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Determination of the Nonlinear Muskingum Model Coefficients Using Genetic Algorithm and Numerical Solution of the Continuity Equation

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Abstract

The optimization method is an appropriate choice for determining optimal parameters in the Muskingum model, in order to increase the speed of computations; coefficients of this model have been computed optimally with assistance of the genetic algorithm. These coefficients were computed from the linear Muskingum and Muskingum-Cunge models using required data and the available relations. In order to evaluate efficiency of the procedure of optimizing coefficients of the nonlinear Muskingum model via the genetic algorithm method compared with the other two methods used for determining these coefficients, outflow hydrographs were computed using the optimal coefficients and solving the continuity equations according to the Runge-Kutta method order 4 and was compared with the two flood routing methods from the Muskingum and Muskingum-Cunge models as well.

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To study the precision of these three methods, square root of sum of squares of difference of discharges computed from each of the three methods and observational discharges obtained from the HEC-RAS RMSE software was used as the objective function and achieved results indicate more proximity of the computed hydrographs from the optimization coefficients in the Runge-Kutta order 4 to the outflow hydrographs obtained to the HEC-RAS software compared with the two Muskingum and Muskingum-Cunge models.

Keywords: Flood routing; Muskingum model; Muskingum-Cunge model; Optimization; Genetic algorithm; HEC-RAS software

1. Introduction

The problem of dispersion of flood wave along a river length and determination of flood discharge at specified sections and times has numerous applications in reducing flood damage, designing of hydraulic structures, and water planning. To estimate the flood wave movement, flood routing can be used. Flood routing consists of computational operations which predict the velocity value and the flood shape as a function of time at one are more points along the waterways [1]. Flood routing is performed in accordance with the procedures that either directly or indirectly are resulted from the Saint Venant Equations. Flood routing operations are divided in to two hydraulic and hydrologic methods. In case flow routing is considered as a function of time and location, along the system it is a hydraulic or distributional routing. Hydraulic routing methods, regarding the terms selected from the movement size equation are divided info the three categories of kinematic, diffusion and dynamic waves. On the other hand if the water flow is only routed as a function of time at a specified location, it is called the hydraulic or concentrated routing. In this method, the flow continuity equation, natural hydrographs and unique, discharge and maximum surface balance of the flood are used. Using the hydrologic method, although lacking precision of the hydraulic procedure, is very easily performed and is used with an acceptable confidence in designing structures and controlling floods [2]. Amongst the hydrologic flood routing methods, the Mastinum, Muskingum -Cunge, Kanox and Atkin methods can be named. Muskingum method is one of the hydrologic methods which have extensive applications for flood routing. In this method, routing parameters are determined based on the upstream and downstream hydrograph of a flood that has previously accruedin the region and by assuming the existence of a linear relation between the river storage and other hydraulic factors including the inflow and outflow discharges. The linearity assumption of the relation between the river storage and other hydraulic factors is not exact therefore alters the method's precision. Another limitation of this method is the need to calibration. Therefore authorities were persuaded to offer more precise and move comprehensive methods. The Muskingum -Cunge method is one of such methods in which flood routing is performed through the kinematic wave with the difference that the kinematic wave equations have been changed to the dispersion wave with the infinite difference method. Cunge [3] showed that the Muskingum method resembles the dispersion transfer equation and its results are similar to those of the liner kinematic method. Through truncating the kinematic wave equation and accommodation of the numerical dispersion with the physical dispersion modified the Muskingum method. In this manner, parameters of the Muskingum -Cunge method are computed based on physical specifications of the river. Ponce and his colleagues [4] studied the Muskingum – Cunge method with two, three and four point's variable parameters. Their results revealed that the two-point method does not show an appropriate precision for computing the maximum discharge and the time of its occurrence.

