



A Novel Higher-Order Sliding Mode Control Scheme for Uncertain Nonlinear Systems: Short-period Missile Control Application

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Abstract

The paper proposes a novel higher-order sliding modes (HOSM) control scheme for a class of uncertain nonlinear systems. The HOSM-based control scheme is developed based on the Filippov's differential inclusion and local properties of affine nonlinear systems with control constraints. The resulting control provides exponential stability and ensures robustness against modeling errors and parameter uncertainties. The proposed HOSM-based control scheme is used to design a short-period pitch-axis flight control system of a short-range tactical missile where performance and robustness are demonstrated via computer simulations.

Keywords: Flight control system design; Higher-order sliding modes; Missile dynamic and control; Nonlinear robust control

1. Introduction

For high tracking performance, the design of a missile's flight control system (FCS) is driven by the characteristics of the guidance commands which are determined by the overall system and homing loop requirements. In many interception and navigation applications, the selection of the performance index changes with the flight phase of the

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missile which requires the change of the autopilot type within the FCS. Because of the short amount of time involved in the endgame interception phase, the basic requirements for a tactical missile's FCS to achieve a successful hit-to-kill engagement are fast response, minimum error, and robustness against disturbances.

In order to alleviate the problems associated to the classical linear autopilot and to respond to the requirements of successful endgame engagement against clever targets, various studies have focused on the problem of nonlinear autopilot design for tactical missiles during the last few years. Researchers have sought to augment the classical autopilots with modern robust controllers yielding many control FCSs schemes [1-10]. Although many of these FCSs ensure good tracking, their application is restrained due to certain conceptual and implementation shortcomings such as linearization, high computational cost, and vulnerability to disturbances.

Since their development, Higher-Order Sliding Modes (HOSM) have been receiving more attention and study from aeronautical and aerospace communities. Having proved their high accuracy and robustness, HOSM have been increasingly used to design high-performance FCSs for advanced missiles [11-22]. HOSMs controllers mitigate the problems associated with standard sliding mode controllers such as high-order dynamics, chattering effect, and control input smoothness.

In this paper a new nonlinear discontinuous HOSM control scheme is derived and used to design an advanced HOSM-based pitch-axis tactical missile FCS. Different from the recursive or nested algorithms presented in [23,24], the proposed approach uses directly the higher-order time derivatives of the sliding variable in one combination. This reduces the complexity of the controllers' architecture for systems with higher relative degrees. Two different pitch-axis autopilots are designed; pitch-rate and angle-of-attack (AOA) autopilots. The performance requirements of these autopilots are demonstrated via computer simulations.

The outline of this paper is as follows. The nonlinear short-mode missile's dynamics are presented in section 2. In section 3 the problem of designing discontinuous HOSM-based control is stated and the proposed HOSM scheme is derived. Computer simulations are conducted in section 4 to demonstrate the efficiency and advantages of the proposed HOSM-based missile's FCS. Section 5 concludes the paper.

2. Missile Longitudinal Short-period Dynamics

The short period mode of the longitudinal motion can be described as follows

- *Missile short-mode dynamics*

$$\begin{cases} \dot{\alpha} = K_{\alpha} M C_n(\alpha, M) \cos(\alpha) + q + \frac{g}{v_s M} \cos(\gamma) \\ \dot{q} = K_q M^2 C_m(\alpha, M) + K_q M^2 e_m q + d_m \delta \end{cases} \quad (1)$$

where the aerodynamic coefficients C_n and C_m are estimated as follows

$$\begin{cases} C_n = \text{sgn}(\alpha)[a_n|\alpha|^3 + b_n\alpha^2 + c_n(2 - M/3)|\alpha|] \\ C_m = \text{sgn}(\alpha)[a_m|\alpha|^3 + b_m\alpha^2 + c_m(-7 + 8M/3)|\alpha|] \end{cases} \quad (2)$$

- *Missile velocity and flight patch angle*

The missile velocity and trajectory are introduced by means of Mach number M and path angle γ as exogenous scheduling parameters generated by the following equations

$$\begin{cases} \dot{M} = -K_z M^2 [C_{D_0} - C_z(\alpha, M) \sin \alpha] - \frac{g}{v_s} \sin(\gamma) \\ \dot{\gamma} = -K_z M C_z(\alpha, M) \cos(\alpha) - \frac{g}{v_s M} \cos(\gamma) \end{cases} \quad (3)$$

- *Missile normal acceleration*

$$\eta = K_z M^2 C_n(\alpha, M) \quad (4)$$

- *Actuator dynamics:*

$$\ddot{\delta} = -\omega_a^2 \delta - 2\xi_a \omega_a \dot{\delta} + \omega_a^2 \delta_c \quad (5)$$

