International Journal of Sciences:
Basic and Applied Research
(IJSBAR)

ISSN 2307-4531
(Print & Online)

http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

Rotational Bands of $Pu^{236\text{-}244}$ Isotopes

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Abstract

By taking into consideration the second order of softness parameter, the rotation vibration model (RVSM) is modified which is denoted by MRVSM. Using the exponential model (EXPOM,) (RVSM), and (MRVSM), the ground rotational bands for **$Pu^{236\text{-}244}$ isotopes** are calculated. The predicted results of MRVSM, RVSM and EXPOM models are compared with experimental data. We find that our calculated results for **$Pu^{236\text{-}244}$ isotopes** are in close agreements with experimental data.

Keywords: rotational bands; variable moment of inertia (VMI); angular momentum; softness parameter (σ)

1. Introduction

The energy states of the ground band for deformed nuclei are described by the formula [1,2,3]

$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \qquad (1)$$

Take into consideration the effect of rotation -vibration, Equation (1) becomes

$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \quad +B[I(I+!]^2 \qquad (2)$$

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The experimental data deviate from predicting results of Equation (2), for that many efforts are made to improve this formula. e.g. R.K .Gupta [4, 5,6,7,8] considered the concept of variation of moment of inertia with angular momentum "softness of nuclear matter" to modify the last formula Equation (2) Also there are many models are proposed to predict the ground state bands like harmonic vibrator model VMI , An harmonic vibrator model AVAM , General vibrator model GVMI, exponential model EXPO [9,10,11], and etc..; In this article we used the concept of softness of nuclear mater up to second order in modifying Equation (2) , which is denoted MRVSM model (in the article [1] it is taken up first order RVSM) . We used the MRVSM, RVSM, and EXPOM models to calculate the ground state energies for **$Pu^{236\text{-}244}$ isotopes**. We find that the predicting results of MRVSM, RVSM and EXPOM models are in close agreement compared with the experimental data.

2. Results and discussion

According to references [5,6] suggestion the moment of inertia $\theta(I)$ can be written as

$$\theta(I)=\theta_0(1+o_1 I+o_2 I^2+o_3 I^3+--+\text{—}) \qquad (3)$$

Where θ_0 is the moment of inertia at I=0 , and σ_n is the softness parameter

$$\sigma_n=\frac{1}{n!}\frac{\delta^n\theta(J)}{\delta J^n}\bigg|_{J=0} \qquad (4)$$

$n=1,2,3,\ldots$

Considering the $\theta(I)$ up second order, then $\theta(I)$ becomes

$$\theta(I)=\theta_0(1+o_1 I+o_2 I^2\ ) \qquad (5)$$

Substituting $\theta(I)$ from Equation (5) in Equation (2); we get:

$$E(I)=\frac{AI(I+1)}{\theta_0(1+o_1 I+o_2 I^2\ )}\quad +B[I(I+!]^2 \qquad (6)$$

Where $A=\frac{\hbar^2}{2\theta_0}$, B , o_1 *and* σ_2 *are fitting paramters*

We also calculate the ground state of rotational band of **$Pu^{236\text{-}244}$ isotopes** by using the exponential model EXPOM [8]. This is written as:

$$E(I)=\frac{\hbar^2}{2\varphi_0}I(I+1)\exp-\left[\Delta_0\left(1-\frac{I}{I_c}\right)\right]^{1/2} \qquad (7)$$

Where $\frac{\hbar^2}{2\varphi_0}$, Δ_0 .and I_c are fitting parameters.

The predicted energies as given by Equation (6) and Equation (7) are compared with the experimental data. By using least square fitting, and excitation energies of experimental data, the parameters A, B, σ_1 and σ_2. for MRVSM model Equation (6) are given as in Table (1) for **$Pu^{236-244}$ isotopes.**

Table1: Fitted parameters of MRVSM as shown in Equation (6) for **$Pu^{236-244}$ Isotopes**

A	B	σ_1	σ_2	Dev	Nucleus
7.952E-03	-9.1765E-012	4.956E-010	-8.819E-03	3.7716E-05	Pu^{244}
7.703E-03	4.931E-012	9.528E-10	0.847E-02	-1.996E-04	Pu^{242}
7.381E-03	5.0574E-12	6.1045E-10	-8.026E-03	-1.615E-04	Pu^{240}
7.566E-03	3.1844E--12	6.797E-10	-7.163E-03	-1.4968E-04	Pu^{238}
7.552E-03	2.782E-12	2.119E-11	-5.492E-03	-2.106E-05	Pu^{236}

Table 2: Fitted parameters for EXPOM as in Equation (7) for **$Pu^{236-244}$ Isotopes.**

