International Journal of Sciences: Basic and Applied Research (IJSBAR)

ISSN 2307-4531
(Print & Online)

http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

Description of Rotational Bands for some Even-Even Nuclei in Actinide Region

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Abstract

In the frame of rotation vibration model (RVSM) and exponential model (EXPOM) the ground rotational bands for some even- even nuclei in Actinide region are calculated. Our result of RVSM and EXPOM models are compared with experimental data and with the predicting results of variable moment of inertia model (VMI) . The predicted results of RVSM and EXPOM are in close agreements with experimental data and with the resolve VMI model .

Keywords: rotational bands; variable moment of inertia (VMI) ; angular momentum, softness parameter (σ)

1. Introduction

We know that the ground state bands of deformed nuclei are described by the formula [1,2]

$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \qquad (1)$$

Considering the effect of rotation -vibration, Equation becomes

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$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \quad +B[I(I+!]^2 \qquad (2)$$

The experimental data deviate from predicting results of equation (2)

R.K .Gupta[3,4, 5,6,7] introduced the concept of variation of moment of inertia with angular momentum "softness of nuclear matter" to modify the last formula Eq.(2) Also there are many models are proposed to predict the ground state bands e.g. harmonic vibrator model VMI , An harmonic vibrator model AVAM , General vibrator model GVMI, exponential model EXPO [8,9,10], and etc..;

In this article we used the concept of softness of nuclear mater in modifying Eq.(2) , which is denoted RVSM model. We use the RVSM model to calculate the ground state energies for some even- even nuclei in Actinide region ,also we are repeat the above calculations by using the EXPOM model . We find that the predicting results of RVSM and EXPOM models are in close agreement compared with the results of VMI model and experimental data.

2. Results and discussion

R.K .Gupta [4,5] introduced the concept of variation of moment of inertia with angular momentum ,So the moment of inertia $\theta(I)$ can be written as

$$\theta(I)=\theta_0(1+o_1 I+o_2 I^2+o_3 I^3+--+--) \qquad (3)$$

Where θ_0 is the moment of inertia at I=0 , and σ_n is the softness parameter

$$\sigma_n=\frac{1}{n!}\frac{\delta^n\theta(J)}{\delta J^n}\bigg|_{J=0} \dots\dots \qquad (4)$$

$$n=1,2,3,\dots$$

Considering ,the $\theta(I)$ up to first order ,then $\theta(I)$ becomes

$$\theta(I)=\theta_0(1+o_1 I+) \qquad (5)$$

Substituting $\theta(I)$ from Eq.(5) in EQ.(2). One gets:

$$E(I)=AI(I+1) \quad +BI^2[I(I+!]+C\lfloor I(I+1\rfloor^2 \qquad (6)$$

Where $A = \frac{\hbar^2}{2\theta_0}$ $, B = Ao_1$ *and* C *are fitting paramters*

We also are calculate the ground state of rotational band of chosen even- even deformed nuclei in Actinide region by using the exponential model EXPOM from [7].

Which is written as:

$$E(I) = \frac{\hbar^2}{2\varphi_0} I(I+1) \exp.\left[\Delta_0\left(1 - \frac{I}{I_c}\right)\right]^{1/2} \qquad (7)$$

Where $\frac{\hbar^2}{2\varphi_0}$, Δ_0 .and I_c are fitting parameters

The predicted energies as given by Eq(6) and Eq (7) are compared with the experimental data and with the results of VMI model from [7] . which is Witten as

$$E(I) = \frac{\hbar^2}{2\theta_0} I(I+1) + \frac{1}{2} C(\theta(I) - \theta_0)^2 \quad .$$

where $\frac{\hbar^2}{2\theta_0}$ and C are fitting parameters and the variable moment of inertia $\theta(I)$ is determined through use of the variational condition.

$$\frac{\partial E(I)}{\partial \theta}\Big|_I = 0$$

By using least square fitting ,and excitation energies of experimental data , the parameters A,B and C RVSM model Eq (6) are given as in Table (1) for chosen nuclei.

