

Backbending Phenomena for Some Even Nuclei in Medium Light Region

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Abstract

The backbending phenomena for the nuclei Se^{72} , S^{74} , Kr^{74} , $Kr^{76}Zn^{62}$, Ge^{64} and Ge^{66} is studied in the frame work of Broken polynomial model (BM) and Exponential model (EXPM). The predicted results of the two models are compared with the experimental data . The two models describe the backbending phenomena in good manner. But the BM model describes the phenomena in finest one comparing with EXPM model.

Keywords: rotational bands; backbending; angular momentum; softness parameter.

1. Introduction

The are many attempts are carried to explain the backbending phenomena in deformed nuclei [1,2,3,4] to describe the rapid change of the moment of inertia (JM) versus the square of angular momentum $(\hbar \omega)^2$. There are many reasons for this phenomena like as rotation alignment [5], pairing collapse [1], centrifugal stretching and etc., the change of the JM against $(\hbar \omega)^2$. Reaches its maximum where the backbending occurs. On the reduction of pairing correlation Sood P. C and Jain A.

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Authors in reference [1] proposed rotational energy levels of the ground band the exponential form as:

$$E(I) = \frac{\hbar^2}{2\varphi_o} I(I+1) \exp \Delta_o \left(1 - \frac{I}{I_c}\right)^{\frac{1}{2}}$$

Where φ_O, Δ_O and I_C are free parameters. This formula fitted well the experimental in deformed region $(150 \rangle A \rangle 190$ [4]. The authors in [5] improved the previous equation to be as:

$$E(I) = \frac{\hbar^2}{2\varphi_o} I(I+1) \exp \Delta_o \left(1 - \frac{I}{I_c}\right)^{\frac{1}{V}}$$

Where introduce the new parameter V. R. K. Gupta [2,3] proposed that moment of inertia as broken polynomial In this article we calculated, the moment JMP(I), energy levels EP (I) and the square angular frequency $(\hbar \omega)^2$, the predicting results JMP(I),Ep(I) and $(\hbar \omega)^2$ are tabulated in Table (1-b) and Table (2a), the fitted parameters are tabulated in Table (1-a) and Table (2-a). Also we plotted the moment of inertia JM(I)/JMP(I) versus $(\hbar \omega)^2$ in figures 1 and 2

2. Methods of calculations

We carried out the calculations on the chosen nuclei which are Se^{72} , S^{74} , Kr^{74} , $Kr^{76}Zn^{62}$, Ge^{64} and Ge^{66} by using the following two methods:

1. Broken Polynomial Model:

For ground-state bands of even-even nuclei, the γ -transition energy is characterized as E2 transition, where:

$$\Delta E(I) = E(I) - E(I-2) \tag{1}$$

The angular frequency ω and the moment of inertia are defined by the following equations [2,3]:

$$\hbar \omega_{I} = \frac{\Delta E(I)}{\Delta \sqrt{I(I+1)}}, \qquad \frac{\hbar^{2}}{2\theta_{I}} = \frac{\Delta E(I)}{\Delta [I(I+1)]}$$

$$(\hbar \omega)^{2} = \left(I^{2} - I + 1\right) \left[\frac{E(I) - E(I-2)}{2I - 1}\right]^{2}$$

$$\omega_{I} \theta_{I} = \hbar J_{I}, \qquad J_{I} = \frac{1}{2} \left[\sqrt{I(I+1)}^{+} \sqrt{(I-2)(I-1)}\right]$$

$$(3)$$

One can reproduce both the known features of the spectra and the expected asymptotic behavior by broken expressions in terms of the angular momentum,

$$E(I) = \frac{P[I(I+1)]}{Q[I(I+1)]} \qquad \stackrel{\text{or}}{=} \frac{\hbar\omega_I}{J_I} = \frac{\hbar^2}{2\phi_I} = \frac{P(J_I^2)}{q(J_I^2)} \tag{4}$$

Where P, Q, p, q are assumed to be polynomials with few terms.

In systematic fits to the data with Equation (4) it is suitable to choose the coefficients in the numerator polynomial as varied parameters, while the denominator polynomial is used to achieve a prescribed asymptotic behavior.

