



Numerical and Experimental Investigation of Carbon Fiber Composites Subjected to a Discrete Source of Damage

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Abstract

The paper investigates the effect of a discrete source of damage on the loading capacity and failure strength of composite materials. First, the effects of varying fiber orientation on the distribution of the circumferential stress around the source of damage and on the failure strengths of continuous fiber-reinforced composites were investigated using theoretical approaches developed for infinite-plate with a central hole. Numerical simulations were conducted and spatial graphical illustrations were plotted using fiber orientation angle and angular location on the circumference of the hole as analysis variables. Tsi-Wu failure criterion was employed to predict the failure strength of a composite layer subjected to a discrete damage with a varying fiber orientation. Second, experimental tests using specimens with and without damage were carried out to find out the impact of discrete damage on the loading capacity and failure strength of woven Carbon-epoxy composite laminates. A qualitative analysis was performed to evaluate the effect of the size of damage on the structural performances of laminated composites.

Keywords: Continuous fiber-reinforced composites; discrete damage; stress concentration; failure strength; woven composites.

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1. Introduction

The increased use of composite materials in aeronautical and aerospace vehicles such as aircraft, spacecraft, helicopters, and missiles requires in-depth analysis of the structural performances of these materials under potential failure conditions. Structures using high performance continuous fiber-reinforced composites are vulnerable to mechanical performance degradation in many aspects including interlaminar delamination, loading rate, in-service impact damage, destruction by shock, and vibration.

It has been well known that composite materials have a significant damage sensitivity, which can have an effect on their structural performances mainly their failure strengths. Hence, the most important tasks of composite design calculation are the damage assessment and failure strength prediction.

In recent years, there have been several interesting investigations and continuous efforts in developing new damage mechanics models, stress concentration prediction approaches, and failure criteria for damage assessment of composite materials. In literature, there exist a large number of these models, approaches, and criteria [1-10]. These contributions are still an open issue for engineering and a comprehensive evaluation of their accuracy and their ability to reveal the mechanisms behind damage and failure of composite materials seems to be not yet completely addressed. Hence, further theoretical, numerical, and experimental investigations are typically required for good understanding and assessment of failure and damage of continuous fiber-reinforced composites.

There are three major elements in the analysis of damaged composite materials, i.e., stress distribution around damage source, lamina failure, and laminate stiffness reduction. To deal with these issues, a three-part process is adopted in the present study. First, using the theoretical formulations developed in [11,12] for plates with a central hole, the effect of fiber orientations on the circumferential stress distribution around a discrete damage source is investigated simulating this source as a hole. Second, Tsi-Wu failure criterion is employed to predict the failure strength of a composite layer subjected to a discrete damage source with varying fiber orientations. Comparatively to Tsai-Hill criterion used in [11], Tsi-Wu failure criterion is more general due to its ability to distinguish between tensile and compression strengths. Two composite laminas are used for comparison purposes, i.e. Carbon-epoxy and Glass-epoxy composite sheets. Third, to understand the global behavior of damaged woven composite materials, two experiments are set up, one for undamaged laminates to evaluate the loading capacity of woven Carbon-epoxy materials, and one for damaged laminates to verify the effect of damage on this capacity. It is worth noting that experimental damage assessment of woven fiber reinforced composites is one of the strongest methodologies for the evaluation of damaged material loading capacity and for the study of the phenomena behind failure under damage [13-18].

2. Analytical Modeling of a Composite Plate with Hole

2.1. Lamina Representative Unit Cell and Macroscopic Properties

Composite materials are commonly defined as materials formed using continuous or discrete fibers (e.g. Carbon, Glass, aramid, boron, and silicon) combined with matrix materials (e.g. metallic, ceramic, and polymer). In this

study, the focus is on the continuous fiber-reinforced composite materials due to their oriented mechanical properties and highest structural performances due to the mechanism of spatially fiber varying orientations. A uniaxial composite unit cell configuration is given in Figure 1.

