



An Investigation of the Statistical Distribution of Total Station Measured Data

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Abstract

It has been investigated and discovered that the Global Positioning System (GPS) measured quantities are normally distributed [1]. Can we say the same of other digital equipment used for measurement in surveying? In this paper, an attempt is made to investigate whether or not the Total Station, which is digital equipment, measured quantities are normally distributed. The data used are the repeated Total Station measured distances of a line at the Federal Polytechnic, Ado-Ekiti. The investigation is carried out using both the graphical approach and numerical testing. The chi-square for goodness-of-fit test was used in the numerical testing. The plotted histogram and normal curve shows that the observed quantities of a total station instrument are normally distributed. The computed statistic (T) (5.4655) is neither more than the table statistic (T) (17.53) at the upper limit) nor less than the table statistic (T) (at lower limit). Therefore, the Null hypothesis which states that total station measured quantities are normally distributed is not rejected at five percent level of significance. It is concluded that total station measured quantities are normally distributed.

Keywords: Statistical distribution; Total Station; GPS; Data

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1. Introduction

The most common type of electronic instrument now available in surveying are termed Total Station instrument [2]. Also according to [3] there are significant advantages that made total stations the predominant instruments used in surveying practice today. They are used for all types of surveys including topographic, hydrographic, cadastral, and construction surveys. Total Station is readily available in the market from different make to different specifications. Its availability may not be far from the fact that this instrument, just like the GPS, can do the work of other digital survey instruments.

A Total Station incorporates a theodolite with electronic circles and an EDM. The EDM normally works concentric with the telescope eyepiece and is generally housed in a casing that forms part of the telescope. These devices can automatically observe horizontal and vertical angles, as well as slope distances, from a set up. From these data they can instantaneously compute horizontal and vertical distance components, elevations and coordinates of points sighted, and display the results on a liquid crystal display (LCD). [2] and [4] further discussed the uses and operational procedure of Total Station.

All measurement or observation is looked upon mathematically as a random variable because it includes error components which exhibit random behavior [5]. Whereas systematic variations are dealt with mathematically using functional relationships or models, random variables must use probabilistic models. These scholars further observed that of all existing probabilistic distributions, none is more important than the normal distribution. The normal distribution has widespread application in sciences, technology, and industry; it is used as the basic model for all physical measurements, including measurements in surveying.

Chi-square, a quantity commonly used to test whether any given data are well described by some hypothesized function. Such a determination is called a chi-square test for goodness of fit. The goodness of Fit (GOF) of a statistical model describes how well it fits into a set of observations. GOF indices summarize the discrepancy between the observed values and the values expected under a statistical model. GOF statistics are GOF indices with known sampling distributions, usually obtained using asymptotic methods, that are used in statistical hypothesis testing. [6]. For more on chi-square test for goodness of fit the works of [7] and [8] should be visited.

Like any other digital instrument used for measurement in surveying, the statistical distribution of Total Station measured data forms the basis for testing the reliability of its results. Therefore, the aim of this paper is to investigate the statistical distribution of Total Station measured data. The researcher uses only Total Station measured distances for this study with the assumption that the outcome of the study holds true for all Total Station measured quantities.

2. Methodology

A similar approach to [1] was used that is demonstration of the graphical technique and numerical statistical test to analyze the measured distances. Histogram and distribution curve was plotted from the computed data to do the visual analysis. Also Chi-square test for goodness-of-fit was carried out to assess its reliability based on numerical statistical test.

2.1 Data Acquisition

A South Total Station instrument was used to measure distance between two beacon stations repeatedly fifty times. The two stations are FPA 3S and FPA 6S both located at the Federal Polytechnic, Ado-Ekiti. Table 1 shows the result of the fifty measured distances.

Table 1: Measured distances

62.853	62.846	62.824	62.833	62.832
62.812	62.809	62.787	62.801	62.805
62.794	62.849	62.857	62.847	62.843
62.817	62.806	62.837	62.832	62.828
62.814	62.826	62.809	62.821	62.831
62.830	62.820	62.828	62.812	62.806
62.825	62.839	62.810	62.793	62.773
62.774	62.778	62.777	62.793	62.792
62.808	62.804	62.815	62.810	62.863
62.852	62.849	62.767	62.817	62.824

2.2 Graphical Method

Histogram and the normal distribution curve are the graphical techniques used in the testing. For the histogram, observed frequency was plotted on the y-axis against the class interval on the x-axis. Plotting the normal distribution curve saw the expected frequency (EF) on the y-axis and the median of the class interval (x) on the x-axis. Microsoft excel package was used to plot and display the histogram and normal distribution curve.

