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Modelling Fluid Flow in an Open Rectangular Channel with Lateral Inflow Channel

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Abstract

In this paper, the flow of an incompressible Newtonian fluid through a man-made open rectangular channel with a lateral inflow channel is investigated. We have considered the effects of angle, the cross-sectional area, velocity and length of the lateral inflow channel on the velocity in the open rectangular channel. The equations governing the flow are the continuity and momentum equations of motion, which are highly nonlinear and cannot be solved analytically. Therefore, an approximate solution of these partial differential equations is determined numerically using the finite difference method. The results are then analyzed using graphs. An increase in the area and the length of the lateral inflow channel leads to a reduction in the flow velocity while an increase in the velocity of this channel leads to an increase in the flow velocity of the open rectangular channel. Additionally, an increase in the angle of the lateral inflow channel does not necessarily lead to an increase in the velocity in the open rectangular channel. That is, an angle between 30^0-50^0 exhibits higher values of velocity in the open rectangular channel compared to the other angles of the lateral inflow channel.

Keywords: open channel; lateral inflow channel; finite difference; wetted perimeter

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1. Introduction

When too much water flows from highlands to lowlands, it leads to saturation of the soil, which results in the stagnation of excess water. Moreover, some areas still suffer from floods even when normal rain falls. Thus, designing channels that would control such an environmental disaster and more so divert the same water to agricultural land is very important. Additionally, the findings are applicable in flour or textile production and in the design of water mills where large volumes of high velocity water are required to turn large turbines and also drive mechanical processes

The cross-section of a channel may be closed or open at the top. The channels that have an open top are referred to as open channels while those with a closed top are referred to as closed conduits. The forces at work in open channels are the inertia, gravity and viscosity forces. Chezy equation was one of the earliest equations developed for average computations of the velocity of a uniform flow [1]. However, the formula that is mostly used in open channel problems is the Manning formula [2]. This Manning formula is highly considered and desirable because it takes into account the coefficient of roughness [3]. Fan and Li [4] formulated the diffusive wave equations for open channel flows with uniform and concentrated lateral inflow. In their formulation, they were ableto present continuity and momentum equations of an open channel with a lateral inflow channel that joins the main open channel at a varying angle. Mohammed [5] investigated how the discharge coefficient varied with respect to the side of the channel wall in the flow direction for four different angles using an oblique weir. The four angles were 30^o, 60^o, 75^o and 90^o of which all were varied along the flow direction. The findings were that maximum discharge was achieved at angle 30^0 of the side weir compared to that of other angles. Moreover, Masjedi and Taeedi [6] studied in the laboratory the effect of intake angle on discharge ratio in lateral intakes at 180^0 bend. The investigations were carried out in a laboratory flume under clear water. The investigations showed that the discharge ratio increased at a lateral intake angle of 45 degrees in all locations of the 180 degrees flume bend. Additionally, Yang [7] was able to study flow structures with diversion angles of 90°, 45° and 30°. A diversion angle between 30°- 45° was recommended to get a better flow pattern of the fluid.

Several studies of fluid flow through open channels have been carried out in the laboratory. However, mathematical modelling of open channels with lateral inflow channel has received little attention. This paper seeks to determine the effect of angle, velocity, cross-sectional area and length of the lateral inflow channel on velocity profiles in the open rectangular channel.

2. Mathematical analysis

In the present study, the lateral inflow channel is introduced into the open rectangular channel as illustrated in Fig. 1. The discharge in the open rectangular channel and the lateral inflow channel are denoted by Q and q respectively. L and θ represent the length and the varying angle respectively of the lateral inflow channel. The net volume of fluid that enters through the cell dx is considered at a time interval dt.