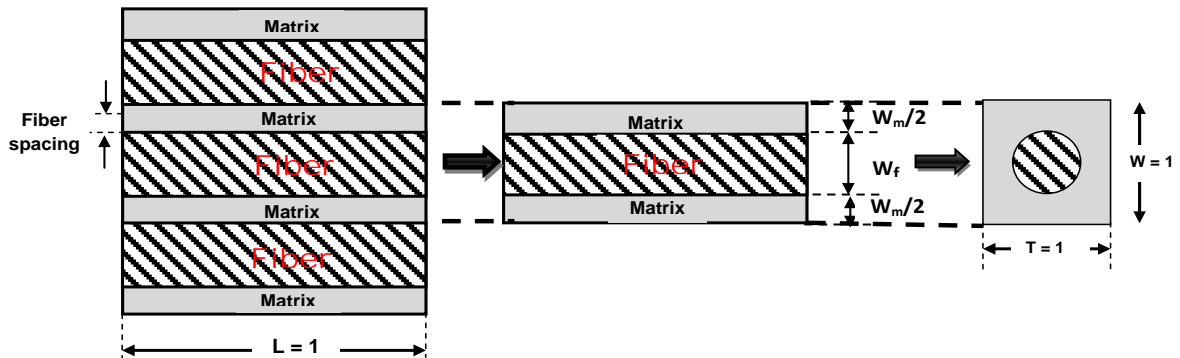


Figure 1: Configuration of unit cell of an uniaxial composite layer.

The different volume units are given as follows

$$\left\{ \begin{array}{l} V_f = \pi \left(\frac{W_f}{2} \right)^2 L \\ V_m = W_m * T * L \end{array} \right. ; \left\{ \begin{array}{l} \eta_f = \frac{V_f}{V_c} = \frac{W_f}{W} \\ \eta_m = \frac{V_m}{V_c} = \frac{W_m}{W} \end{array} \right. \quad (1)$$

L , T , W , V_c denote the length, height, width, and volume of the cell; W_f , V_f , W_m and V_m denote the width and volume of fiber and matrix; η_f and η_m denote the fiber and matrix volume fraction, respectively.

The existing approaches to the micromechanics of composite materials are mainly grouped into two categories; Mechanics of Materials Approach (MMA) and elasticity-based approaches that regroup exact solutions, bounding principles, and approximate solutions. MMA assumes that lamina's properties are different in various directions or orientations, but not different from one location to another. The macroscopic properties of a composite lamina are derived based upon the following assumptions [19,20]

- Fibers and matrix are perfectly bonded and slip-free.
- Fibers are continuous, parallel, and possess uniform strength.
- Geometrical properties such as fiber's diameter, spaces between fibers, etc. are supposed to be uniform.
- Fiber and matrix materials are supposed to be linear.

Macroscopically, a lamina is supposed to be homogenous, orthotropic, linearly elastic, and initially stress-free. As the lamina is considered a biaxial structure (Figure 2), the geometrical deformations and mechanical stresses under tensile loading are determined as follows

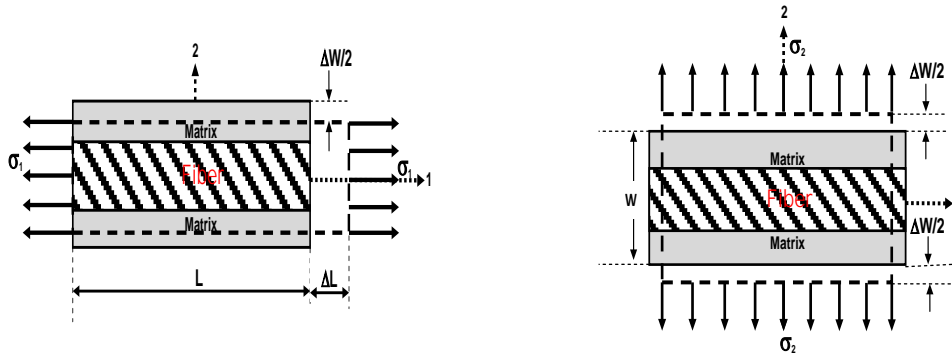


Figure 2: Uniaxial fiber-reinforced composite lamina under biaxial tensile loading

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 & 0 & 0 \\ \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 & 0 & 0 \\ 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & G_{13} & 0 \\ 0 & 0 & 0 & 0 & G_{23} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \quad (2)$$

For a linear elastic orthotropic material based on the C^1 shell elements continuity requirement, the terms associated with the transverse or interlaminar shear effects (i.e., terms associated with γ_{13} and γ_{23}) are eliminated. The corresponding stress-strain relationships for this high-order continuity element are written as:

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \gamma_{12} \end{Bmatrix} \quad (3)$$

With

$$\epsilon_{11} = \Delta L/L, \quad \epsilon_{22} = \Delta W/W, \quad \gamma_{12} = \tau_{12}/G_{12}$$

Where ΔL and ΔW are the longitudinal cell and transverse cell deflection, respectively.