The following expression is considered:

$$\left(\frac{J_{I}}{\hbar\omega_{I}}\right) = \frac{2\phi_{I}}{\hbar^{2}} = \frac{a + bJ_{I}^{2} + cJ_{I}^{\ell}}{1 + kJ_{I}^{\ell}}$$
(5)

Where k is a positive constant and *l* an even integer ≥ 4 . Equation (5) is the simplest possible polynomial ratio which has three linear parameters (*a*, *b*, and *c*) and fulfils the requirements for both low and high spins. We choose to extent the application of broken polynomial expression in terms of the angular momentum for describing the energy levels up to spin values $I > 30^+$, including the crossing in the yrast band.

2. Exponential Model

We also are calculated the ground state of rotational band of chosen by using the exponential model EXPOM [1,4.5] Which is written as:

$$E(I) = \frac{\hbar^2}{2\varphi_0} I(I+1) \exp\left[\Delta_0 \left(1 - \frac{I}{I_c}\right)^{\frac{1}{V}}\right]$$
(6)

Where $\frac{\hbar^2}{2\varphi_0}$, $\Delta_0 \cdot \nu$ and Ic are fitting parameters.

3. Results and Discussion

The experimental moment of inertia JM (I) and square angular frequency $(\hbar \omega)^2$ Are calculate as Equation (2) in the previous section. The predicted energy levels EP(I), and predicted moment of inertia JMP(I) ' for the chosen nuclei (subject of this article) are calculated as predicted upon the bases of the two methods, BM and EXPM as explained in the last section.

Broken polynomial model equation (5) as mentioned in the methods of calculations section was used to calculated the energy levels/moment of inertia (EP(I)/JMP(I)) and was compared with the experimental data (E(I)- value / JM(I) value) as mentioned in the experimental references [10,11]. The results are tabulated in Table (1). The moment of inertia versus $(\hbar \omega)^2$ are plotted in Figure (1).

By similar manner the energy levels and moment of inertia, square angular frequency are calculated using EXPOM exponential model Equation (6). The results are tabulated in Table 2, and also, The moment of inertia versus $(\hbar \omega)^2$ are plotted in Figure (2).

The mean deviation as noted in the last row, column three in tables table (1-b) and table (2-b) is calculated by the formula deviation $= \sum (E(I) - EP(I))/N$

Similarly; the mean deviation as noted in the last row, column fifth in tables table(1-b) and table (2-b) is calculated by the formula deviation $= \sum (JM(I) - JMP(I))/N$, where N is the number of the taken data values and I takes 2,4,6,--,--.2N.

The values of the parameters and a ,b ,c L and K as given in Equation (5) are obtained by fitting experimental energies (E) using best fit method, and the obtained values of the parameters are listed in table (1-a). Also, the parameters $A = \frac{\hbar^2}{2\varphi_0}$, $B = \Delta_0$, I_c and v for exponential model Equation (6), the parameters are given by the

same method and tabulated in table (2-a). The chosen nuclei are around mass number A~60, for Se⁷² the energy levels EP(I) and moment of inertia JMP(I) are calculated up to I⁺=28 for Se⁷⁴ up to I⁺=22, For Kr⁷⁴ up to I⁺=30, for Kr⁷⁶ up to I⁺=20, for Zn⁶² up to I⁺=28 for Ge⁶⁴ up to I⁺=8 and for Gn⁶⁶ up to I⁺=16. We may note that the calculation include the backbending /up bending region for the chosen nuclei in this article.

Nucleus	a	b	с	L	k
Se ⁷²	19.68212	12.08695	-1.269614	4	.0000028
Se ⁷⁴	16.9904	19.32841	2.925625	4	.0000029
Kr ⁷⁴	28.80185	5.268198	-5.422028E-02	4	5E-09
Kr ⁷⁶	18.85097	16.8238	-2.219877	4	.0000024
Zn ⁶²	11.24364	12.38068	-1.439517	4	0.0000004
Ge ⁶⁴	6.155304	52.79163	-60.04033	4	.0000009
Ge ⁶⁶	7.199431	24.26767	-2.916808E-03	4	4E-23

Table 1-a: The parameters of Broken polynomial model as in Equation (5)

Table (1-b): the experimental energy E(I), the predicted energy EP(I) as Equation (5) in Mev , experimentalmoment of inertia JM(I) and predicted moment of inertia JMP(I) in (Mev⁻¹) and (ha)² (I) the square ofangular frequency (Mev²)

