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Safety Analysis of Prestressed Concrete Bridge Beams in Flexure

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Abstract

A probabilistic evaluation of the safety of post-tensioned prestressed concrete simply supported bridge beams at ultimate limit state in flexure as specified in ACI 318 (2002)[1] and EC2 (2008);[2] with due considerations to the loadings recommended in BS 5400 (1978), [3] and AASHTO (2004), [4]; is presented herein and with a review of the relevant design process. Results indicate that the safety of post-tensioned concrete beams is sensitive to the sectional modulus of concrete at the bottom, effective prestress force, profile of eccentricity and the span of the beam in flexure.

Keywords: post-tensioned concrete; highway bridge beams and design safety.

1. Introduction

Prestressed concrete is a particular form of reinforced concrete. Prestressing involves the application of an initial compressive load on a structure to reduce or eliminate the internal tensile forces and thereby control or eliminate cracking. The initial compressive load is imposed and sustained by highly tensioned steel reinforcement reacting on the concrete. With cracking reduced or eliminated, a prestressed section is considerably stiffer than the equivalent (usually cracked) reinforced section.

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Prestressing may also impose internal forces which are of opposite sign to the external loads and may therefore significantly reduce or even eliminate deflection [5]. In general, prestressed concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from external loads are counter-acted to a desired degree. Prestressing is commonly introduced by tensioning the steel reinforcement either prior to concreting called pre-tensioning or after concrete has hardened by keeping steel bars in sheathing and is called post-tensioning. To obtain economy, high strength concrete and steel are used. Prestressed concrete members are free from cracks, more durable, able to resist impact loads, have better fatigue resistance and are well suited to large span structures [6]. The stresses due to eccentric prestressing force alone are generally a combination of direct and bending stresses. The analysis for prestressed members is done by well- known relationships for combined stresses as used in eccentrically loaded columns [7]. Since ends of beams are subjected to heavy stress concentrations, due to prestressing forces acting there, these are especially designed as end blocks [8].

However, external prestressing is one of the latest developments in prestressed concrete technology. It refers to a prestress technique where the prestressing tendons are placed outside the concrete section and the prestressing force is transferred to the concrete by means of end anchorages, deviators and saddles [9]. A prestressed bridge beam can fail in many ways; by crushing, corrosion of the tendons, excessive deformations, exceeding carrying capacity for shear or bending moment, local or overall buckling etc. [9]

The probabilistic design presented herein is towards a numerical approach to the safety analysis of a simply supported post-tensioned concrete bridge beam, within an acceptable probability that the given structure will not fail during it's intended life. The aim of this paper is (i) to evaluate the safety of post-tensioned bridge beams in flexure; (ii) verify the consistency of the safety indices and check to determine whether parameters are within established criteria; (iii) make suggestions and recommendations based on research findings.

2. Background knowledge on concrete and prestressing concrete

Post-tensioning is a method of prestressing in which concrete is cast around hollow ducts which are fixed to any desired profile. The steel tendons are usually in place, unstressed in the ducts during the concrete pour, or alternatively may be threaded through the ducts at some later time. When the concrete has reached its required strength, the tendons are tensioned. Tendons may be stressed from one end with the other end anchored or may be stressed from both ends [10]. The tendons are then anchored at each stressing end. The concrete is compressed during the stressing operation and the prestress is maintained after the tendons are anchored by bearing of the end anchorage plates onto the concrete. The post-tensioned tendons also impose a transverse force to the member wherever the direction of the cable changes. After the tendons have been anchored and no further stressing is required, the ducts containing the tendons are often filled with grout under pressure. In this way, the tendons are bonded to the concrete and are more efficient in controlling cracks and providing ultimate strength. Bonded tendons are also less likely to corrode or lead to safety problems if a tendon is subsequently lost or damaged [11]. It is impossible to measure the tension force of post-tensioned concrete bridges directly. As the tendon load can be transferred continuously to the structure by the bond, the total tendon load cannot be activated by lifting-off the anchor head by means of a press as seen with external tendons [12]. Therefore, several problems such as the possibility of fatigue fracture of post-tensioning steel at coupling joints arises [12]. The probability of fracture can be reduced if some tendons cross the joint.

