Profit Maximization in a Product Mix Company Using Linear Programming Approach to Resource Allocation

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Abstract

This paper demonstrates the use of linear programming methods as applicable in the manufacturing industry. Data were collected as extracts from the records of Bajabure Industrial Complex (Nima Foam), Yola, Nigeria, on five types of sales packages adopted for selling their foam product which includes 15 density, 18 density, 20 density, 25 density and 29 density. Information on selling price and the cost of basic raw materials used as well as quantity of each of the raw material held in stock per month for the production of the foam were available in the records of the company. Based on the costs of raw materials, the maximum profit that would accrue to the company given the product mix was determined. The results showed that the company would attain optimal monthly profit level of about #524,369 if she concentrates mainly on the unit sales of her foam product, ignoring other forms sales packages. The analysis was carried out with R statistical package using library "lpSolve".

Keywords: Linear programming; Simplex Method; Objective function; Product mix; R software.

1. Introduction

Company managers are often faced with decisions relating to the use of limited resources. These resources may include men, materials and money.

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In other sector, there are insufficient resources available to do as many things as management would wish. The problem is based on how to decide on which resources would be allocated to obtain the best result, which may relate to profit or cost or both. Linear Programming is heavily used in Micro-Economics and Company Management such as Planning, Production, Transportation, Technology and other issues. Although the modern management issues are error changing, most companies would like to maximize profits or minimize cost with limited resources. Therefore, many issues can be characterized as Linear Programming Problems [1].

Linear programming (LP) can be defined as a mathematical technique for determining the best allocation of a firm's limited resources to achieve optimum goal. It is also a mathematical technique used in Operations Research (OR) or Management Sciences to solve specific types of problems such as allocation, transportation and assignment problems that permits a choice or choices between alternative courses of action [2]. Linear programming is a term that covers a whole range of mathematical techniques that is aimed at optimizing performance in terms of combinations of resources [3].

Linear Programming being the most prominent OR technique, it is designed for models with linear objective and constraint functions. A LP model can be designed and solved to determine the best course of action as in a product mix subject to the available constraints.

The Simplex method, also called Simple technique or Simplex Algorithm, was invented by George Dantzig, an American Mathematician, in 1947. It is the basic workhorse for solving Linear Programming Problems up till today. There have been many refinements to the method, especially to take advantage of computer implementations, but the essentials elements are still the same as they were when the method was introduced [4,5].

Generally, the Objective function may be of maximization of profit (which is the focus of this paper) or minimization of costs or labor hours. Moreover, the model also consists of certain structural constraints which are set of conditions that the optimal solution should justify. Examples of the structural constraints include the raw material constraints, production time constraint, and skilled labour constraints to mention a few. An optimum solution is a solution that fulfills both the constraints of the problem and the set objective to be met.

In reference [6] Linear programming was used and based on the analysis carried out in this research on Feed Master Limited, Ilorin the result showed that 25kg of Chick mash, 25kg of Layer mash, 25kg of Grower mash, 25kg of Broiler starter mash and 25kg of Broiler finisher mash should be produced but more of 25kg of Layer mash, 25kg of Grower mash and 25kg of Broiler finisher mash in order to satisfy their customers. Also, more of 25kg of Layer mash, 25kg of Grower mash and 25kg of Broiler finisher mash should be produced in order to attain maximum profit because they contribute mainly to the profit earned.

The term "linear", as stated by Akingbade [7], implies proportionality, which means that the elements in a situation are so connected that they appear as straight line when graphed. While the "programming" indicates the solution method which can be carried out by an iterative process in which a researcher advance from one solution to better solution until a final solution is reached which cannot be improved upon.

This final solution is termed the optimal solution of the LP problem. In reference [8] Based on the analysis carried out in this research using SOKAT Soap Industry, Ikotun shows that the company should produce 10kg, 5kg and 2kg of Salem soap but more of 1kg of Salem soap in order to satisfy their customers. Also, more of 1kg of Salem soap should be produced in order to attain maximum profit because it contribute mostly to the profit earned.

In reference [9] the work focuses on soap product manufactured by KASMO Industry Limited (KIL) Osogbo, Nigeria and the showed that the industry should produce 1 soap tablet, 3 soap tablet, 12 soap tablet and 120 soap tablet but more 1 soap tablet in order to optimize their profit.