Se ⁷⁴					
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)	$(\hbar\omega(I))^2$
2	0.663477	0.3472288	9.043267	17.27967	0.1467339
4	1.36521	1.073994	19.95061	19.26344	0.1306445
6	2.2315	2.050726	25.39566	22.5241	0.1922661
8	3.1984	3.16613	31.02699	26.89608	0.2368403
10	4.2563	4.353045	35.92022	32.01575	0.2821132
12	5.5443	5.582798	35.71428	37.40588	0.4170881
14	6.7356	6.854083	45.32864	42.47674	0.356259
16	8.1167	8.186666	44.89175	46.52615	0.478348
18	9.6805	9.622869	44.76276	48.73962	0.6128647
20	11.36012	11.24145	46.43908	48.19034	0.7066718
22	13.2023	13.20316	46.68382	43.83928	0.8497835
Deviation		6.111389E-02		6.068836E-07	
Kr ⁷⁴	I				
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)	$(\hbar\omega(I))^2$
2	0.455623	0.20775	13.16878	28.88087	6.92E-02
4	1.01333	0.6834108	25.10279	29.43274	8.25E-02
6	1.7814	1.407562	28.64322	30.38039	0.1511395
8	2.7479	2.352674	31.03983	31.74228	0.2366443
10	3.3892	3.486882	59.25464	33.50355	0.1036708
12	5.1796	4.77789	25.69258	35.63107	0.8059276
14	6.5157	6.196589	40.41613	38.06303	0.4481274
16	7.8584	7.720083	46.17562	40.69593	0.4521178
18	9.3059	9.334142	48.35925	43.36894	0.5250967
20	10.8809	11.0355	49.52378	45.84579	0.62138
22	12.6499	12.83488	48.61504	47.79417	0.7836096
24	14.689	14.76259	46.09878	48.76259	1.040894
26	17.067	16.88076	42.8932	48.15483	1.415351
28	19.859	19.31429	39.39828	45.20178	1.950748
30	23.124	22.3453	36.14087	38.93085	2.667355
32	26.83	26.82405	33.99893	28.1329	3.436203
Deviation		0.1989065		2.622604E-06	
Kr ⁷⁶					

	I^+	E(I)	EP(I)		JM(I)	JMP(I)	$(\hbar\omega(I))^2$
	2	0.424	0.3140896		14.15094	19.10283	5.99E-02
	4	1.03463	0.9860632		22.92714	20.83415	0.0989244
	6	1.869083	1.914451		26.36457	23.69698	0.178394
	8	2.8787	3.002375		29.71424	27.57548	0.2582294
	10	4.0679	4.182724		31.95425	32.19387	0.3564873
	12	5.347	5.419611		35.96279	37.19012	0.4113438
	14	6.647	6.701762		41.53847	42.11672	0.4242385
	16	7.9963	8.036792		45.94974	46.44091	0.4565738
	18	9.396	9.449659		50.01073	49.54464	0.4909893
	20	10.93	10.98737		50.84744	50.72465	0.5894496
	Deviation	n	4.042866E-	02		3.242493E-06	
	Zn ⁶²						
	I^+	E(I)	EP(I)		JM(I)	JMP(I)	$(\hbar\omega(I))^2$
	2	0.9538	0.5249792		6.290627	11.42902	0.3032448
	4	2.1866	1.626834		11.35626	12.70585	0.4032111
	6	3.7078	3.110584		14.46227	14.82729	0.5928556
	10	5.4815	4.803119		16.91379	17.7249	0.7969899
	12	7.6	6.593885		17.93722	21.21996	1.131335
	14	9.4662	8.428182		24.64902	25.07774	0.8756133
	16	10.3751	10.28973		59.41246	29.00809	0.2073748
	18	13.2366	12.18776		21.66696	32.66553	2.053436
	20	15.0498	14.15133		38.60578	35.64931	0.8239364
	22	17.5907	16.23114		30.69778	37.50339	1.617227
	24	20.8589	18.51131		26.31418	37.71646	2.674615
	26	23.344	21.14275		37.82544	35.72194	1.546027
	Deviation	n	1.075505			7.335956E-07	
S	e ⁷²						
I	÷	E(I)	EP(I)	JM	(I)	JMP(I)	$(\hbar\omega(I))^2$
2	,	0.86212	0.3020671	6.9	59588	19.86314	0.2477503
4		1.63692	0.96521	18.	06918	21.11159	0.1592673
6		2.46681	1.913774	26.	50954	23.19296	0.1764483
8		3.4248	3.06531	31.	31557	26.05215	0.2324953
1	0	4.5043	4.352065	35.	20148	29.53165	0.2937512
1	2	5.7097	5.728305	38.	16161	33.42439	0.3653073
1	4	7.0381	7.169287	40.	65042	37.47445	0.4429769
							1