2.3 Numerical Method

Chi-square testing for goodness-of-fit test was used under this method. The goodness of Fit (GOF) of a statistical model describes how well it fits into a set of observations. GOF indices summarize the discrepancy between the observed values and the values expected under a statistical model. See [7], [9] and [6] for more explanation on goodness of fit (GOF). Chi-square is a quantity commonly used to test whether any given data

are well described by some hypothesized function. According to [1], the general procedure is to test the hypothesis that a certain function, $f(x)$ is the distribution of the population from which a sample was taken. The sample distribution, $f_1(x)$ is an approximation of $f(x)$ and if it approximates $f(x)$ very well, the hypothesis that $f_1(x)$ is the distribution function of the population should not be rejected. Therefore the Null hypothesis in this work is that total station measured quantities are normally distributed. The chi-square testing was carried out at 0.05 and 0.10 significance levels for wider evaluation.

3. Presentation of Results

Table 2 below shows all computed results used to analyze graphical technique i.e. to plot the histogram and the normal distribution curve. Column 1 represents the class interval of the observation while the second column represents the observed frequency (OF) in each case, and these two were used to plot the histogram as shown in figure 1. Column 3 represents the median of each class interval while column 4 represents the probability P_z obtained from statistical table. Column 4 and column 5 show the expected frequency (EF) and V. Normal distribution curve of the observation was plotted using column 3 and column 5 as shown in figure 2. The following equation by [10] as stated in [1] were used to obtain the parameters for P_z estimation, mean of the data (μ), standard deviation (σ), EF and V.

$$EF = N(P(Z_{i+1}) - P(z_i))$$

$$z_i = \frac{(x_i - \mu)}{\sigma}$$

$$\sigma$$

$$x = \frac{(x_i + x_{i+1})}{2}$$

$$2$$

$$\mu = \frac{\sum x}{N} = 62.817$$

$$N$$

$$\sigma = \frac{\sum (x_i - \mu)^2}{(N - 1)^{1/2}} = 0.0235$$

$$(N - 1)^{1/2}$$

$P(z_i)$ = probability of z_i obtained from statistical table

N = number of measured distances = 50

x_i = observed distances

The value of statistic T was computed from $T = \sum V$ as 5.4655 (from table 2 column 5).

$$V = \frac{(OF - EF)^2}{EF}$$

EF

Using $k - r - 1$, degree of freedom (df) was found to be 8.

Where: k = number of class interval = 11

r = number of estimated parameters (mean and standard deviation) = 2

From the statistical table (df = 8), $X^2_{0.975} = 17.53$ (Upper limit) and $X^2_{0.025} = 2.18$ (Lower limit).

Table 2: Computed data for the analysis

X_i	OF	x	P(z)	EF	V
62.760-62.770	1	62.765	0.0075-0.0228	0.77	0.069
62.770-62.780	4	62.775	0.0228-0.0582	1.77	2.810
62.780-62.790	1	62.785	0.0582-0.1251	3.35	1.649
62.790-62.800	4	62.795	0.1251-0.2358	5.54	0.428
62.800-62.810	8	62.805	0.2358-0.3821	7.32	0.063
62.810-62.820	8	62.815	0.3821-0.5517	8.48	0.027
62.820-62.830	8	62.825	0.5517-0.7088	7.86	0.002
62.830-62.840	7	62.835	0.7088-0.8365	6.39	0.058
62.840-62.850	5	62.845	0.8365-0.9192	4.14	0.179
62.850-62.860	3	62.855	0.9192-0.9664	2.36	0.174
62.860-62.870	1	62.865	0.9664-0.9881	1.09	0.007

4. Discussion of Results

According to [11], the normal distribution curve is described by a symmetrical bell-shaped curve which is asymptotic to both negative and positive sides of x-axis. The plotted histogram and normal curve in figure 1 & 2 respectively shows that the observed quantities of a total station instrument are normally distributed. This is because the curve is symmetrical about the mean ($\mu=62.817$) and also the mean equals the mode which as well is equal to the median.