The state-space representation of the model (3) is written as follows

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{b}u \\ \mathbf{y} = \mathbf{c}^T \mathbf{x}_d \end{cases} \quad (6)$$

where

$$\begin{cases} \mathbf{x} = [x_1 \quad x_2]^T = [\alpha \quad q]^T, \quad u = \delta \\ \mathbf{A}(\mathbf{x}) = \begin{bmatrix} A_{11}(\mathbf{x}) & 1 \\ A_{21}(\mathbf{x}) & 0 \end{bmatrix}, \quad \mathbf{b} = [0 \quad d_m]^T \\ A_{11}(\mathbf{x}) = \text{sgn}(x_1) K_a M [a_n x_1^2 + b_n |x_1| + c_n (2 - M/3)] \cos(x_1) \\ A_{21}(\mathbf{x}) = \text{sgn}(x_1) K_q M^2 [a_m x_1^2 + b_m |x_1| + c_m (-7 + 8M/3)] \end{cases} \quad (7)$$

$A_{11}(\mathbf{x})$ and $A_{21}(\mathbf{x})$ are uncertain sufficiently smooth functions and $\mathbf{x}_d = [\alpha_d(t) \quad q_d(t)]^T$ is the desired output vector. The variables $-20^\circ \leq \alpha \leq 20^\circ$, $1.5 \leq M \leq 3$, q , γ , and δ denotes Angle-Of-Attack (AOA), Mach number, pith rate, path angle, and elevator deflection, respectively. The parameters $v_s = 1036.4$ (ft/s), $P_0 = 973.3$ (lb/ft²), $I_y = 182.5$ (slug.ft²), $S = 0.44$ (ft²), $d = 0.75$ (ft), $m = 13.98$ (slug), $\omega_a = 150$ (rd/s), and $\xi_a = 0.7$ denotes sound speed, static pressure, moment of inertia with respect to pitch axis, reference area, reference diameter, airframe mass, actuator natural frequency, and actuator damping ratio, respectively. Numerical values of aerodynamic coefficients of a pitch-axis missile model at an altitude of 20,000 ft are listed in Table 1 [7].

Table 1. Aerodynamic coefficients

Coefficient	Value
K_z	$0.7P_0S/mv_s$
K_m	$0.7P_0Sd/I_y$
a_n	19.373
b_n	-31.023
c_n	-9.717
C_{D0}	-0.30
a_m	40.434
b_m	-64.015
c_m	2.922
d_m	-11.803
e_m	-1.719

The control objectives are to design an inner-loop feedback controller with $\mathbf{c} = [0 \ 1]^T$ to track a pitch rate command $q^c(t)$ and an outer-loop feedback controller with $\mathbf{c} = [1 \ 0]^T$ to track an AOA command $\alpha^c(t)$.

3. Discontinuous HOSM-based Control Design

3.1. Problem statement

Consider a certain class of dynamical systems characterized by smooth nonlinear dynamics and represented by the following closed-loop state-space feedback

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{b}(\mathbf{x})u \\ \mathbf{y} = \sigma(\mathbf{x}) \end{cases} \quad (9)$$

where $\mathbf{x} \in \mathcal{R}^n$, $\sigma \in \mathcal{R}$, and $u \in \mathcal{R}$. The functions A_{ij} and b_i are uncertain continuous sufficiently differentiable functions. Using discontinuous higher-order sliding mode (HOSM) control approach, the control problem considered in this study aims to design high-performance dynamic controllers for real-time tracking missions in spite of internal and external disturbances. The discontinuous HOSM-based control approach is formulated under the following assumptions [23,24].

Assumption 1: The matrix $\mathbf{A}(\mathbf{x})$ and the vector $\mathbf{b}(\mathbf{x})$ are partitioned into nominal and uncertain parts as follows

$$\begin{cases} \mathbf{A}(\mathbf{x}) = \mathbf{A}_{nom}(\mathbf{x}) + \Delta\mathbf{A}(\mathbf{x}) \\ \mathbf{b}(\mathbf{x}) = \mathbf{b}_{nom}(\mathbf{x}) + \Delta\mathbf{b}(\mathbf{x}) \end{cases} \quad (10)$$

with

$$\begin{cases} \|\Delta\mathbf{A}(\mathbf{x})\| \leq A_{max} \triangleq \max_{\mathbf{x} \in \mathbb{X}} |\Delta\mathbf{A}(\mathbf{x})| \in \mathbb{R}^+ \\ \|\Delta\mathbf{b}(\mathbf{x})\| \leq b_{max} \triangleq \max_{\mathbf{x} \in \mathbb{X}} |\Delta\mathbf{b}(\mathbf{x})| \in \mathbb{R}^+ \end{cases} \quad (11)$$

where \mathbb{X} is the operation space of the system (9).