42.52399

43.77736

44.90243

41.3771

44.77882

47.27738

0.5331005

0.6407656

0.7558667

8.4961

10.0951

11.8322

8.6677

10.23094

11.88078

16

18

20

22	13.7422	13.65684	45.02618	48.42179	0.9135046
24	15.8962	15.62698	43.63974	47.71238	1.161505
26	18.216	17.91393	43.96932	44.60085	1.346919
28	20.798	20.77179	42.60262	38.49032	1.668335
Deviation		0.176685		2.656664E-06	

C = ⁶⁴								
Ge								
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)				
2	0.9017	0.8653427	6.654098	6.933669	0.271021			
4	2.0526	2.070476	12.16439	11.61697	0.3514167			
6	3.4067	3.396759	16.24695	16.58772	0.4697619			
8	5.1752	5.172892	16.96353	16.89062	0.7923235			
Deviation		7.682651E-0	3	1.430511E-06				
Ge ⁶⁶	Ge ⁶⁶							
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)	[]+(I)			
2	0.95693	0.7932891	6.270051	7.563447	0.3052383			
4	2.1733	2.178602	11.50965	10.10602	0.3925353			
6	3.564	3.698186	15.81937	14.47764	0.4954994			
8	5.3584	5.141312	16.71868	20.78821	0.8157007			
10	6.5021	6.449913	33.22549	29.03863	0.32973			
12	7.727	7.623818	37.55408	39.18545	0.3772224			
14	8.8013	8.699446	50.2653	50.20323	0.2897174			
Deviation		7.120924E-02		1.158033E-06				





Figure 1: The moment of inertia " J" versus square angular frequency" $(h\omega)^2$ " for chosen nuclei dashed line for predicted values Equation 5 and "O" for experimental values

Nucleus	$A = \hbar^2$	$B = \Delta_{\Omega}$	I _C	v
	$2\varphi_0$			
S ⁷²	.454793E-02	1.82067	30	0.375074
S ⁷⁴	2.333085E-02	1.569316	28	0.446761
<i>Kr</i> ⁷⁴	2.167943E-02	1.084939	40	0.428975
<i>Kr</i> ⁷⁶	2.338118E-02	1.28275	80	0.124932
Zn^{62}	3.581866E-02	1.664784	38	0.313542
Ge ⁶⁴	6.352502E-02	1.417035	18	0.234353
Ge ⁶⁶	4.156311E-02	1.751311	20	0.4012190

Table 2-a: parameters for exponential Model Equation (6)

*The parameters I_C and v are taken from ref.5

Table (2-b): the experimental energy E(I),the predicted energy as .Equation (6), EP(I) in Mev ,experimentalmoment of inertia JM(I) and predicted moment of inertia JMPI) in (Mev⁻¹⁾ and square of angularfrequency($\hbar\omega(I)$)² (Mev²)

Se ⁷²					
I^+	E(I)	EP(I)	JM(I)	JMP(I)	$(\hbar\omega(I))^2$
2	0.86212	0.6699151	6.959588	8.956358	0.2477503
4	1.63692	1.701977	18.06918	13.56508	0.1592673
6	2.46681	2.814609	26.50954	19.77294	0.1764483
8	3.4248	3.919216	31.31557	27.15897	0.2324953
10	4.5043	5.007863	35.20148	34.90574	0.2937512
12	5.7097	6.105074	38.16161	41.92445	0.3653073
14	7.0381	7.247663	40.65042	47.26109	0.4429769
16	8.4961	8.476704	42.52399	50.44587	0.5331005
18	10.0951	9.834583	43.77736	51.55096	0.6407656

