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Spectral and Cross-spectral Analysis of Weather Patterns: An Applied Perspective

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Abstract

In this work, four time series, representing the average monthly maximum and minimum temperatures of the two Western cities of Sudan; Nyala and Al Fasher, are studied through the use of Frequency Domain Analysis. After checking for stationary level of all of the time series, a spectral analysis is carried out, to explore and shed light on any presence of cyclical patterns in the series. Cross-spectral Analysis is used as well, to examine and reveal any linear relationships between the different temperature series. Consequently, Spectral analysis demonstrated a dominance of a single cycle of length of about nine months in maximum and minimum temperatures of both cities. Meanwhile, the cross-spectral analysis regarding maximum temperatures showed that the two cities are strongly linearly related. Meanwhile, the phase relationship reflected a leading behavior of city of Nyala on the other one, El Fasher, in the mentioned period. Regarding the case of minimum temperatures, both cities are also strongly linearly related, and no city was leading the other.

Keywords: Spectral Analysis; Cross-spectral Analysis; Stationary ; Cross Power Spectral Density; Cospectral Density; Cross Amplitude; Phase Spectrum; Gain; Coherency; Quadrature Spectrum.

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1. Introduction

Spectral analysis is widely known in detecting and revealing cyclical patterns in a time series, and examining any linear relationships between different ones. It is a representation for a time series in terms of a linear combination of sinusoids (sins & cosines) with different frequencies and amplitudes. This type of representation is called a Fourier representation, as stated by [1,2,3]**.** On the other hand, cross spectral analysis allows one to determine the relationship between two time series as a function of frequency. Normally, one supposes that statistically significant peaks at the same frequency have been shown in two time series and that we wish to see if these periodicities are related with each other and, if so, what phase relationship is between them , as noted by the authors [4,5]. One may extend this concept a bit by considering whether it may make sense to do cross spectral analysis even in the absence of peaks in the power spectrum.

The main objectives of this paper are to carry out spectral and cross spectral analysis for the weather patterns represented in the average monthly maximum and minimum temperatures of cities of El Fasher and Nyala, Sudan. And to detect and examine cyclical patterns and uncover the correlations between each pair of series at a time and at different frequencies. El Fasher is the capital city of North Darfur . It is a large town in the Darfur region of north-western Sudan, and it is 120 miles (195 km) northeast of Nyala , Which is also located in the north-west of Sudan. Thus frequency domain analysis is carried out to see if there is any presence of cyclical patterns in the different series, and if these cities are affecting each other as claimed by their residents, or just weather patterns pass through them.

Figure1: Nyala and El Fasher, Sudan, Africa.

The study shows that there is a single cycle of nine months dominating maximum and minimum temperatures of El Fasher and Nyala . This is besides the evidence for the presence of strong linear relationships between the two cities temperature series. In the average monthly maximum temperatures series, Nyala is leading El Fasher positively. While in the average monthly minimum temperatures no city is leading the other, leaving us with the fact that the pattern is the same in both of the two cities.

2. Materials and Methods

El Fasher and Nyala maximum and minimum temperatures by meteorological stations (in centigrade degrees) are presented in four series, covering the period from January 2007 to December 2009, as illustrated in table (1). Stationary status and sequence plots are done first, then followed by a simple review and application of the main concepts and methods of Single Spectrum, along with cross-spectrum.

Table 1: Al Fasher and Nyala Maximum and Minimum Temperatures in Centigrade degrees for the period of January 2007 – December 2009

Nyala Max. Temperatures	El Fasher Max. Temperatures	Nyala (2) Min. Temperatures	El Fasher (2) Min. Temperatures
27.8	27.0	13.5	10.1
33.4	32.3	16.9	13.3
36.8	35.7	20.8	16.2
39.5	38.4	23.5	21.3
39.8	39.6	25.6	25.3
36.9	38.7	23.8	25.6
32.1	35.0	31.3	23.8
31.4	32.5	31.5	22.5
34.0	35.3	21.4	22.3
37.0	36.6	22.35	20.9
34.7	33.5	22.35	16.5
31.4	30.2	22.35	11.7
30.6	29.3	22.35	12.9
31.3	30.5	16.4	11.8
37.7	36.7	22.5	17.7
37.6	37.3	24.6	22.9
38.4	38.9	24.9	23.1
37.0	38.5	24.5	24.3
33.5	36.4	22.4	23.9
31.3	38.0	21.7	25.5
34.3	35.6	22.6	27.2
35.4	35.3	21.9	12.0
34.7	33.3	20.1	14.0
32.3	31.1	18.5	13.3
32.5	31.6	17.7	12.6
35.0	33.8	19.5	15.5

36.1	34.7	20.1	16.9
40.2	39.4	25.7	23.3
39.0	38.6	25.9	23.0
38.9	39.7	25.5	25.0
34.1	36.1	23.2	23.9
38.6	34.7	25.0	23.7
35.8	37.5	23.0	24.2
36.7	36.7	23.1	22.5
34.3	33.0	21.4	17.5
32.1	30.9	16.7	12.4

Source: *Statistical Year Book-2009*, Central Bureau of Statistics, Khartoum, Sudan.