Assumption 2: The control input u is a Lebesgue-measurable signal (affine scalar function) with

$$|u(t)| \leq u_{max} \in \mathbb{R}^+ \tag{12}$$

Assumption 3: The output constraint $\sigma(\mathbf{x})$ is of r -order differentiability class where r defines the relative degree of the output constraint $\sigma(\mathbf{x})$ with respect to the control input u [25].

Assumption 4: the output function $\sigma(\mathbf{x})$ and its successive time derivatives up to $r - 1$ form a non-empty integral set

$$\sigma^r = \{\mathbf{x} \in \mathbb{X} | \sigma(\mathbf{x}) = \dot{\sigma}(\mathbf{x}) = \dots = \sigma^{(r-1)}(\mathbf{x}) = 0\} \neq \emptyset - \text{integral set} \tag{13}$$

Assumption 5: if the assumption (3) holds and r is constant and known, the r th- order time derivative of the output constraint $\sigma(\mathbf{x})$ satisfies the following equality [25]

$$\sigma^{(r)} = p(\mathbf{x}) + q(\mathbf{x})u \tag{14}$$

where $p(\mathbf{x}) = L_f^r \sigma(\mathbf{x})$ and $q(\mathbf{x}) = L_g L_f^{r-1} \sigma(\mathbf{x}_o) \neq 0$. Due to the assumption 1, the functions $p(\mathbf{x})$ and $q(\mathbf{x})$ are bounded continuous functions on an open set $\mathcal{X}(\mathbf{x}_o) \subset \mathbb{X}$ around a given initial condition \mathbf{x}_o with

$$0 < \rho_- \leq q(\mathbf{x}) \leq \rho_+ \quad |p(\mathbf{x})| \leq \vartheta \tag{15}$$

ρ_-, ρ_+ and ϑ are some positive constants.

Assumption 6: According to the geometric-differential theoretical framework [25], the system (9) is feedback equivalent under a local diffeomorphic coordinate transformation $z_k = \sigma^{(k)}$ ($k = 0, \dots, r$). A controllable Brunovsky canonical form can be constructed as follows

$$\dot{\mathbf{z}} = \mathbf{\Lambda}\mathbf{z} + \mathbf{Y}u \tag{16}$$

where

$$\begin{cases} \mathbf{\Lambda} = [L_f \sigma(\mathbf{x}) \quad L_f^2 \sigma(\mathbf{x}) \quad \dots \quad L_f^{r-1} \sigma(\mathbf{x}) \quad L_f^r \sigma(\mathbf{x})]^T \\ \mathbf{Y} = [0 \quad 0 \quad \dots \quad 0 \quad L_g L_f^{r-1} \sigma(\mathbf{x}_o)]^T \end{cases} \tag{17}$$

3.2. Design of discontinues r -HQSM controllers

Theorem 1 [24]: Let assumptions 1-6 hold, a bounded r -sliding mode feedback controller such as

$$u_r = -G\Psi_{r-1,r}(\sigma, \dot{\sigma}, \dots, \sigma^{(r-1)}) \tag{18}$$

can be constructed in the constraint-space (16) to drive the constraint output $\sigma(\mathbf{x})$ and its successive $(r - 1)$ time derivatives towards their zero level in finite time and in spite of uncertainties and disturbances.

Theorem 2: Let $K_1, \dots, K_r > 0$, the following higher-order sliding mode-based controller

$$\begin{cases} u_r = -G\Psi_r(\sigma, \dot{\sigma}, \dots, \sigma^{(r)}) \\ \Psi_r = \sigma^{(r)} + K_1|\sigma^{(r-1)} + \sum_{i=r-2}^0 K_i\sigma^{(i)}| * \text{sign}(\sigma^{(r-1)} + \sum_{i=r-2}^0 K_i\sigma^{(i)}) \end{cases} \quad (19)$$

stabilizes the origin of the system (16) and ensure the establishment of the r -sliding mode $\sigma \equiv 0$ in finite time provided that the gain G and the design parameters K_i are selected such that

$$G > \vartheta/\rho_-(|\sigma^{(r)}| + K_1|\sigma^{(r-1)} + \sum_{i=r-2}^0 K_i\sigma^{(i)}|) \quad (20)$$

The controller (19) can be implemented using one of the following saturation functions

$$\Psi_r = \begin{cases} \Psi_r/\Delta & \text{if } |\Psi_r| \leq \Delta \\ \Psi_r = \text{sign}(\Psi_r) & \text{if } \Psi_r > \Delta \end{cases} \quad (21)$$

or

$$\Psi_r = \Delta - \exp(-\mu\Psi_r) \quad (22)$$

where Δ and μ are convergence parameters.