On the other hand, the three and four point method is suitable. Victor and his colleagues [5] compared the Muskingum -Cunge model with a dynamic wave model and found out that this model enjoys suitable correctness and precision. Researcher applied the Muskingum -Cunge and Muskingum method to flood routing in the Liqvan river and observed that the results obtained of the routed flow by the two models show a significant difference with the real results registered in the low stream hydrometer station and mentioned as cause of this difference the region being mountainous, presence of underground drains at the interval between the two up-and low stream stations [6]. Ponce and his colleagues [4] studied the efficiency rate of the Muskingum -Cunge method and found out that high correspondence between the results and computational hydrographs and observations indicate high efficiency of this method amongst the flood routing methods. Karahan and his colleagues [7] studied the applicability of the calibrated Muskingum method for flood routing in the watershed basin of the Manahadi river in India and concluded that this method after being calibrated through appropriate modifications, can be used for a location and a situation like that of the Manahadi watershed basin [8]. The crucial point in solving the nonlinear Muskingum equations is the necessity of appropriate estimation of the three parameters present in this equation which highly influence the routing results and their computation via numerical and trial and error methods is difficult. So far, mach studies have been performed on estimating the linear and nonlinear Muskingum Coefficients. Stephanson [9] using the linear planning method optimized the linear Muskingum wave parameters. Nowadays, employment of modern intelligent methods for predicting and optimization has been considered by the researchers. Gill and his colleagues [8] using the least squares method (LSM) optimized the linear Muskingum model parameters. Results of the research revealed high precision of this method compared with the trial and error method. Following them, Tung and his colleagues [10] based on the Hook-Giouse method, combined with the linear regression methods (HJ+CG), applied the concurrent slope and the DFP method for optimizing these parameters results of which indicated that using the (HJ+CG) and (HJ+DFP) methods compared with the Gale work better. Researcher applied the particle set optimization (PSO) algorithm for estimating parameters of Muskingum nonlinear model results revealed that the PSO algorithm enjoys a high precision for estimating parameters of the Muskingum nonlinear model [11]. Muhan [12] showed that all the previous methods do not ensure achieving an absolute optimal response and will be snared in the trap of local optimal responses. He employed GE for estimating the model parameters and results showed that estimation via GA is better that the previous methods and does not need to a primary estimation close the optimal response. In this research, for the purpose of considerably increasing the speed of executing computations of the flood routing model in the rivers, the idea of optimizing coefficients of the nonlinear Mastingum hydrologic method by the assistance of the genetic algorithm has been used. Regarding the fact that determination of the x and k coefficients in the Muskingum method via the conventional methods needs repetitive computations and is very time consuming, the idea of optimizing these coefficients will undoubtedly lead to decreased time for executing the computations. In order to evaluate efficiency of the optimization method in this research, the computed coefficients of the Muskingum and Muskingum -Cunge linear model for flood routing were used and rouging was performed through solving the differential equations of first order by the Fung-Kuta order 4 and results obtained of this method were compared with outflow hydrographs computed by the hydrologic methods like Muskingum and Muskingum -Cunge model in 5 flood outflows generated by the HEC-RAS as well.

1.1. Hydrologic routing based on the continuity equation

1.2. The Muskingum model

One of the flood routing methods was first applied in 1938 by Mc Carty and the group of engineers of the USA army in connection with the flood control plants at the Muskingum River basin in Ohio State and it was introduced. Hydrologic flood routing in the rivers can be performed via various methods amongst which the Muskingum and Muskingum -Cunge methods are worth mentioning. This method is a linear routing method in the rivers which due to scarcity of required parameters has been always considered by the hydrology engineers. This method is based on the continuity equation:

$$\frac{dS}{dt} = I(t) - O(t) \tag{1}$$

in which S stands for water storage volume of the river, I is discharge of the inflow flood, and O is discharge of the outflow flood. In the linear Muskingum model, water storage in divided into two categories of prismatic storage kO and the wedge storage kx(I-O). With the assumption that the storage volume is a linears function of the inflow and outflow flood, in two temporal steps t and t+1 the linear storage equation can be applied as follows:

$$S^t = k\left[xI^t + (1-x)O^t\right] \tag{2}$$

Through writing the form of finite difference equations 1 and 2, and their combination and finally their simplification we arrive to the following equation based on which routing can be done.