2.1 Stationary Status and Sequence Plots

Regarding maximum temperatures first, sequence plots of both series of Nyala and El Fasher are carried out as in figure (2) which illustrates that maximum temperatures of the two cities resemble each other.

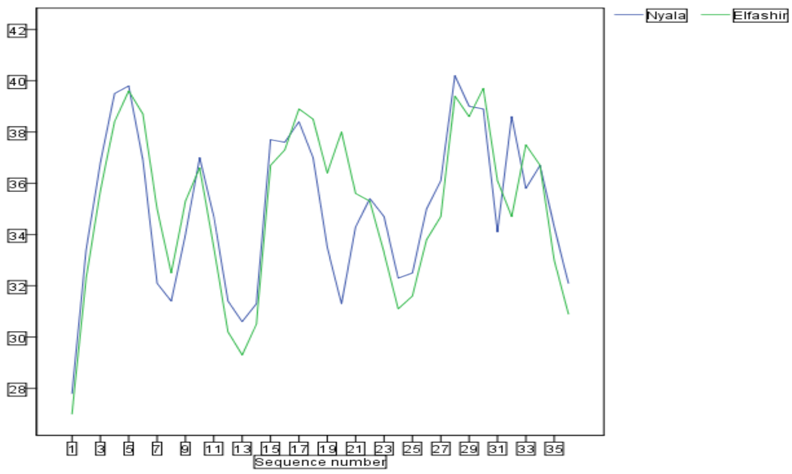

Figure 2: Sequence Plot of Nyala and El Fasher Maximum Temperatures Series

But unfortunately, this may not be simply true, since a separate autocorrelations function plots of the two series, reveal that they are not stationary and thus they may not look alike, and there may be a cyclical pattern affecting the behavior of both series (since seasonality component is not a goal in this paper). While in the case of minimum temperatures, figure (3) represents sequence plots of the two series Nyala (2) & El Fasher (2).

It is obvious that the two series are different from each other and Nyala's minimum temperatures are relatively higher than those of El Fasher. Stationary status is checked by tables (2) and (3). They reflect the fact that both series are not stationary at the moment.

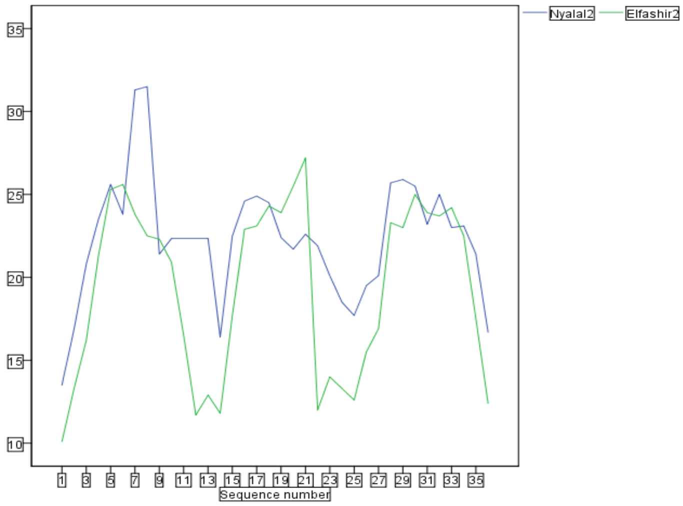

Figure 3: Sequence Plot of Nyala(2) and El Fasher(2) Minimum Temperatures Series

Table 2: Autocorrelations of Series Nyala(2)

Lag	Autocorrelation	Std. Error	Box-Ljung Statistic		
			Value	df	Sig.b
1	.498	.160	9.676	1	.002
2	.136	.158	10.416	2	.005
3	-.029	.155	10.451	3	.015
4	-.196	.153	12.102	4	.017
5	-.257	.151	15.014	5	.010
6	-.521	.148	27.380	6	.000
7	-.435	.146	36.312	7	.000
8	-.089	.143	36.698	8	.000
9	.030	.140	36.745	9	.000
10	.148	.138	37.903	10	.000
11	.228	.135	40.737	11	.000
12	.217	.132	43.430	12	.000
13	.212	.130	46.107	13	.000
14	-.039	.127	46.199	14	.000
15	-.151	.124	47.682	15	.000
16	-.194	.121	50.268	16	.000

a. The underlying process assumed is independence (white noise).

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12	.217	.132	43.430	12	.000
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15	-.151	.124	47.682	15	.000
16	-.194	.121	50.268	16	.000

a. The underlying process assumed is independence (white noise).

b. Based on the asymptotic chi-square approximation.

