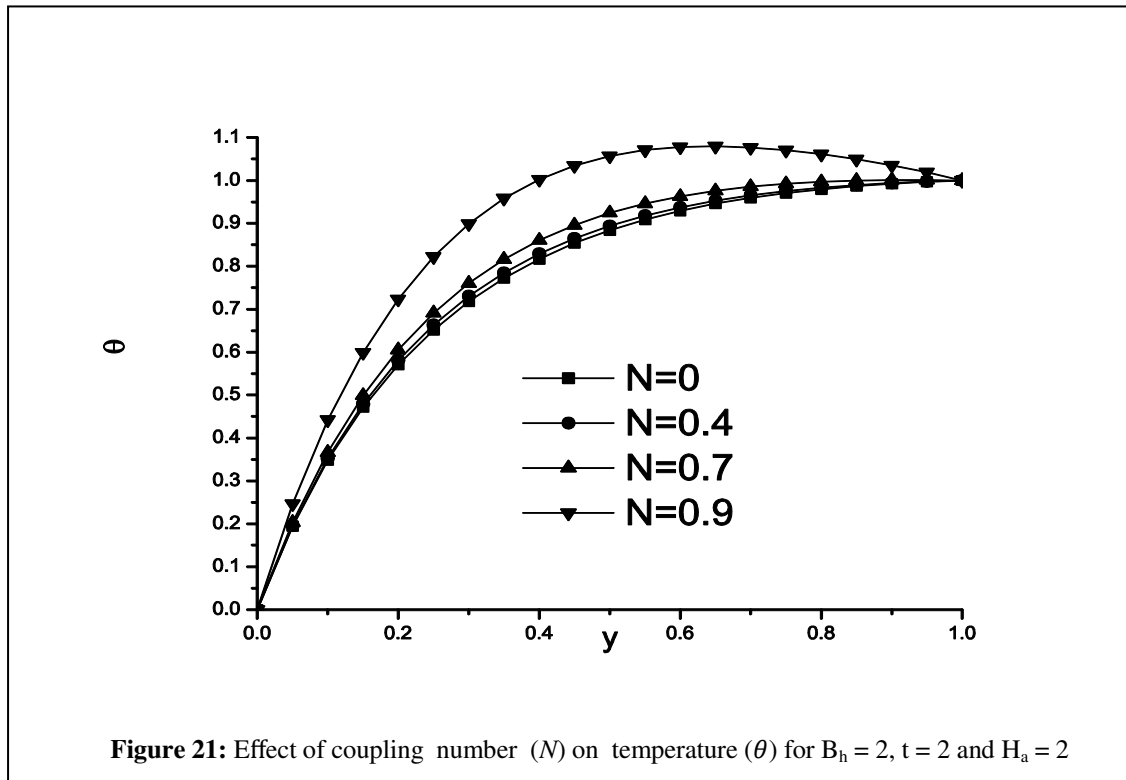
Figure 21: Effect of coupling number (*N*) on temperature (*θ*) for $B_h = 2$, t = 2 and $H_a = 2$

4. Conclusion

In this paper the unsteady micropolar fluid flow between plates when one plate is moving and the other is fixed studied. The transverse magnetic field applied perpendicular to the plates, it is further flow is due to exponential decaying pressure gradient and the Hall Effect taken in to account. The resulting non-dimensional system of equations solved numerically. It is noted that inclusion the hall parameter will enhanced the flow. The more the fluid is micropolarthe fluid motion decelerates as compared to the Newtonian fluid flow. The temperature decreases with increase in the hall parameter B_h and increasing with coupling number or the Hartmann number increases for fixed time level.The article focused only the one dimensional unsteady flow it is hoped that the findings of this investigation will contribute its part for the higher dimensional flows.

References

[1] R.S. Nanda and H.K. Mohanty(1970)."Hydromagnetic Flow in a Rotating Channel", Appl. Sci. Res. 24, pp. 65-78.

[2] J.T. Yen , (1963). "Effect of Wall Conductance on Magnetohydrodynamic Heat Transfer in a Channel", J. Heat Transfer Transac. ASME. 85C, pp.371-377.

[3] S. D. Nigam and S.N. Singh(1960). "Heat transfer by laminar flow between parallel plates under the action

of transverse magnetic field", Quarterly Journal of Mech and App Math 13, pp.85–97.

[4] R. A. Alpher(1961)."Heat transfer in magnetohydrodynamic flow between parallel plates", Int. J. Heat Mass Transfer, 3, pp.108.

[5] H. A. Attia(2006). "On the effectiveness of variation in the physical variables on the MHD steady flow between parallel plates with heat transfer", Int. J. Numer. Methods Eng.,65(2), pp.224–235.

[6] S. D.Nigam and S. N. Singh (1960). Heat transfer by laminar flow between parallel platesunder the action of transverse magnetic field, Q. J. Mech. Appl. Math., pp.13, 85.

[7] I.A. Hassanien and M.A. Mansour(1990)."Unsteady magnetohydrodynamic flow through a porous medium between two infinite parallel plates", Astrophy, Spac. Scie., 163(2), pp.241-246.

[8] M. E. Erdogan and C.E. Imrak(2005)."On unsteady unidirectional flows of a secondgrade fluid", Int. J. Non-Linear Mech., 40(10), pp.1238-1251.

[9] M. E. Erdogan and C.E. Imrak(2007)."On some unsteady flows of a non-Newtonianfluid", Appl. Math. Model., 31(2), pp.170-180.

[10] P.R.Gupta and K.L. Arora(1974)."Hydromagnetic flow between two parallel planes, one is oscillating and the other fixed", Pure. Appl. Geophy. 112(2), pp.498-505.

[11] H. A. Attia(2009)., "Effect of Hall current on the velocity and temperature distributions of Couette flow with variable properties and uniform suction and injection", Comp. and App. Math, 28, N. 2, pp. 195-212.

[12] H. A. Attia (2005)., Unsteady Couette-Poiseulle flow with temperature dependent physicalproperties in the presence of a uniform magnetic field, Malays. J. Sci. Part B}, 24(2).

[13]   S.Muondwe, M. Emmah, J. Njoroge, M. Wafula (2014) ."Finite Difference analysis on magnetohydrodynamic fluid flow past an infinite continuous moving surface embedded in porous medium", International Journal of Sciences: Basic and Applied Research 18, No 1, pp 118-131.

[14] A.C.Eringen(1966)."Theory of Micropolar Fluids". J. Math. Mech. 16: p.1-18,.

[15] T. Ariman, M.A. and Turk, N.D.(1974). "Sylvester, Review article-applications of microcontinuum fluid mechanics", Int. J. Engng. Sci. 12, pp. 273.