$\frac{\hbar^2}{2\varphi_0}$	Δ_o	IC	Dev	Nucleus
5.08E-03	0.4337651	30	-8.42E-04	Pu^{244}
5.13E-03	0.389589	30	2.76E-04	Pu^{242}
4.66E-03	0.4458096	34	-9.46E-05	Pu^{240}
5.34E-03	0.3343265	30	-1.27E-04	Pu^{238}
1.04E-04	0.2125915	18	6.332854E-04	Pu^{236}

Also, using the experimental excitation energies the parameters $\frac{\hbar^2}{2\varphi_0}$, Δ_0 .and Ic are calculated by the same manner using "RVSM" model [1]. We are calculated the energies for **$Pu^{236-244}$ isotopes** which is listed in table (3). The deviation of our results from experimental data are given as

$$\text{Dev} = \frac{1}{N}\sum_{i=1}^{N}\left(E_{Cal} - E_{exp}\right)$$

By similar manner using EXPOM Equation (7) and the given parameters $\frac{\hbar^2}{2\varphi_0}$, Δ_0 .and Ic in *table* (1) , we are calculate the energies for chosen nuclei which is listed also, in table (3).

The calculated results for the ground state rotational bands are given systematically in table 3. From this table we noticed that the calculations are carried out for Pu^{236} up to $J^{\pi} = 16^{+}$, Pu ^{238} up to $J^{\pi} = 22^{+}$, Pu^{240} up to $J^{\pi} = 26^{+}$, Pu^{242} up to $J^{\pi} = 26^{+}$, Pu^{244} up to 28^{+}.

Table 3: Experimental energies "EXP" and predicted energies for the**Pu$^{236\text{-}244}$ Isotopes** calculated by RVSM, M RVSM , and EXPOM (in MeV.).

Pu^{236}				
I	EXP	RVSM	MRVSM	EXPOM
2	0.04463	4.49E-02	0.0480	4.48E-02
4	0.14745	0.1477522	0,14757	0.1472584
6	0.3058	0.3057676	0,30564	0.3049715
8	0.5157	0.5155875	0.51567	0.5149609
10	0.7735	0.7732869	0.77351	0.773694
12	1.0743	1.074374	1.07444	1.076601
14	1.4136	1.413789	1.41348	1.416993

Pu^{238}				
I	EXP	RVSM	MRVSM	EXPOM
2	0.04454	4.52E-02	0.o455	4.48E-02
4	0.1473	0.1485508	0.14904	0.1474152
6	0.3064	0.3071518	0.30777	0.3051983
8	0.5181	0.5180148	0.51833	0.5154929
10	0.7786	0.7779288	0.77793	0.7754253
12	1.0844	1.083486	1.0832	1.081889
14	1.4317	1.431084	1.4307	1.431469
16	1.8167	1.816923	1.8166	1.820334
18	2.236	2.237007	2.2370	2.244059
20	2.686	2.687146	2.6875	2.697329
22	3.163	3.162951	3.1633	3.173381
24	3.662	3.65984	3.6697	3.662834
26	4.172	4.173033	4.17297	4.150667
16	1.786	1.785905	1.78603	1.782254

Pu^{240}				
I	Exp	RVSM	MRVSM	EXPOM
2	0.0442	4.55E-02	0.4690	4.64E-02
4	0.155	0.150547	0.153543	0.15218
6	0.3179	0.313117	0.316945	0.314555
8	0.535	0.53051	0.53387	0.530403
10	0.8024	0.799286	00.80105	0.79645
12	1.1159	1.115315	1,114928	1.109168
14	1.471	1.473736	1.47146	1.464694
16	1.8635	1.86898	1.86580	1.858708
18	2.289	2.294762	2.29202	2.286242
20	2.742	2.744083	2.74293	2.74135

Pu^{242}				
I	EXP	RVSM	MRVSM	EXPOM
2	0.04454	4.52E-02	0.0455	4.48E-02
4	0.1473	0.1485508	0.1493	0.1474152
6	0.3064	0.3071518	0.3077	0.3051983
8	0.5181	0.5180148	0.5183	0.5154929
10	0.7786	0.7779288	0.7779	0.7754253
12	1.0844	1.083486	1.0832	1.081889
14	1.4317	1.431084	1.4306	1.431469
16	1.8167	1.816923	1.8166	1.820334
18	2.236	2.237007	2.2370	2.244059
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18	2.289	2.294762	2.29202	2.286242
20	2.742	2.744083	2.74293	2.74135
22	3.215	3.209232	3.21003	3.216499
24	3.69	3.68178	3.68498	3.701273
26	4.149	4.152587	4.15774	4.179048
28	4.61	4.611799	4.60714	4.614758

3. Conclusion

It Is clear that, by using MRVSM RVSM and EXPOM models the predicted results for the ground state rotational bands of deformed even-even Pu$^{236-244}$ isotopes in close agreement compared with experimental data , and may also be applied to other nuclei

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