2.2 Circumferential Stresses Around a Central Hole

Consider the hollow infinite plate under tensile loading shown in Figure 3. Upon the theory of orthotropic plates, an analytical model for evaluating the circumferential stress around a hole is presented in [1] and adapted and used for composite materials with fiber orientation in [11,12]. For a reinforced composite layer with fibers oriented by an angle θ and subjected to an axial tensile loading σ_x , as shown in Figure 3, the circumferential stress around the hole is computed in the direction α using the following formulation

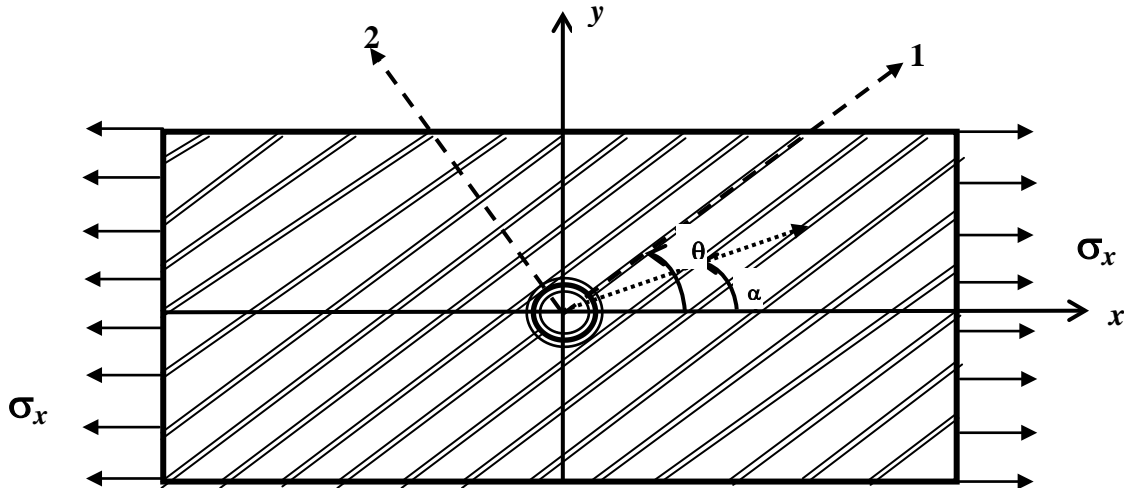


Figure 3: Composite plate with central hole subjected to uniaxial loading

$$\sigma(\theta, \alpha) = \frac{N_1 + N_2 + N_3}{(1 + \gamma_1^2 - 2\gamma_1 \cos 2(\alpha - \theta))(1 + \gamma_2^2 - 2\gamma_2 \cos 2(\alpha - \theta))} \sigma_x \tag{4}$$

With

$$\begin{cases} N_1 = (1 + \gamma_1)(1 + \gamma_2) + \cos 2(\alpha - \theta) \sin^2 \theta [1 + \gamma_1 + \gamma_2 - \gamma_1 \gamma_2 - 2 \cos 2(\alpha - \theta)] \\ N_2 = -4 [\gamma_1 + \gamma_2 - (1 + \gamma_1 \gamma_2) \cos 2(\alpha - \theta) \sin^2 \theta] \\ N_3 = -4(\gamma_1 \gamma_2 - 1) \sin 2(\alpha - \theta) \sin \theta \cos \theta \end{cases} \tag{5}$$

And

$$\left\{ \begin{aligned} \gamma_1 &= \frac{\sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right) + \sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right)^2 - \frac{E_2}{E_1}} - I}{\sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right) + \sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right)^2 - \frac{E_2}{E_1}} + I} \\ \gamma_2 &= \frac{\sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right) - \sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right)^2 - \frac{E_2}{E_1}} - I}{\sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right) - \sqrt{\left(\frac{E_2}{2G_{12}} - \nu_{12}\right)^2 - \frac{E_2}{E_1}} + I} \end{aligned} \right. \quad (6)$$