Table 3: Autocorrelations of series : El Fasher(2)

Lag	Autocorrelation	Std. Error	Box-Ljung Statistic		
			Value	df	Sig.b
1	.655	.160	16.751	1	.000
2	.297	.158	20.293	2	.000
3	-.063	.155	20.458	3	.000
4	-.374	.153	26.424	4	.000
5	-.542	.151	39.368	5	.000
6	-.671	.148	59.927	6	.000
7	-.572	.146	75.388	7	.000
8	-.348	.143	81.314	8	.000
9	-.020	.140	81.334	9	.000

10	.331	.138	87.102	10	.000
11	.539	.135	103.003	11	.000
12	.552	.132	120.346	12	.000
13	.461	.130	132.963	13	.000
14	.216	.127	135.867	14	.000
15	-.087	.124	136.363	15	.000
16	-.244	.121	140.447	16	.000

a. The underlying process assumed is independence (white noise).

b. Based on the asymptotic chi-square approximation.

Consequently, a spectral and a cross-spectral analysis are carried out, to detect and examine these cyclical patterns and uncover the correlations between the series at different frequencies, separately. But initially, let us have a quick review of the Fourier analysis that will be used as follows:

2.2 Single Spectrum

Starting by single spectrum (Fourier analysis), it is widely known that spectral analysis is a very valuable tool in finding out the various kinds of periodic and non-periodic behavior in a series, as stated by [6]. This is through determining the magnitude and phase of periodic variation since it is looked up to the series as a result of waves of sine's and cosines. And in this case, the analysis will be model free, and is purely mathematical and is not based on any theory about a process underling the series, while its main objective will be to know the most effective waves' lengths and frequencies. And a brief summary of the Fourier analysis is given below:

Recalling the autocovariance function, [1] and [5]:

$$\Gamma(j) = \begin{pmatrix} \gamma_{xx(j)} & \gamma_{xy(j)} \\ \gamma_{yx(j)} & \gamma_{yy(j)} \end{pmatrix} \qquad \rightarrow (1)$$

Where: $\gamma_{xx(j)} = \text{cov}(x_t, x_{t-j})$

$\gamma_{yy(j)} = \text{cov}(y_t, y_{t-j})$

$\gamma_{xy(j)} = \text{cov}(x_t, y_{t-j})$

$\gamma_{yx(j)} = \text{cov}(y_t, x_{t-j})$

And if $\sum_{-\infty}^{\infty}|\gamma_{xx(j)}|$, $\sum_{-\infty}^{\infty}|\gamma_{xy(j)}|$, $\sum_{-\infty}^{\infty}|\gamma_{yx(j)}|$, $\sum_{-\infty}^{\infty}|\gamma_{yy(j)}|$, then the matrix valued function

$$f(\omega) = \begin{pmatrix} f_{xx(\omega)} & f_{xy(\omega)} \\ f_{yx(\omega)} & f_{yy(\omega)} \end{pmatrix} = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} e^{-i\omega j}\Gamma(j) \qquad \rightarrow (2)$$

is called the *spectrum* (spectral density) - a plot of the periodogram after it has been smoothed according to certain specifications such as width of the smoothing window i.e. span and weights applied to the neighboring observations - of the bivariate process, as it is noted by [7], [8], and [9]. Thus:

$$f_{xx}(\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} \gamma_{xx(j)} e^{-i\omega j} \quad \rightarrow (3)$$

$$f_{yy(\omega)} = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} \gamma_{yy(j)} e^{-i\omega j} \quad \rightarrow (4)$$

Of $f(\omega)$are just the spectral densities of the univariate processes x_t $t\epsilon z$,y_t $t\epsilon z$.

The function

$$f_{xy}(\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} \gamma_{xy(j)} e^{-i\omega j} \quad \rightarrow (5)$$

is called the *cross spectrum* or *cross density* which can be written in terms of its Cartesian coordinates as follows:

$$f_{xy}(\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} cos(\omega j)\, \gamma_{xy(j)} - i\ \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} sin(\omega j)\, \gamma_{xy(j)} \quad \rightarrow (6)$$

Where:

$\left\{ \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} cos(\omega j)\, \gamma_{xy(j)} \right\}$: is the *cospectrum* $c_{xy}(\omega)$ and $\left\{ \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} sin(\omega j)\, \gamma_{xy(j)} \right\}$: is *quadrature spectrum* $q_{xy}(\omega)$.

The cospectrum at frequency ω indicates which portion of the covariance is due to cycles of frequencyω. It is possible that the cospectrum is positive at some frequencies and negative at others. On the other hand, the *cross-amplitude* is known as the square root of the sum of squared cross density (*cospectrum*) and quadrature density (*quadrature spectrum*) as:

$$A_k = \left(c_{xy}(\omega)^2 + q_{xy}(\omega)^2\right)^{\frac{1}{2}} \quad \rightarrow (7)$$

The cross-amplitude is a measure of covariance between the respective frequency components in the two series.