In reference [10] the research was carried out on Nigeria Bottling Company, Ilorin plant and the analysis revealed that Fanta Orange 50cl and 35cl,Coke 50cl and 35cl, Fanta lemon 35cl, Sprite 50cl,Schweppes,Krest soda 35cl should be produced but more of Coke 50cl and Fanta Orange 50cl in order to satisfy their customers. Also, more of Coke 50cl and Fanta Orange 50cl should be produced in order to attain maximum profit because they contribute usually to the profit earned.

This work demonstrates the pragmatic use of linear programming methods in a manufacturing company in Nigeria Bajabure Industrial Complex (Nima foam). The problem addressed here was to determine the product mix (contribution of sales package) to be adopted by the company for selling her foam product at which the optimal profit level would be attained.

2. Material and Methods

The dataset used for this work was collected as extracts from records of Bajabure Industrial Complex (Nima foam), Yola, Adamawa State, Nigeria on her main product line (Foam product) and five different types of sales packages adopted for selling her Foam product in 2010. These five types of sales packages includes 15 density, 18 density, 20 density, 25 density and 29 density. The marketing strategy of the company is to ensure a relative reduction in the selling price per kg of foam as the number of each kg increases. This was designed to encourage wholesales purchase of the foam product by the users.

Data on the seven basic raw materials used for the manufacturing of Bajabure Industrial Complex (Nima foam) were available in the records of the company. These raw materials are Polyel-Tolene, Prop Amine, Discocynate, Methyl Chloride, Silicone, Starmours Octuate and Water. Information on the quantity of each raw material held in stock per month, probably due to space or financial constraint, was obtained. Also, information on the quantity mix of these basic seven raw materials and their costs for effective manufacturing of the foam product was equally available and obtained from the company's records. Finally, data were collected on the quoted selling prices of the five sales packages of soap adopted by the company.

However, information regarding labour cost, sales and marketing expenses and other related overheads were not obtainable, and as such, their effects were ignored in the analysis. Therefore, the only cost element considered for foam manufacturing in this paper is the cost of raw materials.

In a nutshell, the main focus of this research is to determine the quantity of each of the five sales packages that will maximize the profit of the company given the aforementioned raw materials constraints.

The analysis was carried out using linear programming techniques. The linear programming problem developed here is a mathematical program in which the objective function is linear in the unknown variables and the constraints have linear equation or linear inequality or both. The general form of a linear programming problem is stated in matrix form as show below:

In standard form (Canonical form), it is

Objective function: $Maximize\ Z = \sum_{j=1}^{r} C_j X_j$ (1)

Linear Constraints: $Subject\ to\ \sum_{j=1}^{r} a_{ij} X_j + S_i = b_i,\ i = 1, 2, 3, ..., m$ (2)

$Non-negative\ condition: X_j \geq 0,\ j = 1, 2, 3, ..., n$
$S_i \geq 0,\ i = 1, 2, 3, ..., m$ (3)

Any vector X satisfying the constraints of the Linear Programming Problems is called

Feasible Solution of the problem [11, 12, 4].

The C in (1) is a row vector of m-dimension representing the objective function coefficients, X is a $m \times 1$ column vector of the decision variable of the LP model, A is a $m \times k$ matrix of coefficients, and b is a $k \times 1$ column vector of values in the right hand side of the constraint equations in (2) [9].

Based on the data analysed in the research, the number of decision variables is five setting $m = 5$ in (1). The five decision variables $X_1, X_2, X_3, X_4\ and\ X_5$ in vector X in the objective function represent the five types of sales packages of foam adopted by the company with X_1 represent sales of 15 density, X_2 represent sales 18 density, X_3 sales of 20 density, X_4 represent sales of 25 density and X_5 represents sales of 29 density.

Also, since the company uses seven different raw materials for the manufacturing of her foams, therefore, there are seven linear constraints for the LP model, setting $k = 7$ in (2). The whole analysis was performed using R statistical software [13]. A self-contributed library package in R, the 'lpSolve' that implements the LP was adopted in this research.

3. Data Analysis

The data collected from Bajabure Industrial Complex (Nima foam), Yola, Nigeria on her main product line-foam were analysed to determine the best sales package that would yield maximum profit to the company.