2.3 Failure Strength

The failure strength of a lamina composite is investigated using Tsi-Wu (T-W) failure criterion. T-W theory is a phenomenologically based failure theory that is widely used to predict the failure strength for anisotropic composite materials. It is worth noting that T-W criterion is more general than Tsai-Hill failure criterion because of its ability to distinguish between tensile and compression strengths. In plane stress, the T-W failure criterion reduces to the following quadratic function

$$a\sigma_f^2 + b\sigma_f - 1 = 0 \quad (7)$$

With

$$\left\{ \begin{aligned} a &= \frac{1}{\sigma_{1t}\sigma_{1c}} \left(\frac{\sigma_1}{\sigma}\right)^2 + \frac{1}{\sigma_{2t}\sigma_{2c}} \left(\frac{\sigma_2}{\sigma}\right)^2 + \frac{1}{S^2} \left(\frac{\sigma_{12}}{\sigma}\right)^2 + \sqrt{\frac{1}{\sigma_{1t}\sigma_{1c}\sigma_{2t}\sigma_{2c}}} \left(\frac{\sigma_1\sigma_2}{\sigma^2}\right) \\ b &= \frac{1}{\sigma_{1t}\sigma_{1c}} \left(\frac{\sigma_1}{\sigma}\right) + \frac{1}{\sigma_{2t}\sigma_{2c}} \left(\frac{\sigma_2}{\sigma}\right) \end{aligned} \right. \quad (8)$$

Where σ_f denotes the failure strength in the uniaxial layer while σ_{1t} , σ_{1c} , σ_{2t} , and σ_{2c} denote the uniaxial tensile and compression failure strengths in the fiber direction and transverse direction, respectively.

3. Numerical Investigation

In this section, the analytical formulations shown in section 2 are used to create three-dimensional mapping of the distribution of the circumferential stress around the hole using fiber orientation θ and the angle α as study parameters. Two composite laminas are used to analyze the effect of discrete source of damage on the composite materials: Carbon-epoxy lamina and Glass-epoxy lamina. The properties of these two composites are given in Table 1.

Table 1: Physical properties of composite materials [11].

Property (GPa)	Lamina	
	Uniaxial Carbon- epoxy	Uniaxial Glass-epoxy
E_1	0.181	0.0549
E_2	0.0103	0.0183
G_{12}	0.00717	0.00914
σ_{1t}	1.5	1.0555
σ_{1c}	1.5	1.0555
σ_{2t}	0.04	0.0281
σ_{2c}	0.246	0.1407
S	0.068	0.0422

Poisson's ratio is taken as $\nu_{12} = 0.28$ for Carbon-epoxy lamina and $\nu_{12} = 0.25$ for Glass-epoxy lamina.

3.1 Failure Strength

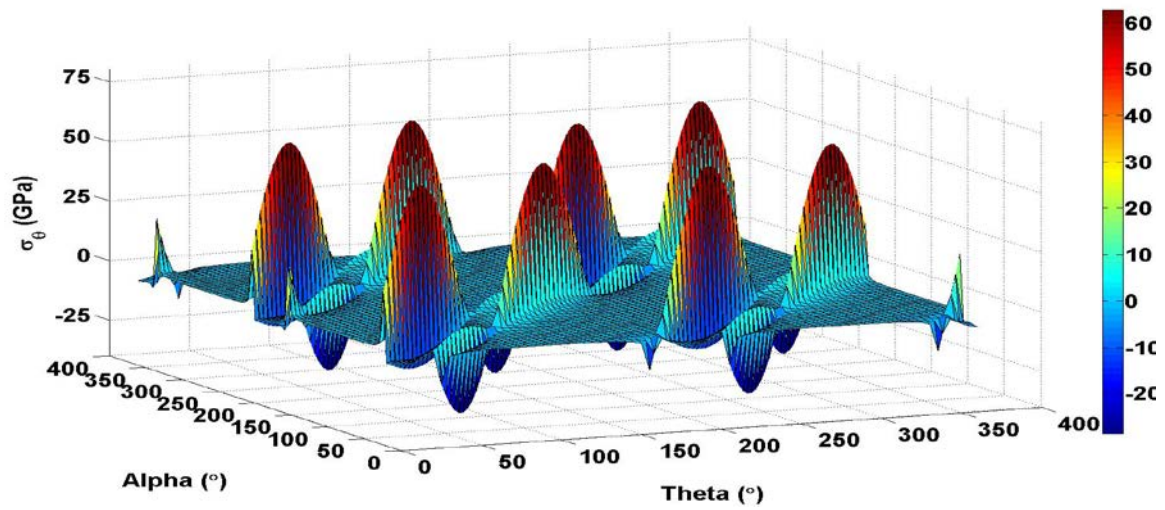
Figure 4 shows the distribution of the circumferential stress around the hole under a unit tensile stress loading.