Alternatively, we can use polar coordinates instead of Cartesian coordinates to represent $f_{xy}(\omega)$,

i.e $f_{xy}(\omega) = R_{xy}(\omega)e^{i\phi xy(\omega)}$and $R_{xy}(\omega) \geq 0, -\pi\phi xy(\omega) \leq \pi$ $\rightarrow (8)$

The real functions of $R_{xy}(\omega)$) and $\phi xy(\omega)$ are called the *amplitude spectrum* (or the *gain*) and the *phase spectrum*, respectively. The gain can be interpreted as the standard least squares regression coefficients. The phase spectrum can be used to determine whether x_t $t\epsilon z$, leads or lags y_t, $t\epsilon z$, at different frequencies.

In general, the polar coordinates are much easier to interpret than the Cartesian coordinates. The amplitude spectrum measures the strength of the linear relationship between x_t and y_t at different frequencies. A scale less measure of this relationship is given by the function:

$$\rho^2{}_{xy}(\omega) = \frac{R^2{}_{xy}}{f_{xx}(\omega)f_{yy}(\omega)} = \frac{|f_{xy}(\omega)|^2}{f_{xx}(\omega)f_{yy}(\omega)} \quad \rightarrow (9)$$

This is called the squared coherency (or *squared Coherence function*) which is the squared correlation between the cyclical components in any considered two series.

2.3 Spectral Analysis plots

Periodograms along with spectral densities by frequency are plotted separately for each pair of series. In drawing the periodogram which is an unsmoothed plot of spectral amplitude plotted on logarithmic scale against frequency or period, plots are produced by frequency ranging from frequency 0 which represents the constant or mean term, to frequency 0.5 which is the term for a cycle of two observations. Then spectral density is plotted, and the chosen spectral window is Tukey-Hamming, as noted by [5] and [6]. This is done after trying all of the other kinds of windows such as: Parzen, Barlett and Daniell. And due to the fact that all of them came up with almost the same density, we choose the most appropriate window i.e. Tukey-Hamming. Furthermore, a short review of the different windows algorithms is given below as:

- Tukey window or Tukey-Hamming window (named after Julius Von Hann) Hamming, R. W [10] and [11], for each frequency, the weights (wj) for the weighted moving average of the periodogram values are computed as:

$$wj = 0.5 + 0.5*\cos(\pi*j/p) \quad \text{(for j=0 to p)}$$
$$w\text{-}j = wj \quad (\text{for } j \neq 0)$$

- Hamming window: In the Hamming (named after R. W. Hamming) window or Tukey-Hamming window, for each frequency, the weights for the weighted moving average of the periodogram values are computed as:

$$wj = 0.54 + 0.46*\cos(\pi*j/p) \quad \text{(for j=0 to p)}$$
$$w\text{-}j = wj \quad (\text{for } j \neq 0)$$

- Parzen window: In the Parzen window Parzen as noted by [12] , for each frequency, the weights for the weighted moving average of the periodogram values are computed as:

$$wj = 1\text{-}6*(j/p)2 + 6*(j/p)3 \quad \text{(for j = 0 to p/2)}$$
$$wj = 2*(1\text{-}j/p)3 \quad \text{(for j = p/2 + 1 to p)}$$
$$w\text{-}j = wj \quad (\text{for } j \neq 0)$$

- Bartlett window: In the Bartlett window Bartlett as stated by [13] , the weights are computed as:

$$w_j = 1-(j/p) \quad (\text{for } j = 0 \text{ to } p)$$
$$w_{-j} = w_j \quad (\text{for } j \neq 0)$$

- Daniell (or equal weight) window: The Daniell window amounts to a simple (equal weight) moving average transformation of the periodogram values, that is, each spectral density estimate is computed as the mean of the m/2 preceding and subsequent periodogram values.

With the exception of the Daniell window, all weight functions will assign the greatest weight to the observation being smoothed in the center of the window, and increasingly smaller weights to values that are further away from the center. In many cases, all of these data windows will produce very similar results.

Regarding the span, a number of them, starting from 3, 5, 7, 9, and up to 11 are tried out. This is done so as to choose the span that makes the spectral density plot easier to read, and in this case, it is span of 5. This is due to the fact that the wider the span, the more bias will be introduced through the missing of spikes corresponding to important periodic variation at narrow frequency ranges.

Moreover, a cross spectral analysis is carried out for the cases of maximum and minimum temperatures of both cities, leading to cross-spectral plots represented in: Cospectral density, Cross amplitude, Phase spectrum, Gain, Coherency, and Quadrature spectrum.