$$O_c^{t+1} = C_0 I^{t+1} + C_1 I^t + C_2 O^t \tag{3}$$

$$C_0 = \frac{\Delta t + 2kx}{\Delta t + 2k - 2kx} \tag{4}$$

$$C_1 = \frac{\Delta t - 2kx}{\Delta t + 2k - 2kx} \tag{5}$$

$$C_2 = \frac{-\Delta t + 2k - 2kx}{\Delta t + 2k - 2kx} \tag{6}$$

In the event of inflow and outflow data being unavailable, k designates the river travers time, and x is an undetermined coefficient which indicates importance degree of I and O. Minimum value of x is zero and it maximum value is 0.5 and on the average changes between 0.2 and 0.4. Δt should not be less than 2kx or more than k. such conditions are considered for the purpose of preventing coefficients to become negative and for avoiding numerical instability so that hydrograph of the flood outflow can be computed. The most important stage of computations is determination of x and k coefficients that in case of inflow and outflow hydrographs of a flood incident at the channel or river interval being available, the can be estimated. One of these methods is the old trial and error graphical method developed by Mc Carty (1938) for the Muskingum linear model. Stages for determination of x and k coefficients in the Muskingum model through the trial and error graphical methods are as follows:

1. Suitable values for x and k coefficients of the Muskingum model are selected and then the [xI+ (1-x)O] value is computed considering values of the x and k coefficients.

2. In each temporal period, (I-O) values of observational inflow outflow hydrographs being the real storage are computed. Since of we subtract the inflow and outflow hydrographs, storage modifications at various Δt are obtained the relation of which will be:

$$S = \int_0^t (I - O)dt \quad (7)$$

3. Now, for each Δt, having S (real storage) and xI+(1-x)O in a system of coordinate axes draw the xI+(1-x)O, it will be observed that the curve obtained from connecting various loop points will be found that is the value selected for x has not been correct once again we select another value for x and the same scenario is repeated.

It will be considered that in case x is appropriately selected, the loop becomes narrower and will resemble a line.

4. The inverse line slope (loop) will be the k value

A crucial and fundamental issue in the Muskingum method in that the x and k parameters have no physical meaning and they can be computed only in the event of inflow and outflow hydrographs of a flood incident being available.

1.3. The Muskingum – Cunge model

This problem has been solved in the Muskingum-Cunge method where the x and k parameters are expressed according to the physical properties of the channel interval:

$$k = \frac{\Delta x}{c_k} = \frac{\Delta x}{\frac{dQ}{dA}} \quad (8)$$

$$x = \frac{1}{2}\left(1 - \frac{Q}{Bc_k s_0 \Delta x}\right) \quad (9)$$

In the above equations, Q stands for the reference discharge, B is width of the upper level of water relevant to the base discharge, and so is the longitudinal length of the channel Δx is length of the channel and ck is the wave transfer. Procedure is so arranged that at first the medium discharge is selected as the base discharge. Selection of the medium discharge as the reference discharge is done through the trial and error method. In the next step, using the Muning relation, value of the normal depth is commuted. Following determining the normal depth, hydraulic parameters of the flow such as hydraulic depth (D), and upper width of the flow (T) at each section can be determined. C0, C1, C2 coefficients are also computed using the 4 to 6 equations and finally outflow discharge can be computed via equation 3.

1.3. Numerical solution of the continuity equation using the Runge-Kutta method order 4.

Although the linear form of the Muskingum model has a broad application for flow routing however, in the majority of rivers, there governs a nonlinear relation between the discharge and the storage. This issue is a motive for employment of the Muskingum linear model to appear inappropriate to a great degree and can bring about significant errors in predicting the flow situation. Gill and his colleagues [8] suggested the relation of the Muskingum nonlinear model at the temporal step t and t+1 in the form of equation 10:

$$S_t = k\left[xI_t + (1-x)O_t\right]^m \qquad (10)$$

$$S_{t+1} = k\left[xI_{t+1} + (1-x)O_{t+1}\right]^m \qquad (11)$$

In the Muskingum nonlinear model, the m parameter has been added as power to the equation, which enables the model to better model the nonlinear relation between the cumulative storage and the flow. Considering the equation 11, outflow discharge can be obtained in the temporal step t+1 from the relation 12.