3.2 Failure Strength from Analytic Predictions

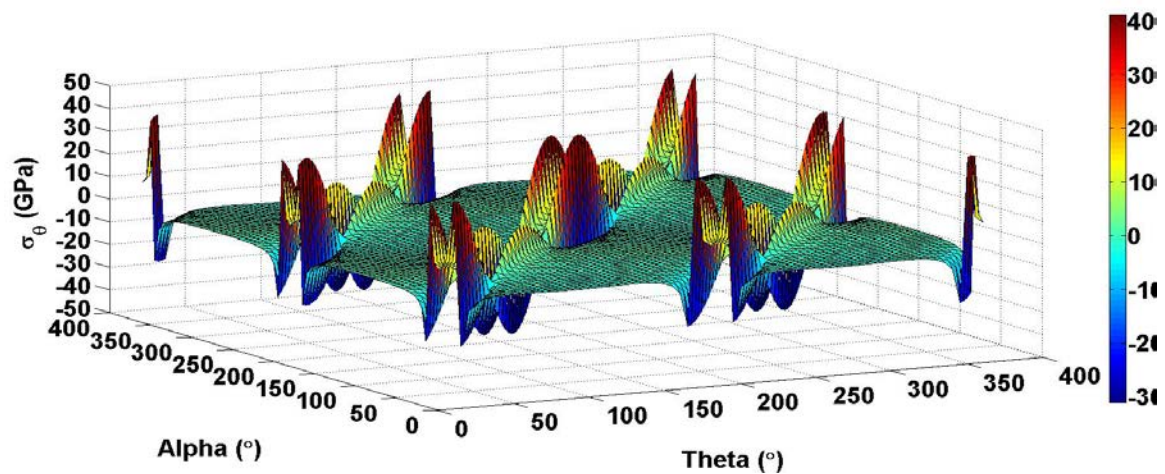
Using the criterion (7), the failure strengths are computed around the hole using the following transformations

$$\begin{cases} \sigma_1 = \sigma \cos^2 \theta \\ \sigma_2 = \sigma \sin^2 \theta \\ \sigma_{12} = \sigma \sin \theta \cos \theta \end{cases} \quad (9)$$

Figure 5 shows the distribution of the tensile failure strength around the hole under a unit tensile stress loading.



(a)

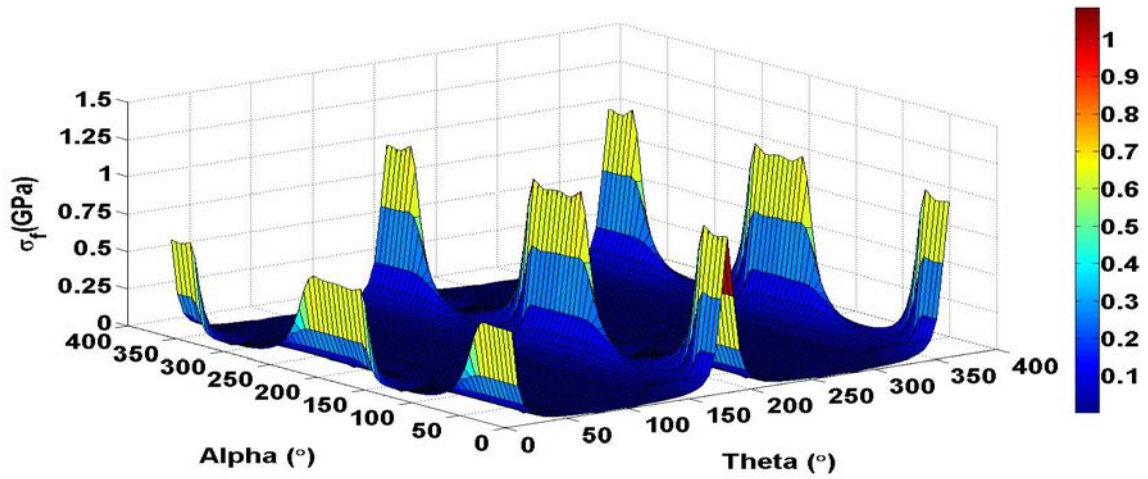


(b)