Proof of theorem 2: Define a sliding function $s = \sigma^{(r-1)}$ and consider the following Lyapunov candidate function

$$V = \frac{1}{2}s^T s \quad (23)$$

Using the equations (14) and (19), the first-order time-derivative of the function V is given as follows

$$\begin{aligned} \dot{V} &= s^T \dot{s} = s^T \sigma^{(r)} \\ &= \sigma^T \mathbf{c}^T [p(\mathbf{x}) + q(\mathbf{x})u] \\ &= \sigma^T \mathbf{c}^T \left[p(\mathbf{x}) - Gq(\mathbf{x}) \left(\sigma^{(r)} + K_1|\sigma^{(r-1)} + \sum_{i=r-2}^0 K_i\sigma^{(i)}| * \text{sign}(\sigma^{(r-1)} + \sum_{i=r-2}^0 K_i\sigma^{(i)}) \right) \right] \\ &\leq |\sigma^T \mathbf{c}^T| \left[\vartheta - G\rho_-(|\sigma^{(r)}| + K_1|\sigma^{(r-1)} + \sum_{i=r-2}^0 K_i\sigma^{(i)}|) \right] \\ &\leq -|\sigma^T \mathbf{c}^T| \vartheta \left[\frac{G\rho_-(|\sigma^{(r)}| + K_1|\sigma^{(r-1)} + \sum_{i=r-2}^0 K_i\sigma^{(i)}|)}{\vartheta} - 1 \right] \\ &\leq -\eta|\sigma^T| \end{aligned} \quad (24)$$

4. HOSM-based Missile's Longitudinal Autopilots Design and Validation

In this section, the proposed HOSM-based control scheme is validated through different nonlinear numerical simulations. For all the scenarios, the sliding variable $\sigma(\mathbf{x})$ is selected as

$$\sigma(\mathbf{x}) = \mathbf{c}^T \mathbf{e} = \mathbf{c}^T (\mathbf{x} - \mathbf{x}_d) \quad (25)$$

with $\mathbf{c} \in \mathbb{R}^n$. The identity (14) is fulfilled as follows

$$\sigma^{(r)} = \mathbf{c}^T \mathbf{e}^{(r)} = p(\mathbf{x}) + q(\mathbf{x})u \tag{26}$$

4.1. Outer-loop AOA autopilot

The control task here is to enforce the missile’s airframe to follow a desired AOA path using the nominal form of the dynamic model (6). According to the assumption 3, from the model (6) it is easy to find that the relative degree of the dynamics $u \rightarrow \sigma(\mathbf{x}) \equiv \delta \rightarrow \alpha$ is $r = 2$ which yields

$$\begin{aligned} \sigma^{(2)} &= [1 \quad 0] \begin{bmatrix} \alpha^{(2)} \\ q^{(2)} \end{bmatrix} \\ &= \left(K_\alpha M \frac{\partial C_n}{\partial \alpha} \cos(\alpha) - K_\alpha M C_n \sin(\alpha) \right) \left(K_\alpha M C_n \cos(\alpha) + q + \frac{g}{v_s M} \cos(\gamma) \right) \\ &\quad + K_q M^2 C_m + K_q M^2 e_m q - \alpha_d^{(2)}(t) + d_m \delta \\ &= p(\mathbf{x}) + q(\mathbf{x})\delta \end{aligned} \tag{27}$$

The controller (19) is given as follows

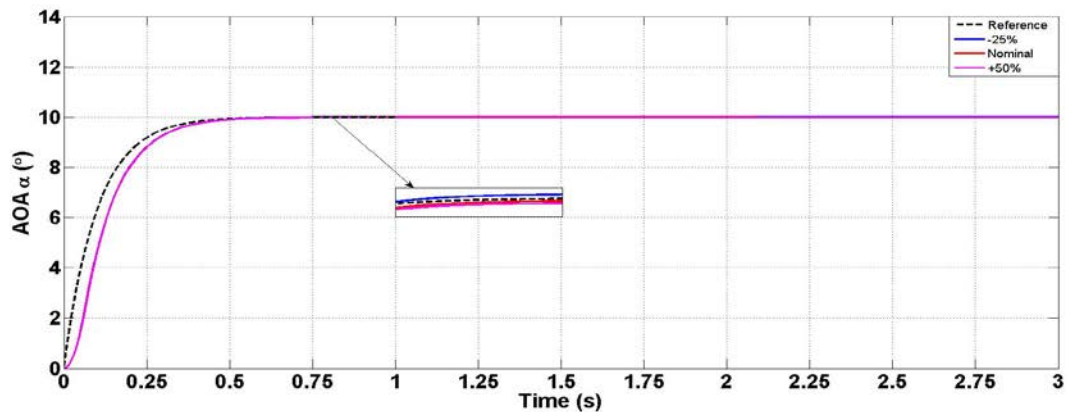
$$\begin{cases} u_2 = -G\Psi_2(\sigma, \dot{\sigma}, \sigma^{(2)}) \\ \Psi_2 = \sigma^{(2)} + K_1|\dot{\sigma} + K_0\sigma| * \text{sign}(\dot{\sigma} + K_0\sigma) \end{cases} \tag{28}$$