Prestressed concrete members must satisfy working-stress requirements under service loads and strength requirements under factored loads [13; 14]. Working stress limits for flexure usually control the selection of cross section and prestressing steel. In particular, the design of precast-prestressed concrete bridge girders is usually controlled by the flexural stress in the bottom flange, f_{bot}.. This is computed from the elastic beam theory as follows [15]:

$$f_{\mathrm{bot}} = -P_i/A(1 + ec_{\mathrm{bot}}/r^2) + M_{\mathrm{DL}}/s_{\mathrm{g}} \tag{1a}$$

under initial conditions and

$$f_{\mathrm{bot}} = -P_e/A(1 + ec_{\mathrm{bot}}/r^2) + M_{\mathrm{DL}}/s_{\mathrm{g}} + M_{\mathrm{SIDL}}/s_{\mathrm{comp}} + M_{\mathrm{LL}}/s_{\mathrm{comp}} \tag{1b}$$

under effective, or service conditions. This stress must satisfy

$$f_{\mathrm{bot}} \geq f_{\mathrm{ci,all}} \tag{1c}$$

$$f_{\mathrm{bot}} \leq f_{\mathrm{te,all}} \qquad \text{(tension is positive)} \tag{1d}$$

where P_e = effective prestress force (after accounting for prestress losses); A = girder cross-sectional area; e = distance from centroid of girder to centroid of prestressing force; c_{bot} = distance from centroid of girder to bottom of girder; r = girder cross-section radius of gyration = $\sqrt{(I/A)}$; M_{DL}= moment due to weight of girder and deck; M_{SIDL} = superimposed dead-load moment (e.g., barriers and diaphragms); M_{LL}= moment due to live-load;

$f_{ci,\mathrm{all}}$, $f_{te,\mathrm{all}}$ = allowable stresses in compression and tension under initial and effective conditions respectively; and S_g ,S_{comp} = section modulus of bare girder and composite section.

At release, Equation (1*c*) places an upper bound on the level of prestress steel. In service, Equation (1*d*) limits the magnitude of the allowable live load. These calculations account for many sources of stress, including the effects of applied loads M_{DL}, M_{SIDL}, M_{LL} and of prestress Pe, which in turn includes estimates of losses attributable to steel relaxation and to elastic shortening, shrinkage, and creep of the concrete. By contrast, the specification does not require the effects of temperature variations to be included, so they are commonly ignored, even though they can be significant [16].

Variations in temperature distribution in bridge members can be described in terms of a uniform component and a temperature gradient. The average (uniform) temperature change only causes changes in axial length of the member, while the temperature gradient causes bending deformations [15]. In structures that are externally statically determinate, such as single-span bridges, these deformations occur without inducing external forces, and the main design consequence of temperature variations is that these deformations must be accommodated. For example, the designer needs to provide bearings with adequate displacement and rotation capacities.

Indeterminacy also occurs over the cross section and can be referred to as internal indeterminacy. If the temperature distribution is nonlinear over the height of the girder, thermally induced stresses will develop during curing and in service. By contrast, stresses due to external indeterminacy are unlikely to occur during curing because the members are simply supported at that time. Both types of indeterminacy (external and internal) contribute to the concrete stress. The relative magnitudes of the contributions depend on the bridge properties and the temperature distribution [16].

In the design of a typical prestress beam, a value of prestress force which will permit all stress conditions to be satisfied at the critical section must be determined. It is necessary to determine the eccentricity at which this force must be provided, not only at the critical section but also throughout the length of the member. At any section along the length of the member, *e,* is the term which defines the effect of variations of moment, prestress force and section properties along the member. The design expressions can be written as [8]:

At transfer,

$$e \leq \left[\frac{Z_t}{A} - \frac{f'_{min} Z_t}{P_o}\right] + \frac{M_{min}}{P_o}, \tag{2a}$$

$$e \leq \left[-\frac{Z_b}{A} + \frac{f'_{max} Z_b}{P_o}\right] + \frac{M_{min}}{P_o}, \tag{2b}$$

At service,

$$e \geq \left[\frac{Z_t}{A} - \frac{f_{max} Z_t}{kP_o}\right] + \frac{M_{max}}{KP_o}, \tag{2c}$$

$$e \geq \left[-\frac{Z_b}{A} + \frac{f_{min} Z_b}{KP_o}\right] + \frac{M_{max}}{KP_o}, \tag{2d}$$