$$O_{t+1} = \left(\frac{1}{1-x}\right)\left(\frac{S_{t+1}}{k}\right)^{\frac{1}{m}} - \left(\frac{x}{1-x}\right)I_{t+1} \qquad (12)$$

In combination with the continuity equation we have:

$$\frac{dS}{dt} = -\left(\frac{1}{1-x}\right)\left(\frac{S_t}{k}\right)^{\frac{1}{m}} + \left(\frac{1}{1-x}\right)I_t \qquad (13)$$

Relation 15 is a differential equation of first order that can be solved via various methods of which is the Runge-Kutta order 4 as follows:

$$S_{t+1} = S_t + \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right) \qquad (14)$$

In the above said relation k1 to k4 are values of the Runge-kutta order 4 coefficients which are obtained of the following relations:

$$K_1 = -\Delta t\left[\left(\frac{1}{1-x}\right)\left(\frac{S_i}{k}\right)^{\frac{1}{m}} + \left(\frac{1}{1-x}\right)I_i\right] \tag{15}$$

$$K_2 = -\Delta t\left[\left(\frac{1}{1-x}\right)\left(\frac{S_i + \frac{K_1}{2}}{k}\right)^{\frac{1}{m}} + \left(\frac{1}{1-x}\right)\frac{I_i + I_{i+1}}{2}\right] \tag{16}$$

$$K_3 = -\Delta t\left[\left(\frac{1}{1-x}\right)\left(\frac{S_i + \frac{K_2}{2}}{k}\right)^{\frac{1}{m}} + \left(\frac{1}{1-x}\right)\frac{I_i + I_{i+1}}{2}\right] \tag{17}$$

$$K_4 = -\Delta t\left[\left(\frac{1}{1-x}\right)\left(\frac{S_i + K_3}{k}\right)^{\frac{1}{m}} + \left(\frac{1}{1-x}\right)I_{i+1}\right] \tag{18}$$

Therefore for determining the outflow hydrograph, the following steps should be taken.

Step one: Computation of S based on relation 10 and in exchange for Ot and It.

Step two: Calculation of St+1 from the relation14. Step three: Calculation of Ot+1 based on relation 12. Therefore, through this procedure in the event of values of the parameters x, k and m being known, the hydrograph can be determined. Since the outflow hydrograph has been computed by the HEC-RAS software, unknowns of the Muskingum nonlinear model in the above relations will be the coefficients of this model which, for each developed outflow hydrograph of the software which bear particular geometric and hydraulic specifications, optimal coefficients of the nonlinear Muskingum model are obtained. As it was pointed, the genetic algorithm (GA) is an appropriate choice too determination of optimal parameters in the Muskingum nonlinear model. Results of comparison of this method with other methods of estimating nonlinear parameters of the Muskingum model. Like optimization in excel software has shown that the genetic algorithm method is sensibly superior to these methods. Therefore, the genetic algorithm was used to determine optimal coefficients of the Muskingum nonlinear model. Such stages were performed using programming in MATLAB software.

2. Determination of the Muskingum nonlinear model coefficients via the genetic algorithm

Regarding importance of optimization in the field of engineering sciences and other disciplines, during the recent years much attempt has been in this regard and numerous methods have been employed and in determining the Muskingum model parameters too optimization has been used. X, k and m parameters through conventional procedures require repeated and very time consuming computations. Employment of optimization methods including algorithm lead to reduce of computations and increased precision. Amongst intelligent algorithms, the genetic search method the idea of which has been adapted from the system of natural evolution of living creatures (gene and chromosome), as the modern method of optimizing nonlinear models, is very appropriate and has frequent testes application [8].