3. Results

The major objectives of this paper are to carry out spectral and cross spectral analysis for the patterns of temperatures of cities of El Fasher and Nyala, Sudan, and to detect and examine and reveal any presence of cyclical patterns. To this end, the basic findings of the paper can be divided into two parts as follows:

3.1 Single Spectrum Analysis

Nyala's maximum temperature series is considered as the independent series, while the other of El Fasher is the dependent one. Accordingly, periodograms along with spectral densities are plotted for the two series. The smoothing window is Tuky-Hamming with a span of 5 and with weights as follows:

- W (-2) 2.137
- W (-1) 2.214
- W (0) 2.240
- W (1) 2.214
- W (2) 2.137

And in fact, both spectral densities reveal the dominance of a strong periodic cycle at frequency (0.11), which approximates to a period of 9 months long for maximum temperatures of the two cities. There exist other peaks (amplitudes) in both periodograms of the two cities and which are considered as harmonic peaks, as noted by [14]. The spectral densities are highest at frequency (0.11). The two series are affected by trend effect. This is obvious since there are two single peaks in the periodograms near frequency (0.0).

Periodograms and spectral densities for minimum temperatures of the two cities are plotted. Nyala is also the independent series leaving El Fasher as the dependent one. The chosen smoothing window is also Tuky-Hamming with a span of 5 weights which have been mentioned above. And again, the spectral densities demonstrate a single cycle of 9 months long at frequency (0.11).

3.2 Cross-Spectrum Analysis

Figure (3) represents the cospectral density function (the real part of the cross-spectrum) of both cities maximum temperatures series.

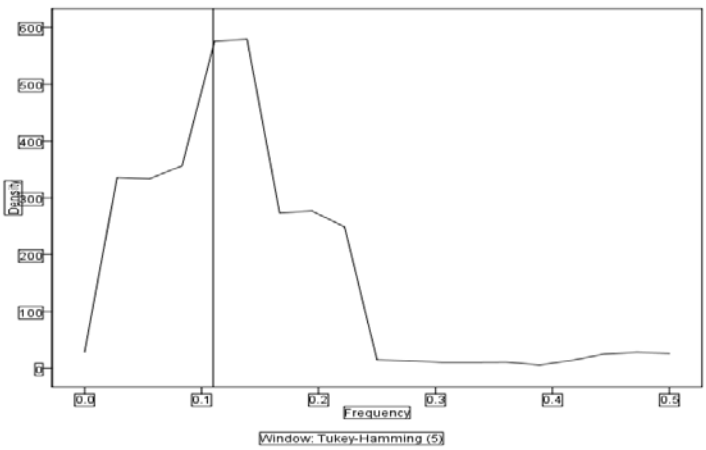

Figure 4: Cospectral Density of Nyala and El Fasher by Frequency

Figure (4) indicates which portion of the covariance is due to cycles of frequency ω (i.e. 0.11) since its highest peak is on this frequency. And it starts to decline immediately after this frequency, till it reaches density zero.

Figure (5), quadrature spectrum is about the (imaginary part) of the cross-spectrum. And it can be seen that the curve is high till it reaches frequency (0.11) after which it starts to decline quickly.

Regarding Figure (6), the cross-amplitude is demonstrated. It reaches its highest value which is the degree of covariance between the respective frequency components of the two series, at frequency (0.11).

Figure (7) is about the phase spectrum of the two cities temperatures by frequency. Its highest values around frequency (0.11) shows that it is approximately linear with a tendency to have a negative slope. This illustrates the fact that Nyala's maximum temperature is leading El Fasher one, confirming a negative correlation between the two series.