Maximize the objective function (profits):

$$Z = 1898.07X_1 + 2253.84X_2 + 2467X_3 + 3027.01X_4 + 3561.69X_5$$

Subject to (raw material constraints):

Polye-Tolene: $\quad 85X_1 + 95X_2 + 108X_3 + 95X_4 + 95X_5 \leq 14340$

Prop Amine $\quad 0.18X_1 + 0.14X_2 + 0.14X_3 + 0.13X_4 + 0.17X_5 \leq 26.60$

Disocynate $\quad 49.50X_1 + 51.40X_2 + 46.40X_3 + 49.59X_4 + 35.52X_5 \leq 8134.35$

Methyl Chloride $\quad 8.77X_1 + 2.85X_2 + 0.00X_3 + 1.77X_4 + 4.66X_5 \leq 631.75$

Silicon $\quad 1.00X_1 + 0.90X_2 + 0.97X_3 + 1.20X_4 + 1.04X_5 \leq 178.85$

Starmours Octuate $\quad 0.26X_1 + 0.24X_2 + 0.22X_3 + 0.22X_4 + 0.18X_5 \leq 39.20$

Water $\quad 4.25X_1 + 4.37X_2 + 3.87X_3 + 3.99X_4 + 2.85X_5 \leq 676.55$

In order to represent the LP model in canonical form, seven slack variables $X_i \left(i = 6, 7, 8, 9, 10, 11, 12 \right)$ were introduced into the model. This changed the inequalities signs in the constraints aspect of the model to equality signs. A slack variable will account for the unused quantity of raw material (if any) it represent at the end of the production. As a result, the above LP model becomes that of

Maximize

$$Z = 1898.07X_1 + 2253.84X_2 + 2467X_3 + 3027.01X_4 + 3561.69X_5$$

Subject to (raw material constraints):

Polye-Tolene: $\quad 85X_1 + 95X_2 + 108X_3 + 95X_4 + 95X_5 + X_6 = 14340$

Prop Amine $\quad 0.18X_1 + 0.14X_2 + 0.14X_3 + 0.13X_4 + 0.17X_5 + X_7 = 26.60$

Disocynate

$$49.50X_1 + 51.40X_2 + 46.40X_3 + 49.59X_4 + 35.52X_5 + X_8 = 8134.35$$

Methyl Chloride

$$8.77X_1 + 2.85X_2 + 0.00X_3 + 1.77X_4 + 4.66X_5 + X_9 = 631.75$$

Silicone

$$1.00X_1 + 0.90X_2 + 0.97X_3 + 1.20X_4 + 1.04X_5 + X_{10} = 178.85$$

Starmours Octuate

$$0.26X_1 + 0.24X_2 + 0.22X_3 + 0.22X_4 + 0.18X_5 + X_{11} = 39.20$$

Water

$$4.25X_1 + 4.37X_2 + 3.87X_3 + 3.99X_4 + 2.85X_5 + X_{12} = 676.55$$

With non-negativity constraint that $X_1, X_2, X_3, ..., X_{12} \geq 0$.

For analysis proper, the Simplex method proposed by George B. Dantzig as published in [14] was adopted to solve the above LP problem. The Simplex method has been found to be more efficient and convenient for computer software implementation in many instances [4].The R statistical package was employed to fit the LP model using the 'lpSolve' library.

4. Results

The results from the analysis carried out on the LP model in the previous section using Simplex method estimated the value of the objective function to be 524,369. The contribution of the five decision variables $X_1, ..., X_5$ into the objective function are 0, 0, 0, 24.79749, and 126.14988 respectively. This shows only $X_4 \ and \ X_5$ variable contributed meaningfully to improve the value of the objective function of the LP model.

It is observed from the result's output that the optimal solution to the linear programming model fitted was attained at the first iteration. This simply shows the goodness of the structure of the data collected for the analysis.

5. Discussion and Conclusion

The appropriateness of the linear programming methods for optimal resource allocation in the company has been demonstrated in this research. This is evident from the result obtained from the profit maximization type of the LP model fitted to the data collected on foam manufactured by Bajabure Industrial Complex (Nima Foam), Yola, Nigeria.

From the results of the LP model, it is desirable for Bajabure Industrial Complex (Nima foam) to concentrate on the sales of 25kg density and 29kg density of her foam products. By this, total sales of 25 and 126 would be sold by the company per month. This would fetch the company an optimal profit of about #524,369 per month based on the costs of raw materials only.

The results of the LP model fitted to data collected from Bajabure Industrial Complex (Nima foam) are only based on the cost of raw material used for foam production. Therefore, it is quite instructive to remark that if information on the other elements of cost of production such as labour and overhead cost is available and incorporated into the LP model formulation and analysis, the results reported here might be remarkably different. Nonetheless, findings from this research could still server as useful guides to the management of Bajabure Industrial Complex (Nima foam) in the formulation of the production and marketing strategies for their foam products.

Acknowledgements

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