3. Genetic algorithm

Genetic algorithm tests and examines the algorithms developed based on the meaning of natural structures and their natural genetics. This method has wide applications in engineering issues such as optimization of the flow network in the pipes, constructional structures, determination of culture optimized model, determination of flow discharge of composite sections, calibration of rain-run off and underground waters. [8, 4]. Also these methods have been widely applied in hydraulic models. The optimization process in genetic investigation is such performed that at first, a primary population is created and then it is multiplied, bears mutation and genetic exchange is performed on this population. Firstly some responses within the limit of modification of parameters are estimated and through converting these numbers to zero and one chains, values of objective function in lieu of this response value are obtained. In this research x, k and m parameters of the Muskingum nonlinear model will be optimized. The modification limit for x is between 0 and 0.5. For k however, no limit has been defined. Then using the rotating wheel (roulette) method, the chains for which the objective function is minimized remain and the rest are omitted. Following selection of Chains and through choosing the probability percentage of the appropriate combination, loci of the function of chains are selected for exchange of information. In the manner that a point of the data chain is selected and thereafter all the next zero and ones from this point will be exchanged in two combining chains. Also through selection of appropriate percent for the mutation probability, locus or loci of the responses chain are selected and the numbers within these loci are changed from zero to one and vice versa. Then once again the objective function for the new population is computed and this trend is so much repeated that all the responses are directed toward the optimal point. This response (chromosome) is introduced as the best estimation for parameters of the flow discharge at any time. In this research for comparing the methods for determination of the Muskingum coefficients, from the HEC-RAS software, we attempted to create observational hydrographs of 5 suggested floods and the Muskingum coefficients with required data in each method were determined and as a result, flood routing via the three Muskingum, Muskingum-Cunge, and Runge-Kutta order 4 were performed. The objective function for evaluation of optimal x, k and m parameters in the nonlinear Muskingum model, minimization of sum of squares of the remainders between observational and routed outflows SSQ has been considered. The objective function too, can be compared with the relation 19.

$$SSQ = \sum_{i=1}^{n} \left(O_c - O_m\right)^2 \qquad (19)$$

Also, for more precise study of the hydrographs computed of the methods studied in this research, the factor of square root of difference of sum of squares of discharge, computed from the RMSE observed for the occurrence time was used.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(O_c - O_m)^2}{n}} \qquad (20)$$

Procedure used for computing this parameter is as follows: In which Oc stands for the computed discharge, Om is the observational discharge of the HEC-RAS software and n is the number of observed and computational pair discharges of each model. The SSQ and RMSE parameters for results of each of the three routing methods are presented in tables 1 to 3. In order to control the results obtained from the previous comparison and identifying the probable mistake or error factors, the observational and computational values for each flood have been compared separately.

4. Results and conclusion

Standard errors and optimized coefficients of the Muskingum nonlinear model obtained of the genetic algorithm method, along with the coefficients obtained of the Muskingum linear hydrologic models and Muskingum – Cunge models are presented in the tables 1 to 3.

Table 1: The comparison between non-linear Muskingum model error and optimization coefficient in *Runge*-Kutta 4th.

Standard error for estimated parameters	Flood 1	Flood 2	Flood 3	Flood 4	Flood 5
SSQ (m^3/s)	20.75	27.43	24.23	72.78	179.18
RMSE (m^3/s)	0.74	1.17	0.85	1.46	2.79
x	0.055	0.063	0.15	0.15	0.2
K (hours)	0.67	0.77	1.84	1.75	3.02
m	1.03	10.3	1.001	1.2	1.3

Table 2: The comparison between linear Muskingum-Cunge model coefficient and error value

Standard error for estimated parameters	Flood 1	Flood 2	Flood 3	Flood 4	Flood 5
SSQ (m3/s)	56.24	59.8	31.35	479.35	573.62
RMSE (m3/s)	1.23	1.72	0.97	3.75	4.99
x	0.42	0.42	0.47	0.45	0.47
K (hours)	0.41	0.109	1.82	2.46	5.88

Table 3: The comparison between linear Muskingum model coefficient and error value

Standard error for estimated parameters	Flood 1	Flood 2	Flood 3	Flood 4	Flood 5
SSQ (m3/s)	80.47	33.4	714.68	889.45	779.06
RMSE (m3/s)	1.47	1.29	4.65	5.11	5.82
x	0.1	0.1	0.1	0.1	0.1
K (hours)	0.32	0.32	0.32	0.32	0.32