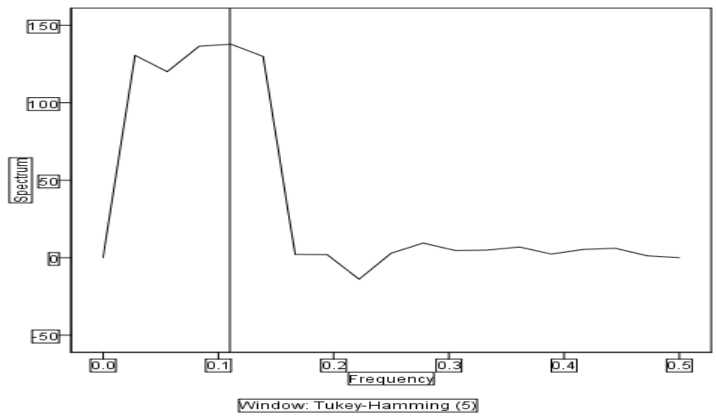

Figure 5: Quadrature Spectrum of Nyala and El Fasher by Frequency

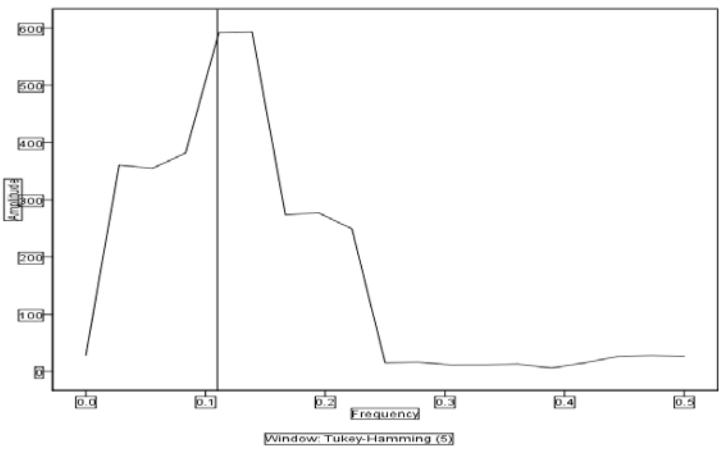

Figure 6: Cross Amplitude of Nyala and El Fasher by Frequency

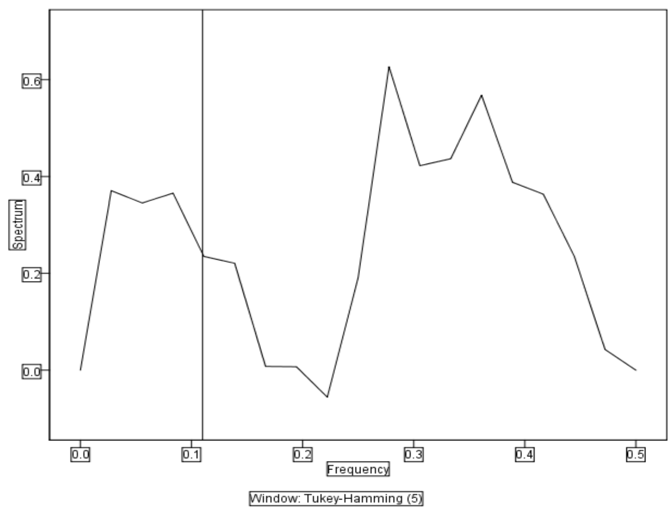

Figure 7: Phase Spectrum of Nyala and El Fasher by Frequency

Figure (8) is concerning the gain of Nyala from El Fasher and vice-versa. There is a slight tendency that the amplitude of variations at Nyala relative to that of El Fasher decreases slightly as frequency increases within a range where there exists a linear relationship between the two series (around frequency 0.11).

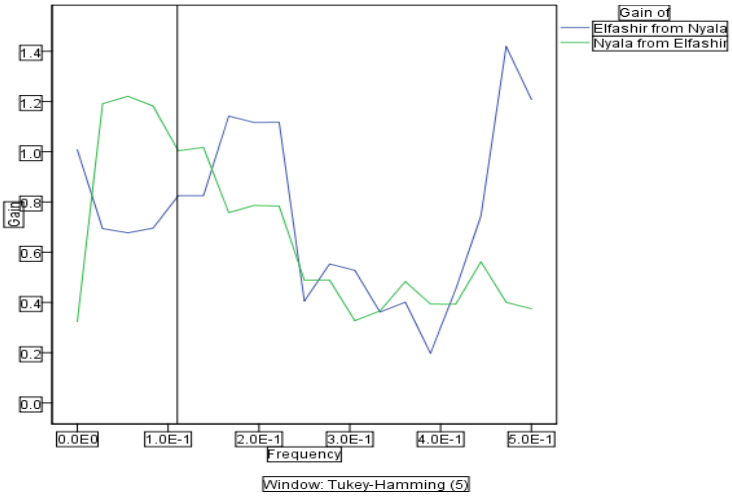

Figure 8: Gain of Nyala and El Fasher

The same information is deducted from Figure (9) which is about the coherency of the two series [15]. It reaches its maximum (slightly above 0.8) before it declines immediately after frequency (0.11). But generally, it can be said that the coherency is high between the two series indicating the presence of a strong linear relationship.

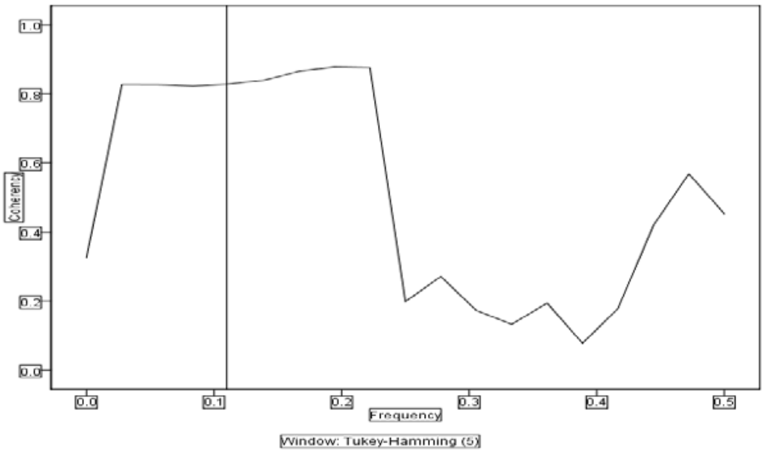

Figure 9: Coherency of Nyala and El Fasher by Frequency

Figures (10), (11) and (12) are about the cospectral, quadrature and cross-amplitude, of the minimum temperatures series. The three plots show obvious peaks at the frequency (0.11) after which the curves declines quickly, and then gradually to frequency (0.5).