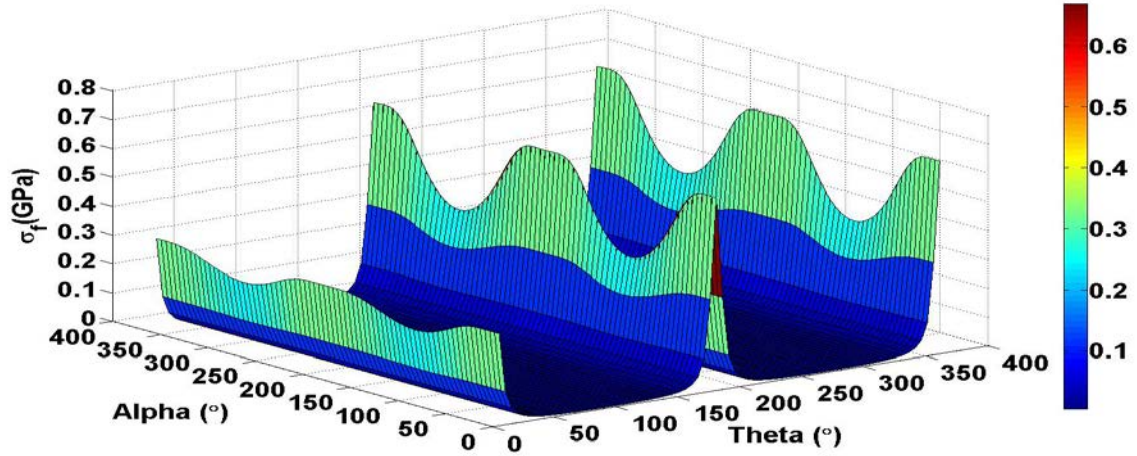
Figure 4: Circumferential stress distribution around the hole for a) Carbon-epoxy composite, (b) Glass-epoxy composite

4. Experimental Investigation

A number of experiments were conducted to assess the effect of a discrete source of damage on the tensile failure strength of a woven composite material. The Carbon-epoxy composite laminates were manufactured with five plies of (0°/90°) woven prepreg. The plies were obtained with 40% of Carbon fibers. Table 2 shows the properties of the carbon fibers and epoxy used in this experimentation. Specimens were divided into three groups; without hole, with central hole of diameter of 10mm, and with central hole of diameter of 25 mm. Figure 6 shows 250.0×50.0×1.8mm woven Carbon-epoxy samples without and with a central hole.



(a)



(b)

Figure 5: Tensile failure stress distribution around the hole: (a) Carbon-epoxy layer, (b) Glass-epoxy layer.

Table 2: Properties of Woven Carbon fibers and epoxy

Property	Carbon fiber	Epoxy
E_1	2.757903e11	3.447379e9
E_2	2.757903e11	3.447379e9
G_{12}	9.65266e10	1.378951e9
ν_{12}	0.33	0.33