The above equations can be evaluated at any section to determine the range of eccentricities within which the resultant force P_o must lie. The moments M_{max} and M_{min} are those relating to the section being considered. For a member of constant cross-section, if minor changes in prestress force along the length are neglected, the terms in bracket in the above expressions are constant. Therefore the zone within which the centroid must lie is governed by the shape of the bending moment envelopes [8]. In the case of uniform loading, the bending moment envelopes are parabolic, hence the usual practice is to provide parabolic tendon profiles if a straight profile will not fit within the zone [5]. At the critical section the zone is generally narrow and reduces to zero if the value of the prestress force is taken as the minimum value from the Magnel diagram. At sections away from the critical section, the zone becomes increasingly greater than the minimum required.

2.1 Design Criteria for Bridge Beams

To calculate the capacity of a typical post-tensioned bridge beam, basic principles of engineering structural analysis/structural mechanics were used. In this case, the bridge cross-section used for design was reviewed and moment capacities computed. The determined values were taken as nominal resistance for probabilistic modeling. It should be noted that in the calculation of capacities, no resistance factors (i.e., strength reduction factors) were applied. However a reliability-based design of a single span simply supported post-tensioned bridge beam is analyzed. The disposition of bridges involves clear economical and safety implications. To avoid high costs of replacement or repair, the evaluation must accurately reveal the present load carrying capacity of the structure and predict loads and any further changes in the capacity (such as deterioration) in the applicable time span.

In this work, the reliability analysis is performed for the bridge beam. The load and resistance models and the limit state functions are defined. In determining the loads to be considered, BS 5400 (1978) part 2, divides the nominal loads into two groups namely; permanent and transient loads. The permanent loads are defined as dead loads, superimposed dead loads, loads due to filling materials, differential settlement and loads derived from the nature of the structural material. The transient loads on the other hand refers to all loads other than the permanent loads: these consist of wind loads, temperature loads, exceptional loads, erection loads, the primary and secondary highway loadings, footway and cycle tract loadings, and the primary and secondary railway loading, if any. Primary highway and railway loadings are vertical live loads, whereas the secondary loadings are the live loads due to changes in speed or direction. Hence the secondary highway loading include centrifugal, braking, skidding and collision loads and the secondary railway loadings include lurching, nosing, centrifugal, traction, and braking loads [17].

The general philosophy governing the application of the loads is that the worst effects of the loads should be sought. In practice, this implies that the arrangement of the loads on the bridge is dependent upon the load effect being considered, and the critical section being considered. The applied loading (LRFD) used on the flexural optimisation of the bridge and the span considered is as shown in Figure 1.

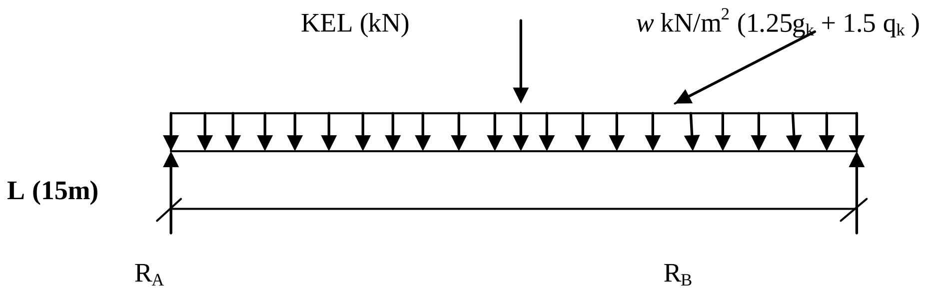

Figure 1: Optimised bridge beam system.

In a prestressed concrete beam, failure can occur when either the prestressing steel or the concrete fails. In most cases however, the amount of prestressing steel (or non-prestressing steel at the bottom is large enough to prevent failure of steel before failure of concrete. Hence, in practice, the failure of a prestressed concrete beam occurs when the concrete at the compression zone fails. For estimating the ultimate load, or load corresponding to failure, it is assumed that failure occurs when the strain in the extreme compression fiber in concrete reaches a limiting value ε_u. The ultimate load therefore, may be thought of as a load that by definition corresponds to a strain in concrete at the top fiber ε_u. The quantity ε_u does not necessarily correspond to the complete collapse of the beam. However, a beam with an extreme fibre strain of ε_u has deformed beyond usefulness [18].