Scenario 1: tracking of an asymptotic path

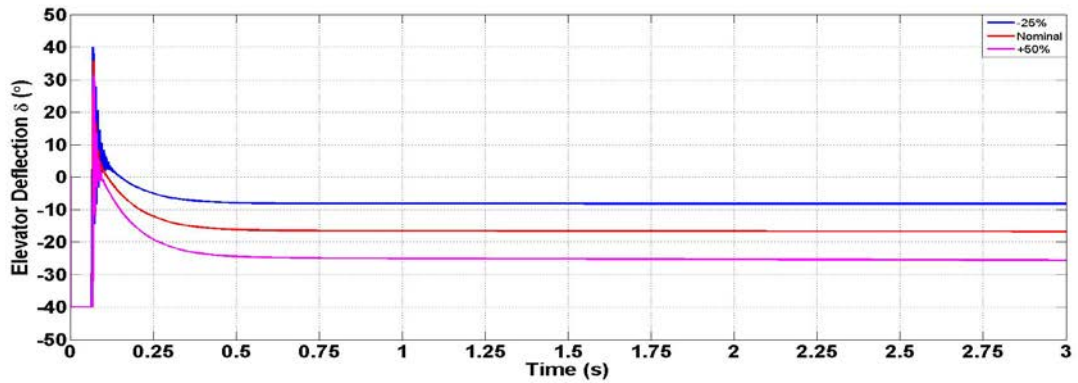
In this scenario, the performance and robustness of the proposed HOSM-based control scheme are evaluated through a smooth tracking of an asymptotic path. Nominal-, 25% underestimated-, and 50% overestimated-form of the model (6) were used in this simulation as shown in Figure 1.

Scenario 2: tracking of sinusoidal path

The control objective of this scenario is to track a sinusoidal reference trajectory at the extreme AOA conditions with $\alpha_d(t) = 20 \sin(2t)$ (deg). Figure 2 shows the time-history of the AOA response and its corresponding control effort.

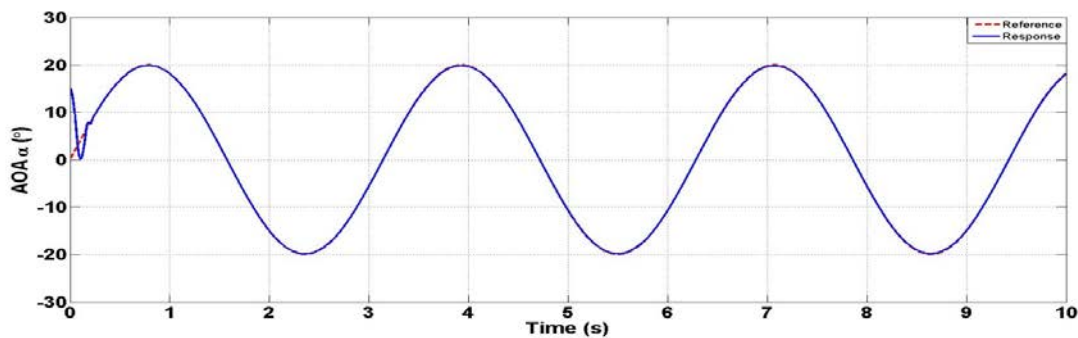


(a)

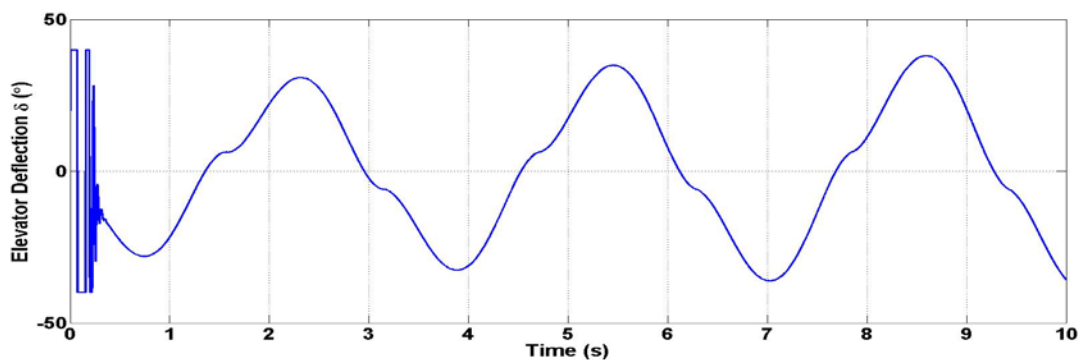


(b)

Figure 1. Time-histories of a tracking of asymptotic path scenario: (a) AOA response, (b) corresponding aerodynamic control.