Regarding the methods used for determining coefficients of the Muskingum model, in the graphical method, due to the trial and error method, precision of this method is low and values of the Muskingum coefficients have been approximate therefore these computed values in the standard error as well, has been more compared with other studied methods. Also the Muskingum-Cunge model for determining the Muskingum coefficients requires data on cross sections, longitudinal stop of the river, roughness coefficient of the river bed, and also the physical specifications of the river including length of the river, upper width of the flow, and the wetted perimeter which is very complex and time consuming. Compared with these two methods, the genetic algorithm in spite of the long time required for optimizing the considered parameters, it doesn't need collection of a lot of complex data and only through availability of inflow and outflow hydrographs and writing the intended program in MATLAB, optimizes coefficients of the Muskingum nonlinear model. In order to determine precision of the three developed models and the Muskingum and the Muskingum-Cunge models in estimating values of the peak discharge and time of it occurrence, computation error percentage of each parameter relative to observational values was determined and its results are presented in the following diagrams. In these diagrams, the factor of discharge error percent or the peak time computed from the all three methods were compared with the real value in the observed hydrograph of that incident and its dispersion relative to line zero has been obtained. In studying the results of RMSE and SSQ error values it reveals that the Runge-Kutta order 4 routing regarding the minimum error value in the five occurred flood, compared with the two Muskingum and Muskingum-Cunge methods, has computed the outflow hydrograph closer to the values of the observational outflow hydrographs obtained of the HEC-RAS software. In the study of results relevant to the factor of error percent for estimation of the peak discharge, and value of this parameter in two Runge-Kuttaorder 4 and the Muskingum-Cunge model and for each of the floods a negative number has been achieved. This means that both two methods have always estimated the peak discharge value less than the real value. In the Muskingum model however, the peak discharge error percentage has been obtained as positive for most of the floods which in this model, always the peak discharge value has been estimated to be more than the real value. In comparison, diagrams relevant to dispersion around the zero line for the peak discharge error percentage factor, in the overall dispersion for the Runge-Kutta order 4 method and the Muskingum – Cunge model is less and thus, these two methods have computed the peak discharge value with more precision. In the diagrams relevant to dispersion around the zero line for the factor of peak discharge occurrence time error percentage, it is seen that the Muskingum-Cunge model in three of the five floods have correctly computed the peak discharge occurrence time (regarding the zero error percentage for three cases of the floods). Also, in the Range-Kutta order 4 method results have been estimated very close to estimations of peak discharge time by the Muskingum – Cunge model. In the

Muskingum model however, times for the peak discharge have been estimated less that the real time. Therefore, the Muskingum-Cunge and the Runge-Kutta order 4 models are more precise for estimating the peak discharge time estimation.

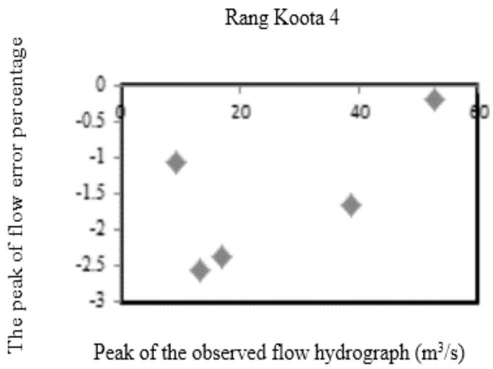

Figure 1: The distribution around 0 line for peak of flow error percentage.

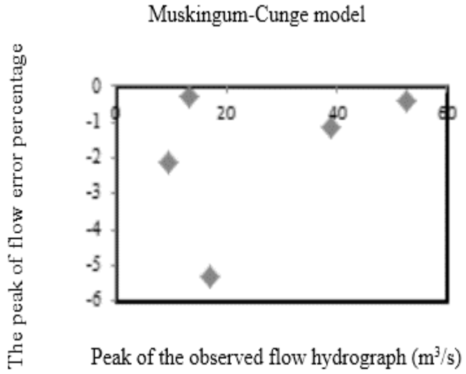

Figure 2: The distribution around 0 line for peak of flow error percentage.