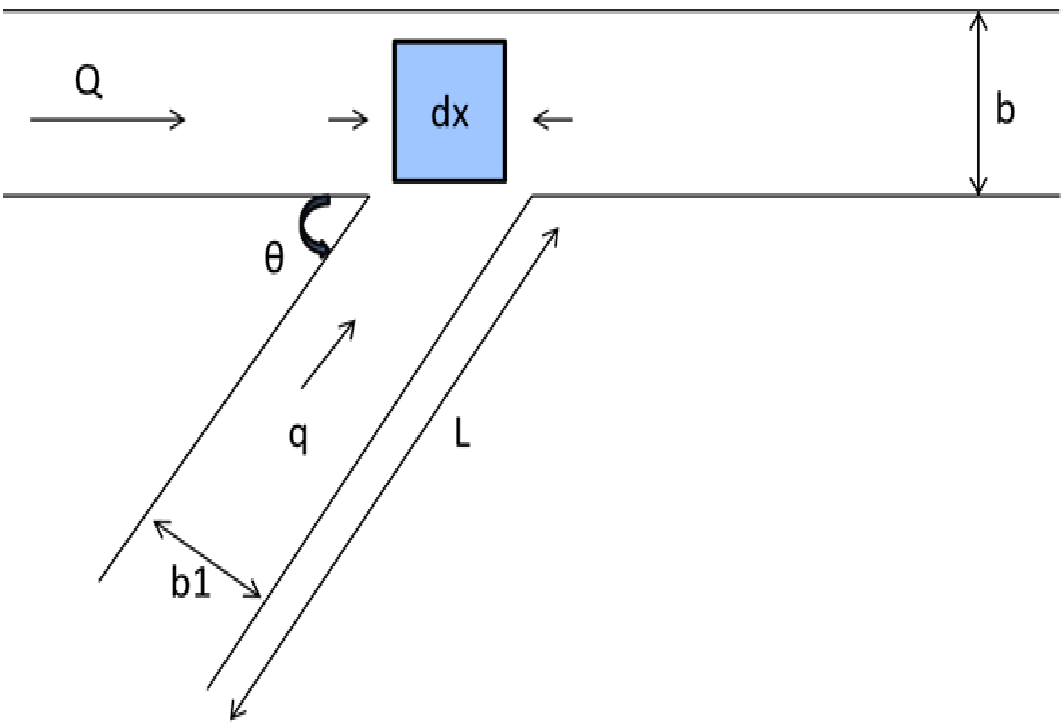

Fig. 1. Model of the open rectangular channel with a lateral inflow channel at an angle

Since the open channel is rectangular in shape, the wetted perimeter P of this channel is given by,

$$P = 2y + b \tag{1}$$

The cross-section area of this channel is given by,

$$\mathrm{A} = \mathrm{b.y} \tag{2}$$

Where y is the depth of flow and b is the width of this channel.

The discharge for the open rectangular channel is defined as,

$$Q = V.A \tag{3}$$

Where$V$ is the velocity and A is the cross-sectional area of this channel.

Now, for the lateral inflow channel, the area will be given by,

$$a = b1.y1 \tag{4}$$

Where $b1$ is the width and $y1$ is the depth of flow of the lateral inflow channel.

The term q represents the discharge in the lateral inflow channel and it can be defined by,

$$q = u.a \tag{5}$$

Where u is the velocity of this lateral inflow channel.

The equations governing the flow are the continuity and momentum equations respectively, for an open channel with the lateral inflow channel. They are given by,

$$\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x} + \frac{A}{b}\frac{\partial V}{\partial x} = \frac{q}{b\,L}\sin\theta \tag{6}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g\left(S_f - S_0\right) = \frac{q}{A\,L}\sin\theta\,(u\cos\theta - V) \tag{7}$$

Where S_f and S_0 are the friction slope and slope of the channel bottom respectively of the open rectangular channel.

The initial and boundary conditions have the form,

$$V(x,0) = 0, \quad y(x,0) = 0, \qquad for\ all\ x > 0 \tag{8a}$$

$$V(0,t) = V_0, \quad y(0,t) = y_0, \qquad for\ all\ t > 0 \tag{8b}$$

$$V(x,t) = V_0, \quad y(x,t) = y_0, \qquad for\ all\ t > 0 \tag{8c}$$

3. Method of solution

Since equations (6) and (7) are nonlinear, they cannot be solved by an analytical method. Therefore, we use the finite difference method to solve these two equations subject to the initial conditions (8a) and boundary conditions (8b) and (8c).

The equations (6) and (7) respectively in their finite difference form become,

$$\frac{y(i,j+1) - 0.5\big(y(i-1,j) + y(i+1,j)\big)}{\triangle\,t} + V(i,j)\frac{y(i+1,j) - y(i-1,j)}{2\triangle\,x}$$

$$+\frac{A}{b}\frac{V(i+1,j) - V(i-1,j)}{2\triangle\,x} = \frac{q}{b\,L}\sin\theta \tag{9}$$

$$\frac{V(i,j+1) - 0.5\big(V(i-1,j) + V(i+1,j)\big)}{\triangle\,t} + V(i,j)\frac{V(i+1,j) - V(i-1,j)}{2\triangle\,x}$$

$$+g\frac{y(i+1,j)-y(i-1,j)}{2\,\Delta\,x}+g\left(\frac{S_f(i-1,j)+S_f(i+1,j)}{2}-S_0\right)$$

$$=\frac{q}{A\,L}\sin\theta\left(u\cos\theta-V(i,j)\right) \qquad (10)$$

In equations (9) and (10), the index i refers to the distance along the channel while j refers to time. Now taking the velocity V_o =10 m/s and depth of the channel to be y_o = 0.5m, the initial and boundary conditions in finite difference form become,