(a)



(b)

Figure 2. Time-histories of a tracking of a sinusoidal path scenario: (a) AOA response, (b) corresponding aerodynamic control.

4.2. Inner-loop pitch rate autopilot

In the case of pitch rate control, the missile's airframe is enforced to follow a desired pitch rate path as inner-loop control. According to the assumption 3, from the model (6) the relative degree of the dynamics $u \rightarrow \sigma(x) \equiv \delta \rightarrow q$ is $r = 1$ which results in

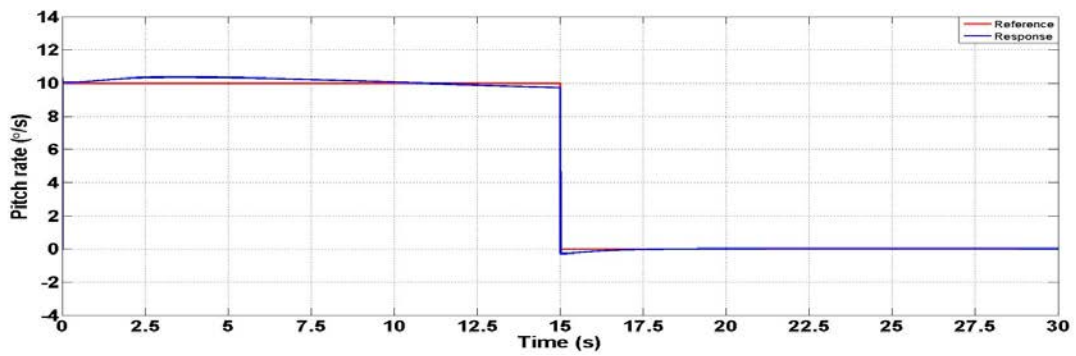
$$\begin{aligned}
 \sigma^{(1)} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha^{(1)} \\ q^{(1)} \end{bmatrix} \\
 &= K_q M^2 C_m + K_q M^2 e_m q + d_m \delta \\
 &= p(x) + q(x)\delta
 \end{aligned}
 \tag{29}$$

The controller (19) is given as follows

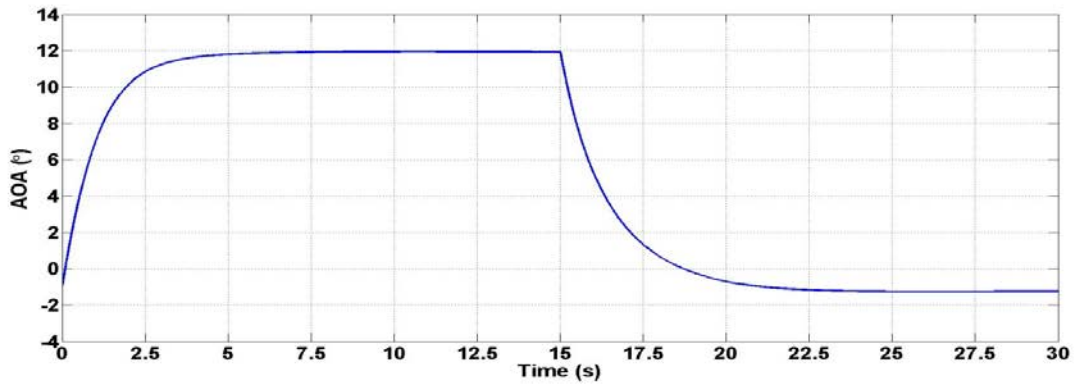
$$\begin{cases} u_1 = -G\Psi_2(\sigma, \dot{\sigma}) \\ \Psi_1 = \sigma^{(1)} + K_1|K_0\sigma| * \text{sign}(K_0\sigma) \end{cases}
 \tag{30}$$

Scenario 3: tracking of pitch rate command

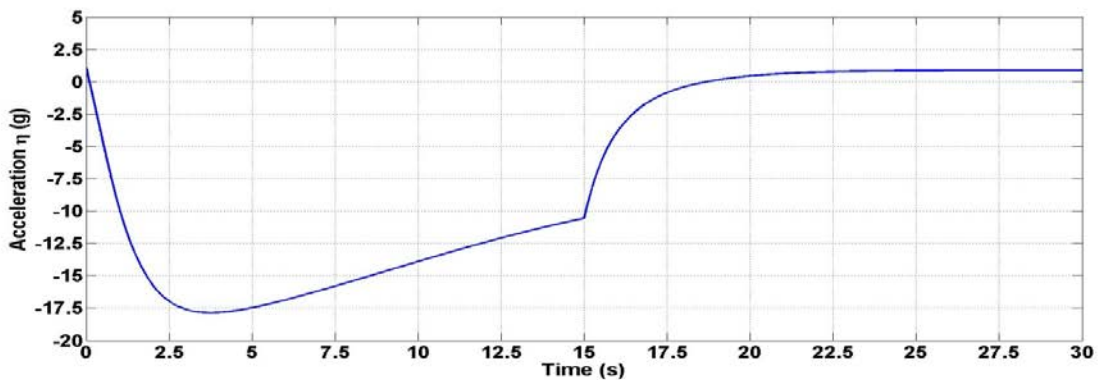
Using the nominal form of the model (6), the missile's airframe is enforced to track a pitch rate pattern as shown in Figure 3.