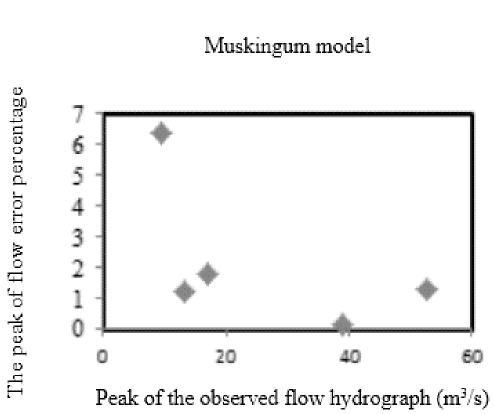

Figure 3: The distribution around 0 line for peak of flow error percentage.

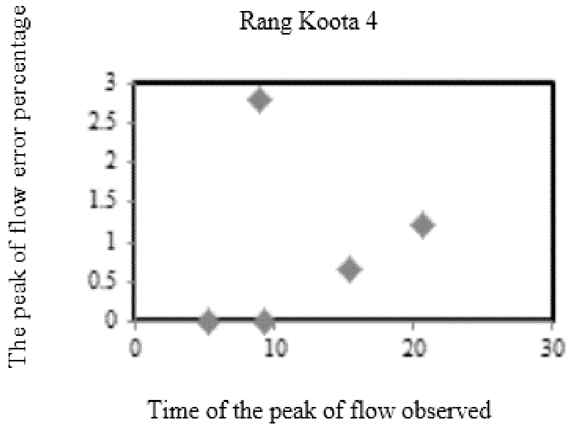

Figure 4: The distribution around 0 line for peak of flow error percentage.

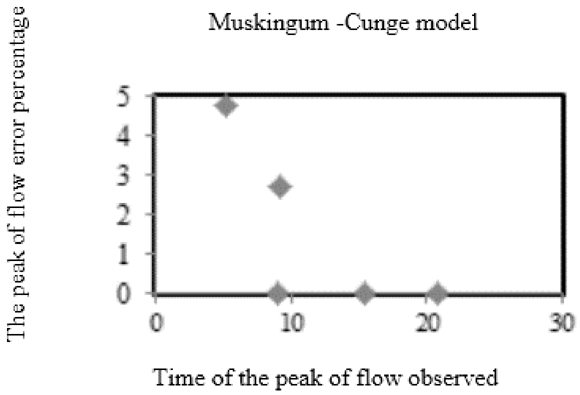

Figure 5: The distribution around 0 line for peak of flow error percentage.

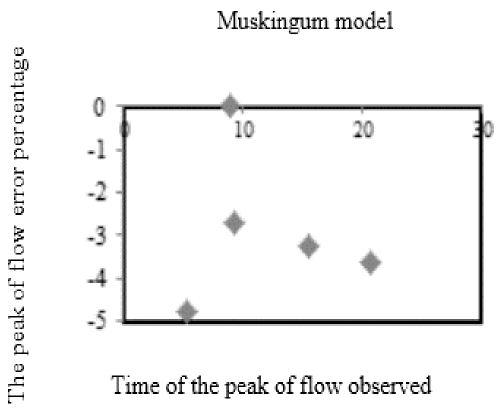

Figure 6: The distribution around 0 line for peak of flow error percentage.

5. Conclusion

In this research efficiency and precision of the three genetic algorithm, Muskingum and Muskingum – Cunge methods for determination of the Muskingum model coefficients of all the three methods with solution of the relevant equations, and flood routing using these coefficients in all these three methods, were investigated. Parameters relevant to comparison of results obtained of routing of five floods due to proximity of the computed values to the observational values from the HEC-RAS software with the genetic algorithm optimization methods relative to the other two methods indicate high precision of this method which is close to that of the HEC-RAS software for flood routing. Also the genetic algorithm and the Muskingum-Cunge model enjoy a high potential for determining the value and time of occurrence of the peak discharge. The advantage of the genetic algorithm however, in comparison with the Muskingum – Cunge model is its easy and rapid application of the Maskingum nonlinear model optimized coefficients over the genetic algorithm in flood routing by the Runge-Kutta order 4 and also its lack of need to use trial and error approximation methods and collection of much data from the river like the Muskingum and the Muskingum-Cunge models, therefore, employment of this method for flood routing is recommended.

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