Also, using the experimental excitation energies. the parameters $\frac{\hbar^2}{2\varphi_0}$, Δ_0 .and Ic are calculated by the same manner. Using Eq. (6) "RVSM" model and the given parameters in Table(1) . we are calculated the energies for chosen nuclei which is listed in table (3). The deviation of our results from experimental data are given as Dev $= \frac{1}{N}\sum_{i=1}^{N}(E_{Cal} - E_{exp})$

By similar manner using EXPOM Eq.(7) and the given parameters $\frac{\hbar^2}{2\varphi_0}$, Δ_0 .and Ic in $Table(1)$, we are calculate the energies for chosen nuclei which is listed also, in table (2).

Table (1) Fitted parameters of RVSM as shown in Eq,(6) for Chosen Nuclei

A	B	C	Dev	Nucleus
7.63E-03	-1.56E-05	-1.86E-06	8.66E-03	Pu^{244}.
7.64E-03	-5.16E-05	-6.89283E-07	-1.60E-03	Pu^{242}.
7.49E-03	-7.20E-05	2.93E-07	-5.05E-03	Pu^{240}
7.58E-03	-5.31E-05	-1.79E-07	-2.69E-03	Pu^{238}
7.57E-03	-3.75E-05	-1.48E-06	-3.81E-04	Pu^{236}
7.88E-03	-8.13E-05	-6.02E-08	-7.25E-03	U^{238}
7.96E-03	-8.32E-05	-6.69E-08	-7.24E-03	U^{236}
7.61E-03	-8.90E-05	8.38E-08	-2.71E-03	U^{234}
8.30E-03	-9.91E-05	2.00E-08	-2.47E-03	U^{232}
0.0088887	-9.57633E-05	-1.30E-06	-3.24E-04	U^{230}
8.31E-03	-2.23E-05	-3.86E-06	-3.86E-06	Th^{234}
8.89E-03	-1.50E-04	2.65E-07	-0.009270065	TH^{232}
9.36E-03	-1.46E-04	8.39E-07	-2.68E-03	Th^{230}
1.03E-02	-1.46E-04	8.39E-07	-2.68E-03	Th^{230}
1.27E-02	-0.000389517	5.84E-06	8.88E-03	Th^{226}
1.68E-02	-8.17E-04	1.86E-05	8.30E-03	Th^{224}

The calculated results for the ground state rotational bands are given systematically in table 3. From this table we noticed that the calculations are carried out for Pu^{236} up to $J^{\pi}=16^{+}$, Pu^{238} up to $J^{\pi}=22^{+}$, Pu^{240} up to $J^{\pi}=26^{+}$, Pu^{242} up to $J^{\pi}=26^{+}$, Pu^{244} up to $J^{\pi}=28^{+}$, Th^{224} up to $J^{\pi}=16^{+}$, TH^{226} up to $J^{\pi}=18^{+}$, Th^{230} up to $J^{\pi}=24^{+}$, ,TH^{232} up to $J^{\pi}=30^{+}$, Th^{234} up to $J^{\pi}=12^{+}$, U^{230} up to $J^{\pi}=16^{+}$, U^{232} up to $J^{\pi}=20^{+}$, U^{234} up to $J^{\pi}=20^{+}$, U^{236} up to $J^{\pi}=26^{+}$, and U^{238} up to $J^{\pi}=28^{+}$.

As can be seen, the results are excellent for all nuclei, in the ground rotational bands of majority nuclei, results of RVSM and EXPOM are in close agreement with the results predicted by the VMI model compared with experimental data

Table (2) Fitted parameters for EXPOM as in Eq.(7) for Chosen Nuclei

$\frac{\hbar^2}{2\varphi_0}$	Δ_o	IC	Dev	Nucleus
5.08E-03	0.4337651	30	-8.42E-04	Pu^{244}
5.13E-03	0.389589	30	2.76E-04	Pu^{242}
4.66E-03	0.4458096	34	-9.46E-05	Pu^{240}
5.34E-03	0.3343265	30	-1.27E-04	Pu^{238}
1.04E-04	0.2125915	18	6.332854E-04	Pu^{236}
4.34E-03	0.5705118	34	7-3.804519E-04	U^{238}
4.78E-03	0.4794887	30	-3.80E-04	U^{236}
4.94E-03	0.4062729	24	-1.06E-04	U^{234}
5.27E-03	0.4292345	24	-9.28E-05	U^{232}
6.83E-03	0.2520388	14	-1.51E-05	U^{230}
6.91E-03	0.1935971	14	-1.72E-05	Th^{234}
3.97E-03	0.7452663	36	-1.19E-04	TH^{232}
4.97E-03	0.596007	28	-8.42E-05	Th^{230}
5.32E-03	0.614448	22	6.98E-05	Th^{228}
4.88E-03	0.9010366	24	-1.19E-04	TH^{226}
4.01E-03	1.389346	22	4.16E-04	Th^{224}