(a)



(b)



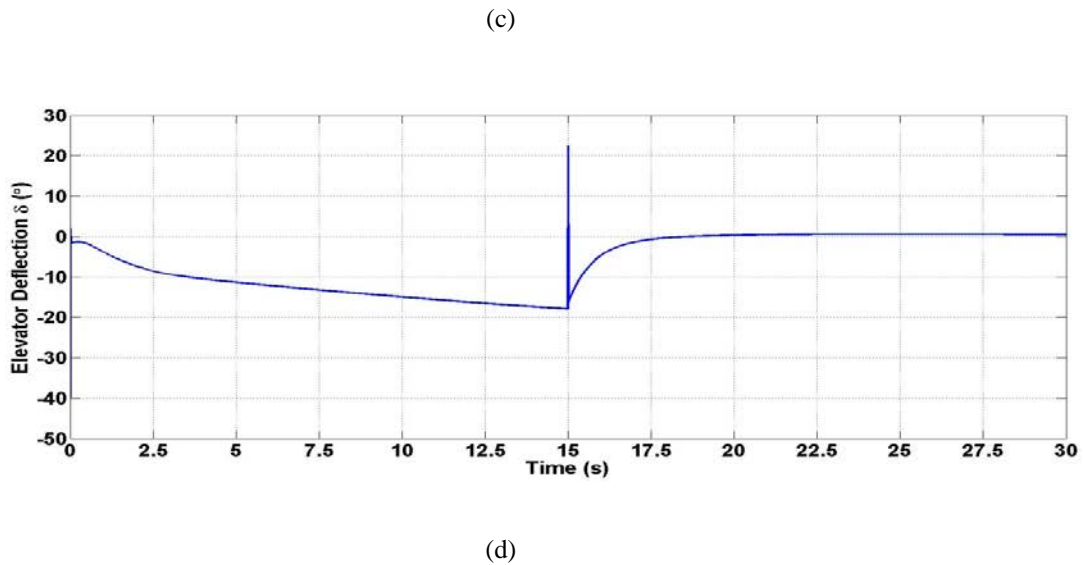


Figure 3. Time-histories of a tracking of a pitch rate pattern scenario: (a) pitch rate response , (b) AOA response, (c) Normal acceleration latex, (d) aerodynamic control.

5. Conclusion

A robust higher-order sliding mode (HOSM) control scheme for nonlinear uncertain systems driven by an affine control input is proposed. The main contributions of this paper are the design of robust finite-time convergent HOSM-based controllers without resort to recursive procedures and chattering effect reduction even for first-order dynamics. The proposed HOSM-based control scheme was applied to designing pitch-axis autopilots for a short-range tactical missile during the endgame interception. High-tracking precision and robustness against heavy uncertainty conditions were achieved while the control input was maintained smooth.

Similar to many existing discontinuous sliding mode control algorithms, the proposed HOSM-based control scheme does not require an exact dynamic model of the controlled system or process. The only requirements for real-time implementation are the measurements of the sliding variable, the relative degree of the system output with respect to its input, and the bounds or restrictions (15). The unbounded uncertainties or disturbances, undefined relative degree, and high measurement noises remain unsolved problems and main challenges for the extension of the proposed research.

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References

- [1] F.W. Nesline, B.H. Wells, and P. Zarchan., "Combined Optimal Classical Approach to Robust Missile Autopilot Design". *Journal of Guidance, Control, and Dynamics*, vol. 4, no. 3, pp. 316–322, 1981.