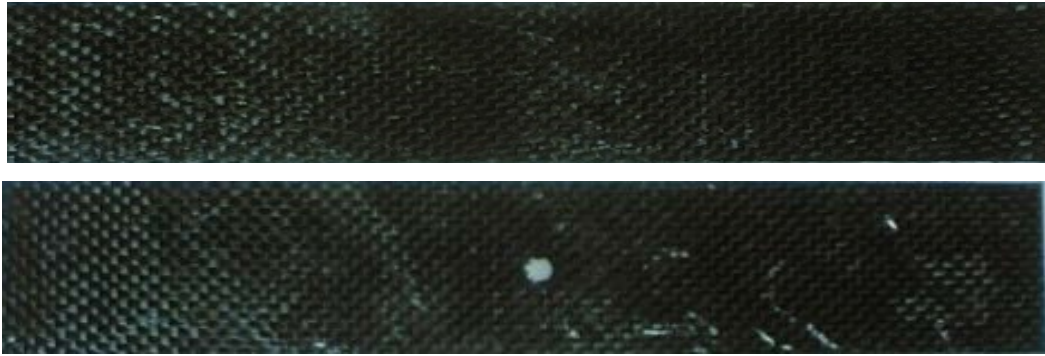
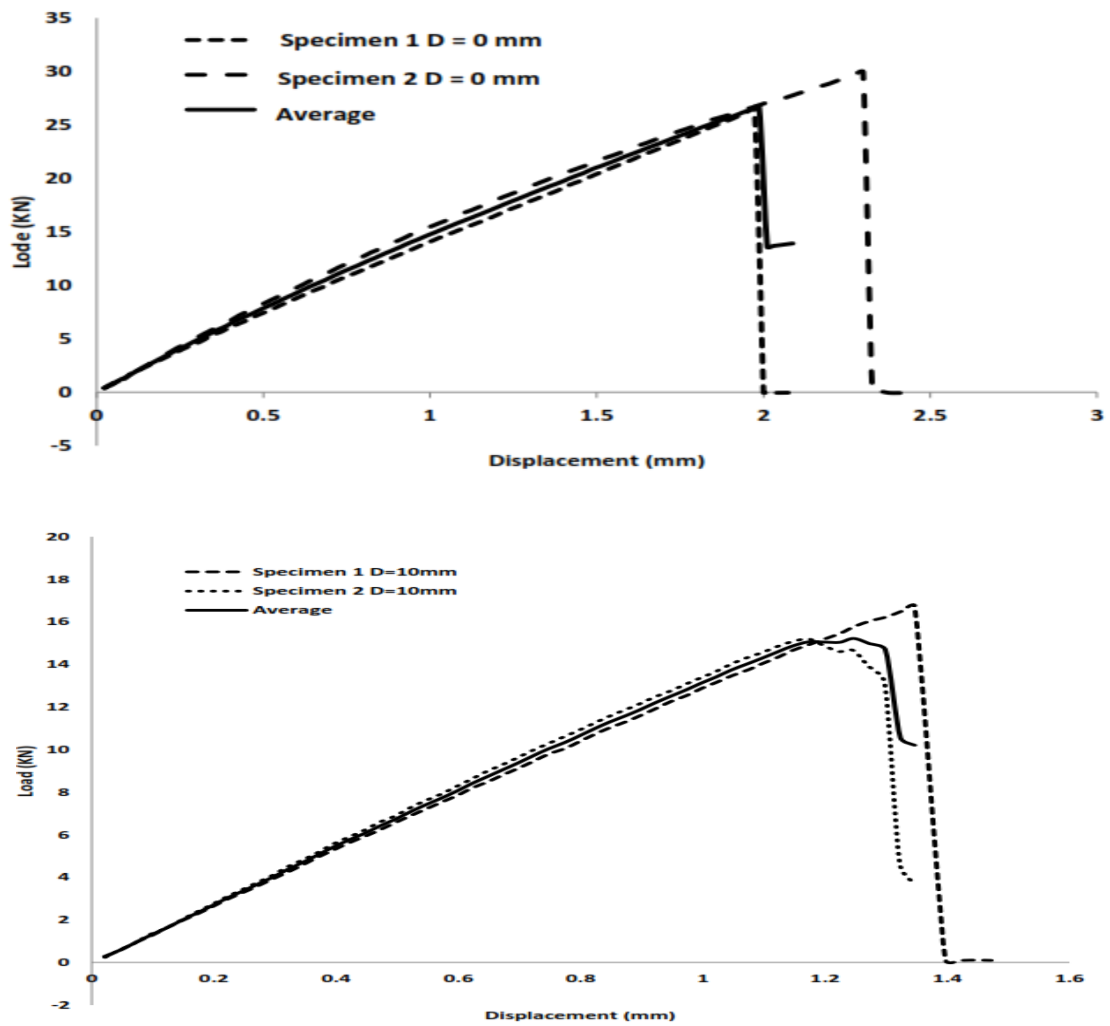


Figure 6: Woven carbon composite specimens without and with a central hole

The experimental results for different specimens with average values are shown in Figure 7.