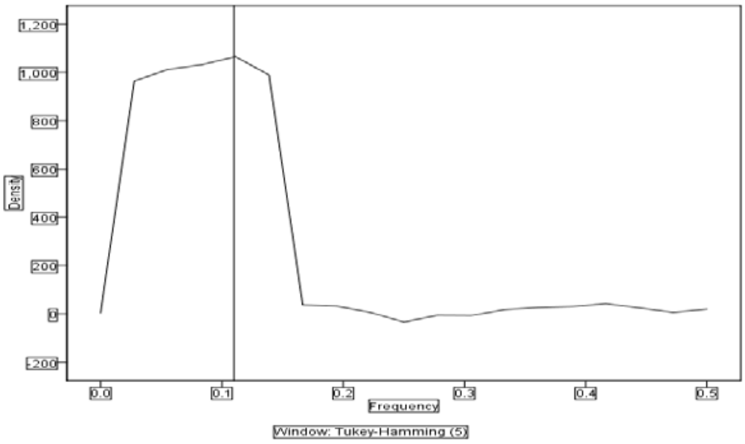

Figure 10: Cospectral Density of Nyala2 and El Fasher2 by Frequency

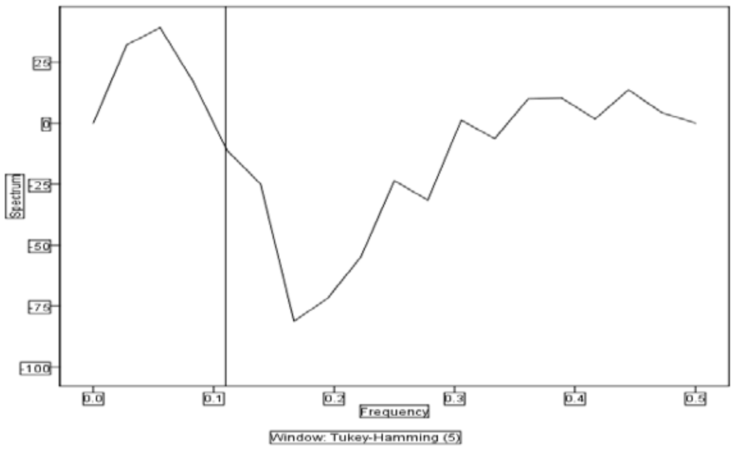

Figure 11: Quadrature Spectrum of Nyala2 and El Fasher2 by Frequency

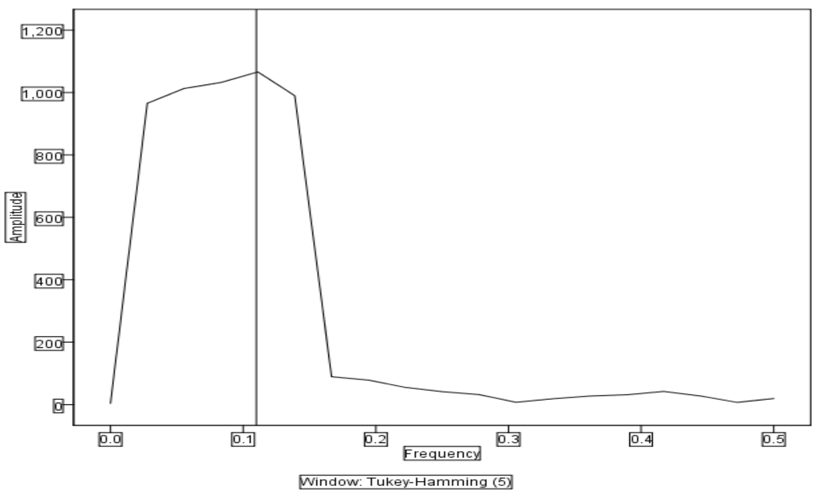

Figure 12: Cross Amplitude of Nyala2 and El Fasher2 by Frequency

Figure (13) is about the phase spectrum, which is nearly a horizontal line and not significantly different from zero, indicating that Nyala is not leading El Fasher in minimum temperatures or vice versa.