On the other hand, the result of the numerical test shows that the computed statistic (T) which is 5.4655 is not more than the table statistic (T) which is 17.53 at the upper limit. So also the computed (T) is not less than the table statistic (T) at lower limit. One can therefore say that the Null hypothesis which states that total station measured quantities are normally distributed is not rejected at five percent level of significance. Looking at both the graphical testing and the numerical testing of the measured quantities of total station, there is a confirmation that such quantities are normally distributed.

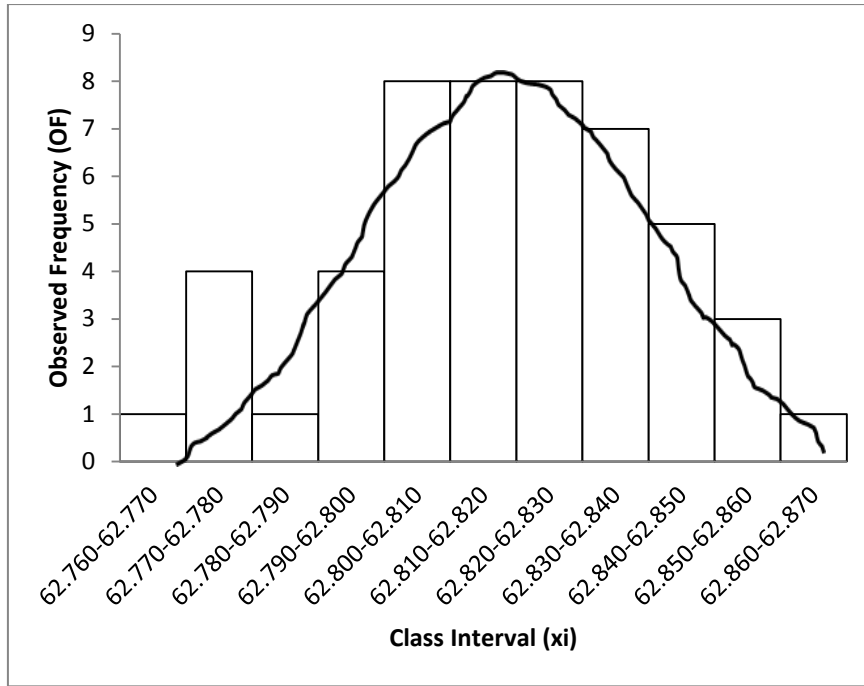


Figure 1: Histogram of the Distribution

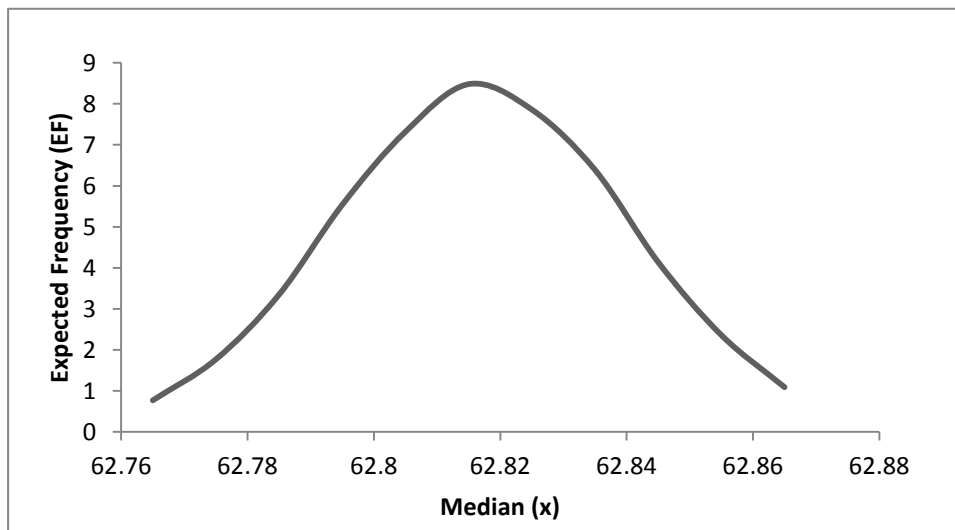


Figure 2: Normal Distribution Curve

5. Conclusion

This work has attempted to investigate the statistical distribution of total station measured data. The graphical method and numerical testing, which are two techniques of statistical analysis were reviewed and used to investigate the statistical distribution of total station measured quantities. It is concluded that total station

measured quantities are normally distributed. Both statistical and graphics techniques used point to this inference.

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