$V(i,0)$= 0, $y(i,0)=0$, for all x>0 *(11a)*

$V(0,j)=10$, $y(0,j)=10$, for all t>0 *(11b)*

$V(N,j)=10$, $y(N,j)=10$, for all t>0 *(11c)*

Equations (9) and (10) together with the conditions 11 (a, b, c) are computed using very small values of $\Delta\,t$. In this paper, we set $\Delta\,x=$ 0.1 and $\Delta\,t=0.0001$. The finite difference method is known to be convergent and numerically stable whenever $\frac{\Delta t}{(\Delta x)^2}<\frac{1}{2}$.The number of sub-divisions along the length of the channel is taken to be N =100, while along the time is taken to be K =10000 sub-divisions. From equation (8), at the end of the time step $\Delta\,t$, the depth *y(i+1,j)* is computed in terms of depths and velocities in earlier time steps. In a similar way, the velocity *V(i+1,j)* is computed from the equation (9).

4. Results and Discussion

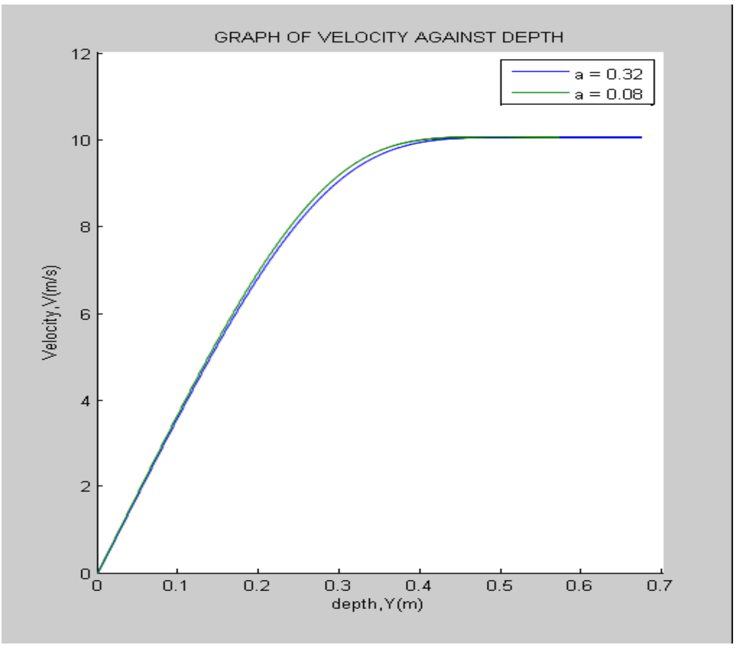

Fig. 2. Velocity profiles versus depth for the varying cross-sectional area of the lateral inflow channel at angle 40^0.

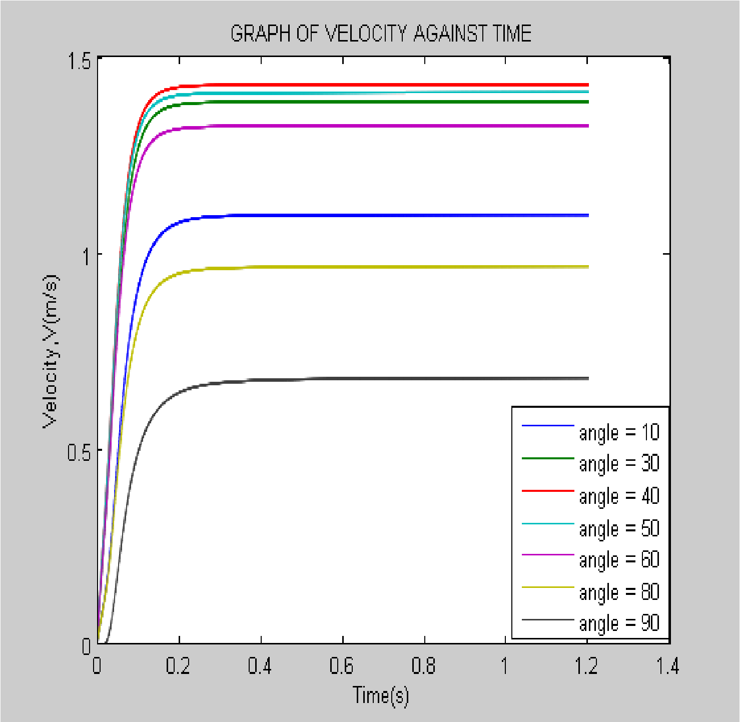

Fig. 3. Velocity profiles versus time along the channel for varying angle of the lateral inflow channel.