Table (3) Experimental "EXP" energies and predicted energies for the chosen nuclei calculated by RVSM,EXPOM and VMI models (in MeV.)

Pu^{236}				
I	EXP	RVIM	EPOM	VMI
2	0.04463	4.49E-02	4.48E-02	0.0449603
4	0.14745	0.1477522	0.1472584	0.148339
6	0.3058	0.3057676	0.3049715	0.306929
8	0.5157	0.5155875	0.5149609	0..516682
10	0.7735	0.7732869	0.773694	0.77333
12	1.0743	1.074374	1.076601	1.07305
14	1.4136	1.413789	1.416993	1.412111
16	1.786	1.785905	1.782254	1.78742
Pu^{238}				
I	EXP	RVIM	EXPOM	VMI
V2	4.48E-02	0.044076	4.42E-02	0.0449603
4	0.1473094	0.145952	0.1457232	0.148339

6	0.3047257	0.30338	0.3023048	0.306929
8	0.5143569	0.51358	0.5116812	0.516682
10	0.7733976	0.77348	0.7713886	0.773383
12	1.078974	1.0801	1.078752	1.07305
14	1.428143	1.4291	1.430817	1.41211
16	1.817893	1.8185	1.824253	1.78742
18	2.245144	2.2449	2.255198	2.19624
20	2.706748	2.7057	2.718984	2.63621
22	3.199487	3.1988	3.209651	3.10527
24	3.720076	3.7208	3.71889	3.60161
26	4.265158	4.2652	4.233345	4.12364
Pu^{240}				
I	EXP	RVIM	EXPOM	VMI
2	0.042824	4.41E-02	4.31E-02	0.042811
4	0.14169	0.1440936	0.1415794	0.141614
6	0.294319	0.2968222	0.2931247	0.294043
8	0.49752	0.499108	0.4951587	0.496966
10	0.7478	0.747919	0.7450105	0.746947
12	1.0418	1.040336	1.039846	1.04058
14	1.3756	1.373551	1.376626	1.37466
16	1.7456	1.744872	1.752045	1.74628
18	2.152	2.151715	2.162449	2.15266
20	2.591	2.591613	2.6037	2.59208
22	3.061	3.062209	3.070974	3.0619
24	3.56	3.56126	3.558402	3.56052
26	4.088	4.086634	4.058407	4.08632
Pu^{242}				
I	EXP	RVIM	EXPOM	VMI
2	0.04454	4.52E-02	4.48E-02	0.0456841
4	0.1473	0.1485508	0.1474152	0.15072
6	0.3064	0.3071518	0.3051983	0.311837
8	0.5181	0.5180148	0.5154929	0.524908
10	0.7786	0.7779288	0.7754253	0.785642
12	1.0844	1.083486	1.081889	1.08999
14	1.4317	1.431084	1.431469	1.43431
16	1.8167	1.816923	1.820334	1.81541
18	2.236	2.237007	2.244059	2.23052
20	2.686	2.687146	2.697329	2.67723
22	3.163	3.162951	3.173381	3.15345
24	3.662	3.65984	3.662834	3.65734