20	11.8322	11.36385	44.9	90243	51.0	00485	0.75586	667	
22	13.7422	13.1062	45.0	02618	49.3	5866	0.91350)46	
24	15.8962	15.10052 43.63		53974	47.1	3375	1.16150)5	
26	18.216	17.37899	43.9	96932	44.7	6686	1.34691	19	
28	20.798	19.95949	42.6	50262	42.6	5274	1.66833	35	
Deviation		0.1451348			1.93	37425			
Se ⁷⁴									
I ⁺	E(I)	EP(I)	JM((I)	JMF	P(I)	(ħω(I)) ²	2	
2	0.63477	0.5289958	9.45	52243	11.3	34225	0.1343	1	
4	1.36521	1.417833	19.1	16653	15.7	5092	0.14155	522	
6	2.2315	2.44584	25.3	39566	21.4	0063	0.19226	561	
8	3.1984	3.517174	31.02699 28.00		00249	0.2368403			
10	4.2563	4.600772	35.92022 35.0		06833 0.2821		132		
12	5.5443	5.699185	35.71428 41.3		41.8	37862 0.4170		381	
14	6.7356	6.832669	45.32864 47		47.6	.64073 0.356		59	
16	8.1167	8.031395	44.89175		51.7	2157	0.478348		
18	9.6805	9.331937	44.76276		53.8	53.82372		0.6128647	
20	11.36012	10.77594	46.4	43908	54.01668		0.7066718		
22	13.2023	12.40966	46.0	58382	52.64053		0.8497835		
Deviation		.0667552			2.59	01318			
Kr ⁷⁴	·								
I ⁺	E(I)	EP(I)		JM(I)		JMP(I)		$(\hbar\omega(I))^2$	
2	0.455623	0.340628		13.16878	8	17.61455		6.92E-02	
4	1.01333	1.013094		25.10279	9	20.8188	7	8.25E-02	
6	1.7814	1.913773		28.64322	2	24.4260	4	0.1511395	
8	2.7479	2.974803		31.03983		28.27439		0.2366443	
10	3.3892	4.153343		59.25464	59.25464 32		9	0.1036708	
12	5.1796	5.424153		25.69258	8	36.1973	8	0.8059276	
14	6.5157	6.774491		40.41613 39.9		39.99		0.4481274	

16	7.8584	8.200593	46.17562	43.47515	0.4521178
18	9.3059	9.705267	48.35925	46.52171	0.5250967
20	10.8809	11.29624	49.52378	49.02649	0.62138
22	12.6499	12.98505	48.61504	50.92345	0.7836096
24	14.689	14.78626	46.09878	52.18732	1.040894
26	17.067	16.7169	42.8932	52.83208	1.415351
28	19.859	18.79608	39.39828	52.90548	1.950748
30	23.124	21.04452	36.14087	52.48088	2.667355
32	26.83	23.48404	33.99893	51.64951	3.436203
Deviation		,.2336007		2.315303	
Kr ⁷⁶					
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)	$(\hbar\omega(I))^2$
2	0.424	0.3998683	14.15094	15.00494	5.99E-02
4	1.03463	1.094963	22.92714	20.14113	9.89E-02
8	1.869083	1.952513	26.36457	25.65449	0.178394
10	2.8787	2.923468	29.71424	30.89741	0.2582294
12	4.0679	3.995483	31.95425	35.44726	0.3564873
14	5.347	5.172324	35.96279	39.08772	0.4113438
16	6.647	6.464544	41.53847	41.78855	0.4242385
18	7.9963	7.885131	45.94974	43.64391	0.4565738
20	9.396	9.447402	50.01073	44.80658	0.4909893
Deviation		9.035111E-04		1.105419	
Zn ⁶²			·		·
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)	$(\hbar\omega(I))^2$
2	0.9538	0.872467	6.290627	6.87705	0.3032448
4	2.1866	2.302631	11.35626	9.789084	0.4032111
6	3.7078	3.938158	14.46227	13.45132	0.5928556
8	5.4815	5.64451	16.91379	17.58137	0.7969899
10	7.6	7.38745	17.93722	21.80224	1.131335