- [2] R.T. Reichert. "Dynamic scheduling of modern-robust-control autopilot design for missiles". *IEEE Control Systems Magazine* 12, no 5, pp. 35–42, 1992.
- [3] R.A. Nichols, R.T. Reichert and W.J. Rugh. "Gain scheduling for H_∞ controllers: a flight control example". *IEEE Transactions on control systems technology*. vol. 1, no 2, pp. 69-79, 1993.
- [4] C.P. Mracek, and J.R. Cloutier. "Missile longitudinal autopilot design using the state dependent Riccati equation method". *Proceedings of the first international conference on nonlinear problems in aviation and aerospace*. Daytona, Beach, FL, pp. 387-396, 1996.
- [5] P.C. Pellanda, P. Apkarian, H.D. Tuan, and D. Alazard. "Missile Autopilot Design via a Multi-Channel LFT/LPV Control Method". *15th Triennial World Congress of IFAC*, vol. 15, part 1, pp. 1569-1569, 2002.
- [6] C.L. Lin, and C.L. Hwang. "A Dynamic Fuzzy Gain-Scheduled Design for Missile Autopilot". *The Aeronautical Journal*. vol. 1 no 2, pp. 599-605, 2003.
- [7] M. Xin, and S.N. Balakrishnan. "Nonlinear H_∞ missile longitudinal autopilot design with θ -D method". *IEEE Transactions on aerospace and electronic systems*. vol. 44 no1, pp. 41-56, 2008.
- [8] D. Zhou, and S. Chuntao. "Dynamics and autopilot design for endoatmospheric interceptors with dual control systems". *Aerospace Science and Technology*, vol.13 issue 6, pp. 291-300, 2009.
- [9] B.J.E. Misgeld, M. Darcis, and T. Kuhn. "Robust linear-parameter varying Autopilot design for a tail/thrust vector controlled missile". *Advances in aerospace guidance, navigation and control*, vol. 2, pp. 287-301, 2011.
- [10] W.Q Tang, and Y.L. Cai. "Functional Control-Based Missile Autopilot Design". *Journal Of Guidance, Control, And Dynamics*, vol. 35, no. 5, pp. 1450-1455, 2012.
- [11] E. Devaut, A. Hiret, and H. Siguerdidjane. "Dynamic Inversion and LPV Approach: Application to a Missile Autopilot". *In Proceedings of the IEEE Hong-Kong Symposium on Robotics and Control*, vol. 1, pp. 293-298, 1999.
- [12] Y. Fu-Kuang, K.Y. Cheng, and Li.C. Fu. "Variable structure-based nonlinear missile guidance/autopilot Design with highly maneuverable actuators". *IEEE Transaction on control systems technology*. vol. 12, no 6, pp. 944-949, 2004.
- [13] A. Shang, W. Gu, Y. Yuan, and C. Li. "Missile Autopilot Design Based on Second Order Sliding Modes Control". *Proceedings of the 5th World Congress on Intelligent Control and Automation*. vol. 2, pp. 1167-1170, 2004.
- [14] T. Shima, M. Idan, and O.M. Golan. "Sliding-mode control for integrated missile autopilot guidance". *Journal of guidance, control, and dynamics*. Vol. 29 no 2, pp. 250-260., 2006.
- [15] D.F. Lin, and J.F. Fan. "Missile autopilot design using backstepping approach". *Second international conference on computer and electrical engineering*, pp. 238-241, 2009.
- [16] M. Bahrami, B. Ebrahimi, and G.R. Ansarifar. "Sliding mode observer and control design with adaptive parameter estimation for a supersonic flight vehicle". *International Journal of Aerospace Engineering*, article ID 474537, pp. 1-9, 2010.
- [17] J.F. Fan, and S. Zhong. "Missile Longitudinal Autopilot Design Using Backstepping Approach". *IEEE Aerospace conference proceeding*, paper no. 1416, 2010.

- [18] D.C. Foreman, C.H. Tournes, and Y.B. Shtessel. "Integrated missile flight control using quaternions and third-order sliding mode control". *11th International Workshop on Variable Structure Systems VSS'10*, pp. 370-375, 2010.
- [19] B. Kada, "Outer-Loop Sliding Mode Control Approach to Longitudinal Autopilot Missile Design". *18th IFAC World Congress*, pp. 11157-11164, 2011.
- [20] B. Kada. "Higher Order Sliding Mode Control for Missile Autopilot Design". *Conference on Aerospace, Mechanical, Automotive and Materials Engineering*, vol. 70, 2012.
- [21] B. Kada. "A New Methodology to Design Sliding-PID Controllers: Application to Missile Flight Control System". *IFAC Conference on Advances in PID Control*, 2012.
- [22] B. Kada. "Arbitrary-Order Sliding-Mode-Based Homing-Missile Guidance for Intercepting Highly Maneuverable Target". *Journal of Guidance, Control, and Dynamics*, vol. 37, no. 6, pp. 1999-2013, 2014.
- [23] A. Levant. "Universal SISO sliding-mode controllers with finite-time convergence". *IEEE Transactions on automatic control*, vol. 46 no 9, pp. 1447-1451, 2001.
- [24] A. Levant. "Homogeneity approach to high-order sliding mode design". *Automatica*. vol. 41 no 5, pp. 823-830, 2005.
- [25] A. Isidori. *Nonlinear control systems*. New York: Second edition, Springer-Verlag, 1989.