26	4.172	4.173033	4.150667	4.18729
Pu^{244}				
I	Exp	RVIM	EXPOM	VMI
2	0.0442	4.55E-02	4.64E-02	0.0.50913.
4	0.155	0.150547	0.15218	0.166252
6	0.3179	0.313117	0.314555	0.339602
8	0.535	0.530507	0.530403	0.564046
10	0.8024	0.799286	0.79645	0.833448
12	1.1159	1.115315	1.109168	1.14271
14	1.471	1.473736	1.464694	1.48768
16	1.8635	1.86898	1.858708	1.86496
18	2.289	2.294762	2.286242	2.27178
20	2.742	2.744083	2.74135	2.70581
22	3.215	3.209232	3.216499	3.16509
24	3.69	3.68178	3.701273	3.64795
26	4.149	4.152587	4.179048	4.15295
28	4.61	4.611799	4.614758	4.67881
Th^{224}				
I	EXP	RVIM	EXPOM	VMI
2	0.0981	0.091835	9.05E-02	0.093149
4	0.2841	0.2786495	0.2819308	0.27952
6	0.5347	0.5337484	0.5509954	0.531320
8	0.8339	0.8375973	0.8749785	0.833268
10	1.1738	1.177822	1.231267	1.1762
12	1.5498	1.549211	1.596823	1.55407
14	1.9589	1.953709	1.947172	1.96256
16	2.398	2.400426	2.254273	2.39846

Th^{226}				
I	EXP	RVIM	EXPOM	*VMI*
2	0.0722	0.071827	6.93E-02	*0.0730527*
4	0.22643	0.225476	0.221966	0.229274
6	0.4473	0.446174	0.446862	0.450497
88	0.7219	0.721387	0.732614	0.723647
10	1.0403	1.040823	1.067333	1.03992
12	1.3952	1.396434	1.438339	1.39317
14	1.7815	1.78241	1.83167	1.7789
16	2.1958	2.195183	2.231156	2.19369
18	2.6351	2.633428	2.616488	2.63486

20	3.0971	3.098061	2.958272	3.1002

Th [228]

I	EXP	RVIM	EXPOM	VMI
2	0.057759	5.91E-02	5.73E-02	0.0584579
4	0.186823	0.1885408	0.1853205	0.188576
6	0.378179	0.3789973	0.3769974	0.380053
8	0.6225	0.6221594	0.624775	0.623384
10	0.9118	0.9107017	0.9204345	0.91107
12	1.2394	1.238291	1.254738	1.2374
14	1.5995	1.599583	1.616792	1.59796
16	1.9881	1.990228	1.992714	1.98927
18	2.4079	2.406864	2.362269	2.40858

Th [230]

I	EXP	RVIM	EXPOM	VMI
2	0.0532	5.44E-02	5.29E-02	0.05473
4	0.1741	0.1758145	0.1725157	0.1782
6	0.3566	0.3577093	0.3538715	0.36276
8	0.5941	0.5939825	0.591904	0.60053
10	0.8797	0.8788269	0.881222	0.884728
12	1.2078	1.206757	1.21604	1.20986
14	1.5729	1.572609	1.590031	1.57153
16	1.9715	1.971543	1.99608	1.9662
18	2.3978	2.399038	2.425889	2.39097
20	2.85	2.850897	2.869217	2.8434
22	3.325	3.323245	3.312289	3.3216
24	3.812	3.812529	3.733717	3.8238

TH [232]

I	EXP	RVIM	EXPOM	VMI
2	0.049369	5.16E-02	4.91E-02	0.0512957
4	0.16212	0.1662229	0.1603029	0.167196
6	0.3332	0.3375998	0.3292083	0.340785
8	0.5569	0.559811	0.5514871	0.564827
10	0.827	0.8274566	0.8226445	0.833022
12	1.1371	1.135622	1.138028	1.14022
14	1.4828	1.479879	1.492784	1.48228
16	1.8586	1.856285	1.881792	1.85584
18	2.2629	2.261381	2.299582	2.25815
20	2.6915	2.692196	2.740205	2.687
22	3.1442	3.146245	3.197041	3.1403

24	3.6196	3.621525	3.66249	3.6166
26	4.1162	4.116523	4.127439	4.1144
28	4.6318	4.630208	4.580262	4.6325
30	5.162	5.162039	5.004673	5.17

Th^{234}

I	EXP	RVIM	EXPOM	VMI
2	0.04955	4.94E-02	4.96E-02	0.0498417
4	0.163	0.1628128	0.1628362	0.16383
6	0.3365	0.336472	0.3360998	0.337332
8	0.5648	0.5652706	0.5649844	0.564863
10	0.843	0.8425852	0.8433274	0.841075
12	1.1602	1.160311	1.160286	1.16119