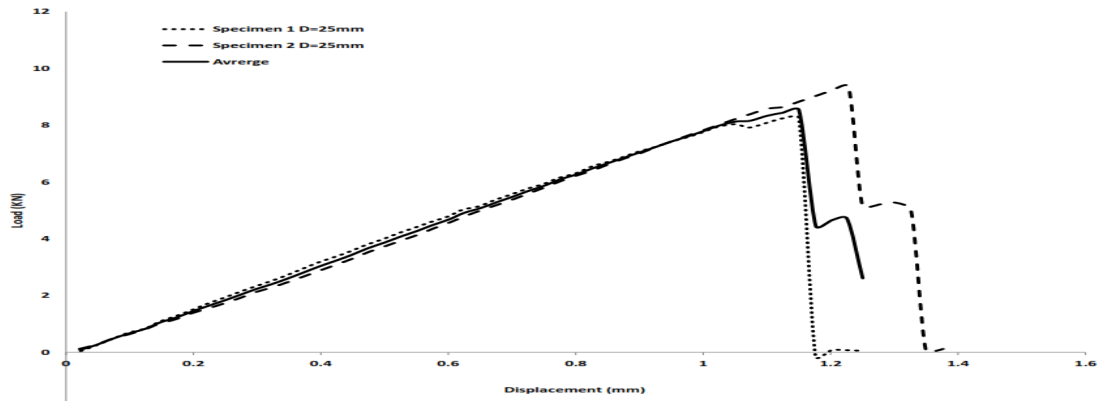


Figure 7: Experimental results for different specimens with average values

The loads and displacements corresponding to the failure of specimens are presented in Table 3 and plotted in Figure 8 using polynomial interpolations.

Table 3: Tensile test results at the failure of specimens

Hole Diameter (mm)	Axial force (kN)	Axial displacement (mm)
0	28.1882	2.1358
10	15.2192	1.3471
25	08.5458	1.2503

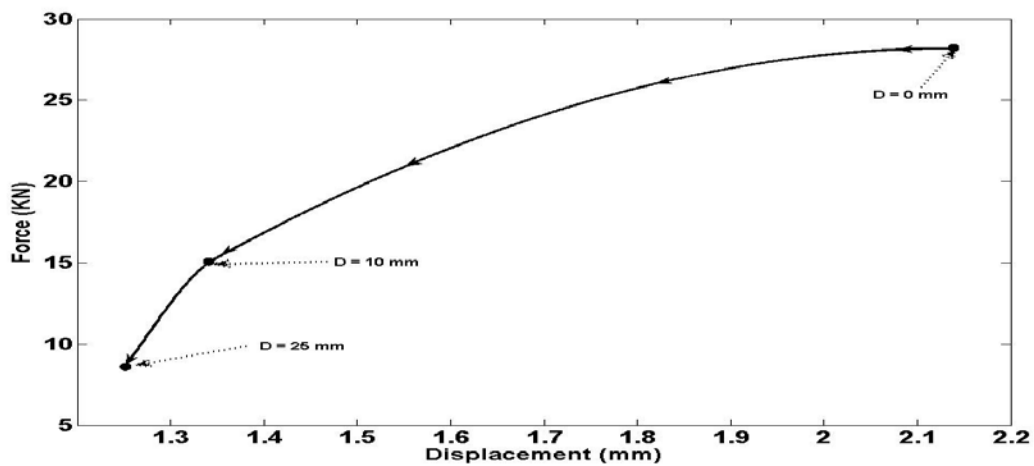


Figure 8: Failure loads versus failure displacements of different specimens

From Figure 8, it is clear that the presence of a discrete source of damage considerably reduce the tensile failure strength of composite materials including woven fabric composites. Results show that the damage size has a proportional effect on the strength of the woven composite materials.

5. Conclusion

This paper presented a numerical and experimental analysis of the effect of a discrete source of damage on the failure strengths of composite materials. In the first part, the effects of fiber orientation on both stress distribution around the damage source and tensile failure strengths were numerically investigated using theoretical approaches. Two composite laminas with different materials i.e. Carbon-epoxy and Glass-epoxy laminas were used to illustrate these effects. Graphical results depicted in Figure 4 show that high circumferential stress values for Carbon-epoxy composite correspond to the fiber orientations around $\theta = \{45^\circ, 140^\circ, 220^\circ\}$ while for Glass-epoxy composite the highest values are for $\theta = \{0^\circ, 180^\circ\}$. In Figure 5, the tensile failure strength values obtained from Tsi-Wu criterion are presented for both composite materials. Results show that higher failure strengths correspond to fiber orientations around $\{0^\circ, 100^\circ\}$ for both composites.

The second part was dedicated to the experimental assessment of the effect of a discrete source of damage size on the strength level of woven Carbon-epoxy composite materials. From Figures 7 and 8, it is clear that the presence of a discrete source of damage considerably reduces the strength level of composite materials even woven fabrics are used. The load resisting capacity of woven Carbon-epoxy materials decreases proportionally to the size of the source of damage.

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