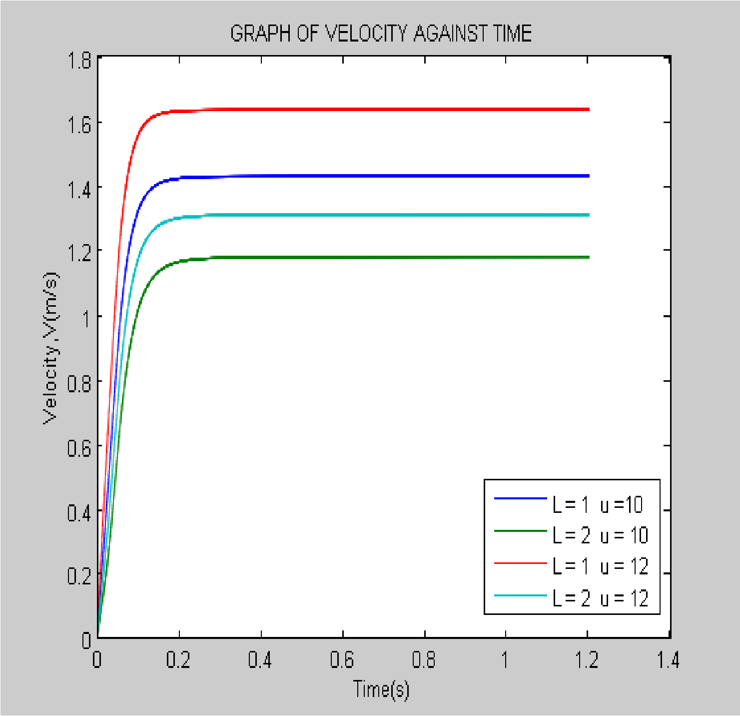

Fig. 4. Velocity profiles versus time along the channel for varying length and velocity of the lateral inflow channel at angle 40^0.

From figure 2, an increase in the cross-sectional area from 0.08 m^2 to 0.32 m^2 of the lateral inflow channel leads to a decrease in the flow velocity of the open rectangular channel. This is because an increase in the area leads to an increase in the wetted perimeter of the lateral inflow channel since the fluid will spread more in the conduit. A large wetted perimeter leads to large shear stress at the sides of the channel, which results in the flow velocity being reduced. In addition, an increase in depth leads to an increase in the velocity.

From figure 3, we note that an increase in the angle of the lateral inflow channel does not necessarily mean an increase in the velocity of the fluid in the open rectangular channel. We see that an angle of 40^0 has a higher velocity value than an angle of 90^0. The angles 10^0, 60^0, 80^0 and 90^0 have lower values of velocity compared to angles 30^0, 40^0 and 50^0. Since the discharge is directly proportional to velocity, this means that for one to get a maximum discharge from the lateral inflow channel, one has to construct it with an angle ranging from 30^0-50^0.

From figure 4, we observe that an increase in the velocity of the lateral inflow channel from 10m/s to 12 m/s leads to an increase in the flow velocity of the open rectangular channel. However, an increase in the length of the lateral inflow channel from 1m to 2m leads to a reduction in the flow velocity in the main open channel. An increase in the flow velocity of the lateral inflow channel means that more fluid particles at a given time will collide with the fluid particles in the main open channel resulting in more random motion of the particles. This random motion leads to bombardments between the fluid particles resulting in an increase in the velocity of the particles, which in turn leads to an increase in the velocity of the fluid. An increase in the length of the lateral inflow channel results in a decrease in the velocity because of the increase in the shear stress on the walls and the bottom of the channel.

5. Conclusion

Investigations of the effects of varying angle, the cross-sectional area, velocity and length of the lateral inflow channel on velocity in the open rectangular channel have been carried out. Increasing the angle of the lateral inflow channel does not necessarily mean an increase in the velocity in the main open channel. Angles in the range of 30^0 and 50^0 exhibit higher values of velocity in the open rectangular channel compared to other angles. Increasing the cross-sectional area of the lateral inflow channel leads to a decrease in the flow velocity in this channel. Increasing the velocity in the lateral inflow channel leads to an increase in the flow velocity in the open rectangular channel, while increasing the length of the lateral inflow channel leads to a decrease in the velocity in this channel.

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