U^{230}

I	EXP	RVIM	EXPOM	VMI
2	0.05172	5.21E-02	5.17E-02	0.0519903
4	0.1695	0.1695912	0.1689897	0.169995
6	0.3471	0.3468921	0.3469869	0.347769
8	0.5782	0.5780649	0.5798434	0.578488
10	0.8564	0.8566345	0.8594517	0.855978
12	1.1757	1.175625	1.171701	1.17505

U^{232}

I	EXP	RVIM	EXPOM	VMI
2	0.047572	4.86E-02	0.0477327	0.0483552
4	0.15657	0.1581597	0.156095	0.158565
6	0.3226	0.3238323	0.321274	0.325521
8	0.541	0.5409189	0.5391701	0.543394
10	0.8058	0.8046932	0.8052962	0.806704
12	1.1115	1.110436	1.114618	1.1107
14	1.4537	1.453438	1.461288	1.45139
16	1.8281	1.828994	1.838154	1.82546
18	2.2315	2.232409	2.235733	2.23011
20	2.6597	2.658996	2.639608	2.66301

U^{234}

I	EXP	RVIM	EXPOM	VMI
2	0.043498	4.46E-02	4.37E-02	0.0443987
4	0.143351	0.1451173	0.143018	0.145705
6	0.296071	0.297345	0.2946758	0.299409

8	0.49704	0.4970945	0.4950952	0.500305
10	0.7412	0.7402129	0.7403625	0.743439
12	1.0238	1.02258	1.026077	1.02448
14	1.3408	1.340106	1.347113	1.33977
16	1.6878	1.688735	1.697187	1.68623
18	2.063	2.064444	2.067945	2.06129
20	2.4642	2.46324	2.446658	2.46277

U^{236}

I	EXP	RVIM	EXPOM	VMI
2	0.045242	0.0467589	4.56E-02	0.0473182
4	0.149476	0.1525151	0.1494647	0.155263
6	0.309784	0.3132284	0.308427	0.318989
8	0.52224	0.5248334	0.519166	0.532918
10	0.7823	0.7832384	0.7781577	0791754
12	1.0853	1.084327	1.081619	1.09087
14	1.4263	1.423955	1.425424	1.42637
16	1.8009	1.797956	1.804977	1.79498
18	2.2039	2.202134	2.215014	2.19397
20	2.6317	2.63227	2.64926	2.62101
22	3.0812	3.084119	3.099796	3.07413
24	3.55	3.553409	3.555715	3.55165
26	4.039	4.035843	3.999665	

U^{238}

I	EXP	RVIM	EXPOM	VMI
2	0.044916	4.63E-02	4.42E-02	0.0473447
4	0.14838	0.1511565	0.1457232	0.155232
6	0.30718	0.3105437	0.3023048	0.318626
8	0.5181	0.5205233	0.5116812	0.531797
10	0.7759	0.7771061	0.7713886	0.789364
12	1.0767	1.07628	1.078752	1.08667
14	1.4155	1.414009	1.430817	1.41981
16	1.7884	1.786236	1.824253	1.78554
18	2.1911	2.188879	2.255198	2.18113
20	2.6191	2.617833	2.718984	2.60429
22	3.0681	3.06897	3.209651	3.05308
24	3.5353	3.53814	3.71889	3.52581
26	4.0181	4.021168	4.233345	4.02106
28	4.517	4.513858		

3. Conclusion

The present models Eq. (6) and Eq.(7) predicted the ground state rotational bands of deformed even-even nuclei in Actinide region, and can also be applied to nuclei where the energies of levels are experimentally available. It includes three parameters which are determined straight forward using linear least squares fitting

Acknowledgment

The authors would like to acknowledge financial support for this work ,from the Deanship of Scientific Research (DSR) , University of Tabuk (Tabuk , Saudi Arabia , under grant no S -1435-0091) .Thanks also to Prof.Nafie,H.O; for its cooperative during calculation and discussion of this work.

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