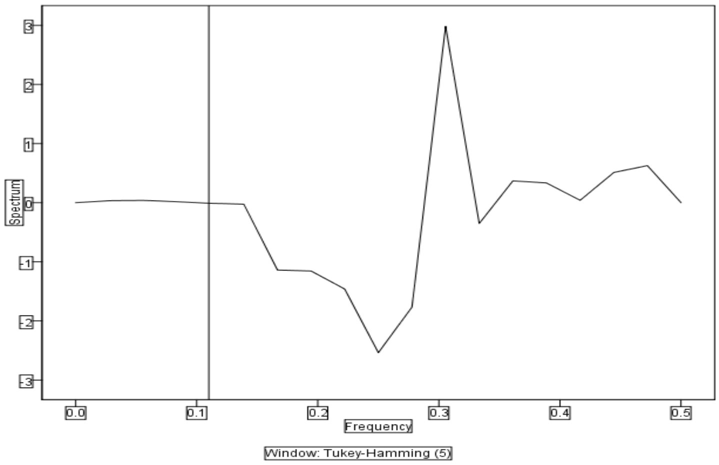

Figure 13: Phase Spectrum of Nyala2 and El Fasher2 by Frequency

The gain and coherency are given in Figures (14) and (15) respectively. Both of the plots are increasing slightly before frequency (0.11) after which they start to decline gradually. The coherency is very high and approaches the value of (0.8), confirming the presence of a very strong linear relationship between the two series of minimum temperatures.

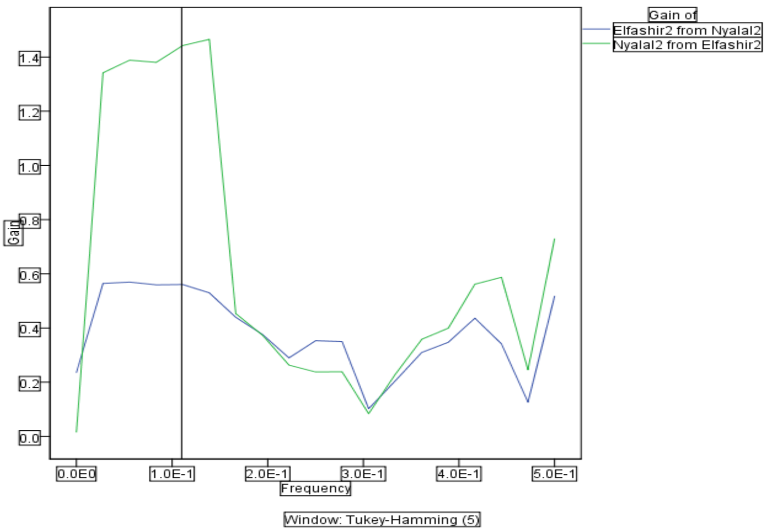

Figure 14: Gain of Nyala2 and El Fasher2

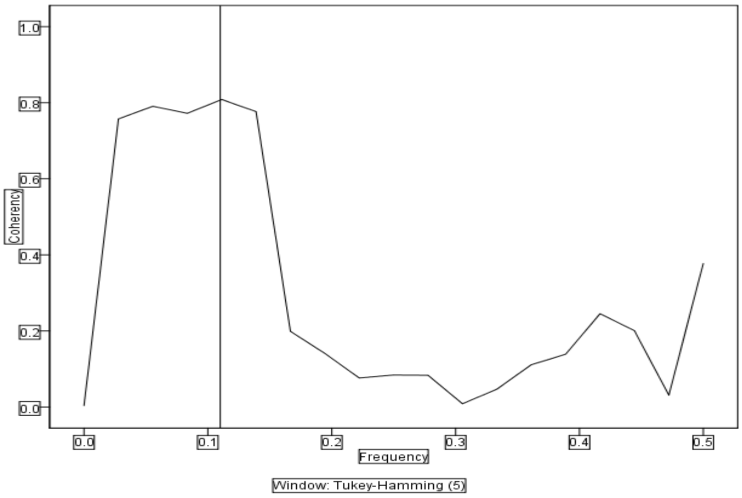

Figure 15: Coherency of Nyala2 and El Fasher2 by Frequency

4. Conclusion

The main objectives of this work were to find out if there is any cyclical pattern and to detect correlation between maximum and minimum temperatures of Nyala and El Fasher cities of Western Sudan. Consequently, the paper highlighted the main features of spectral and cross-spectral analysis, along with their practical applications. This was done through introducing some typical concepts like coherency, phase spectrum, gain and others. By means of coherency, it was proved that the two cities maximum and minimum temperatures are linearly related in a cycle of 9 months long. By phase spectrum Nyala leads El Fasher positively in maximum temperatures, while no city leads the other in minimum ones and thus the two temperature patterns are the same.

The study shows that spectral analysis can provide quantitative and visual information for the analysis of temperature patterns in the two cities. However, the results described in this paper are not necessarily final conclusions, since the data available at present are believed small amount. And thus further research work will be required as soon as the appropriate amount of data is available, to help in extracting more information about the case.

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