12	9.4662	9.178276	24.64902	25.68647	0.8756133
14	10.3751	11.04833	59.41246	28.87621	0.2073748
16	13.2366	13.0372	21.66696	31.17344	2.053436
18	15.0498	15.18733	38.60578	32.55624	0.8239364
20	17.5907	17.54137	30.69778	33.13446	1.617227
22	20.8589	20.14074	26.31418	33.08493	2.674615
24	23.344	23.02433	37.82544	32.59822	1.546027
26	26.1761	26.22709	36.01568	31.84756	2.00751
Deviation		3.824755E-02		3.824755E-02	
Ge ⁶⁴	L	-1			
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)	$(\hbar\omega(I))^2$
2	0.9017	0.898249	6.654098	6.679663	0.271021
4	2.0526	2.063313	12.16439	12.01651	0.3514167
6	3.4067	3.429895	16.24695	16.09856	0.4697619
8	5.1752	5.133157	16.96353	17.61326	0.7923235
Deviation		2.896592E-03		.0947547	
<i>Ge</i> ⁶⁶					
I ⁺	E(I)	EP(I)	JM(I)	JMP(I)	$(\hbar\omega(I))^2$
2	0.95693	0.9589278	6.270051	6.256988	0.3052383
4	2.1733	2.269137	11.50965	10.68532	0.3925353
6	3.564	3.58604	15.81937	16.70586	0.4954994
8	5.3584	4.886048	16.71868	23.07678	0.8157007
10	6.5021	6.241143	33.22549	28.04232	0.32973
12	7.727	7.750611	37.55408	30.47431	0.3772224
14	8.8013	9.522804	50.2653	50.20323	0.2897174
Deviation		1.881165E-02		1.881165E-02	



Figure (2): The moment of inertia "J" versus square angular frequency" $(h\omega)^2$ " for chosen nuclei dashed line for predicted values Equation 6 and "O" for experimental values

From figures 1 and 2, it is clear that the two models Equation (5) and Equation (6) described well the behavior of the moment of inertia JMP (I) versus the square angular frequency (h Ducluding the back bending/up bending phenomena for the nuclei under consideration.

Also by comparing the tabulated data E(I), EP(I), JM(I) and JMP(I) in table (1-b) and table(2-b) and the deviations in the last raw in the same tables we noted that the broken polynomial Equation (5) is better comparing with experimental data than the exponential model Equation (6).

4. Conclusion

The present models broken polynomial Equation (5), and exponential Equation (6) are predict the yrast state rotational bands of the deformed chosen nuclei, and can also describe well the behaviors of moment of inertia versus square angular (h D)2, but the proceeding with the results of exponential model Equation (6).

Acknowledgment

The authors would like to acknowledge financial support for this work, from the Deanship of Scientific Research (DSR), University of Tabuk (Tabuk, Saudi Arabia, under grant no S -1436-0174)

References

[1] P.C.Sood and A.K.Jain " Expoential Model with pairing attenuation and backbending phynomenon " Phys.Eev.C18,1906-(1978)

[2] R. K. Gupta (1971). "Nuclear-softness model of Ground state Bands in even-even nuclei " Phys Rev. Lett. Vol. 36B, No. 3 pp. 173.

[3] J. S. Batra and R. K. Gupta (1991). "Determination of the variable".

[4] S.U.El-Kamesy ,H.H.Alharabi , and H.A.Alhndi"Backbending phenomena in light nucleiat A 60 mass region" arXiv;nucl-th/0509015v1 7 Sep 2005.

[5] H.H.Alharbi,H.A.Alhend,and S.U.El-Kamessy. "Nuclear Structure Of Some Actinide Nuclei" arXiv;nucl-th/0502017v1 6 Feb 2005.

[6] D. Bonatsos and A. Klein. (1984). "Generalized Phenomenological models of yrast band" Phys.Rev.C Vol. 29 pp 1879.

[7] D. Bonatsos and A. Klein (1984)."Energies of Ground-state bands of even-even Nuclei from generalized variable moment of inertia moment of inertia model in terms of nuclear softness" Phys. Rev.C Vol.43 pp. 1725
[8] J.H.Bakeer and S.M.Alaseri (2014) Description of Rotational Bands for Some Even-Even\Nuclei in Actinide Region " IJSBAR, P.88-98.

[9] Klein. (1980)." Perspective in the theory of nuclear collective motion" Nucl. Phys. A Vol. 347, pp. 3-30.

[10] Klein (1980). "Rotation of variable moment of inertia (VMI) Concept with the interacting model" Phys.

Litt.B. Vol. 93No. 1, pp 1 Edition.) Plenum press, New York. models" Nucl. Data Tables Vol. 30, pp. 27.