3. Materials and methods

3.1 Structural Reliability Algorithm

The reliability of a structure is defined here as it's probability to fulfil the safety requirement for a specified period or it's lifetime. Reliability is defined as the probability of a performance function $g(\mathbf{X})$ greater than zero i.e. $P\{g(\mathbf{X}) > 0\}$. In other words, reliability is the probability that the random variables $X_i = (X_1, \ldots\ldots, X_n)$ are in the safe region that is defined by $g(\mathbf{X}) > 0$. The probability of failure is defined as the probability $P\{g(\mathbf{X}) > 0\}$ .Or it is the probability that the random variables $X_i = (X_1, \ldots\ldots, X_n)$ are in the failure region that is defined by $g(\mathbf{X}) > 0$. In a mathematical sense, structural reliability can be defined as the probability that a structure will not attain each specified limit state (ultimate or serviceability) during a specified reference period. An important component of structural reliability is concerned with the calculation or estimation of the probability of a limit state violation for the structure during its lifetime. The probability of occurrence of structural failure or a limit state violation is a numerical measure of the likelihood of its occurrence of the interested event for generally similar structures or using numerical analysis and simulation, based on measurement data for the elements involved in modelling. For example, for highway bridge structures, statistics of data for these elements are used in modelling, such as bridge components, strengths sizes, deterioration rates, truck load magnitudes, traffic volume, etc [19].

Assume that R and S are random variables whose statistical distributions are known very precisely as a result of a very long series of measurements. R is a variable representing the variations in strength between nominally identical structures, whereas S represents the maximum load effects in successive T-yr periods, then the probability that the structure will collapse during any reference period of duration T years is given by Equation (3) as:

$$P_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_s(x) dx \qquad (3.1)\ (3)$$

where, F_R is the probability distribution function of R and f_s the probability density function of S. Note that R and S are statistically independent and must necessarily have the same dimensions. The reliability of the structure is the probability that it will survive when the load is applied, given by Equation (4) as:

$$\Re = 1 - P_f = 1 - \int_{-\infty}^{\infty} F_R(x) f_s(x) dx \qquad (3.2)\ (4)$$

3.2 *Limit State Function*

The performance function g(x) is sometimes called the limit state function. It relates the random variables for the limit-state of interest. The limit state function, gives a discretised assessment of the state of a structural element as being either failed or safe. It is obtained from traditional deterministic analysis, but uncertain input parameters are identified and quantified. Interpretation of what is considered to be an acceptable failure probability is made with consideration of the sequences of failure, which is predetermined [20].

Therefore, the model must also include the load magnitude and frequency of occurrence, rather than just load magnitude as is the case in the ultimate limit states. This thesis is focused on the ultimate limit state of the moment carrying capacity. The limit state, g(x) = R – S, is a function of material properties, loads and dimensions. The state of the performance function g(x) of a structure or its components at a given limit state is usually modelled in terms of infinite uncertain basic random variable $x = (x_1, x_2, \ldots\ldots\ldots, x_n)$ with joint distribution function given as Equations (5) and (6) [19]:

$$F_s(x) = P\{\prod_{i=1}^{n} x_1 \leq x_2)\} \tag{5}$$

$$P_f = P\{g(x) \leq 0\} = \int f_x(x)\, dx \tag{6}$$

where $f_x(x)$ is the joint probability distribution function of x. The region of integration of the function g(x) is stated as: g(x) > 0: represents safety; g(x) = 0: represents attainment of the limit state; g(x) < 0: represents failure. The probability of failure is given by P(g(x) < 0) and therefore the reliability index, β, can be related to probability of failure by the following Equation (7).

$$P_f = 1 - \Phi\,(\beta) \tag{7}$$

A relationship can be drawn between the probability of failure, P_f, and the reliability index, β [21; 19]. However, this position is true only when the safety margin is linear in the basic variables, and these variables are normally distributed. This relationship is stated below:

$$P_f = \Phi\left(-\beta\right) \tag{8}$$

$$\beta = -\Phi^{-1}\left(P_f\right) \tag{9}$$

where Φ is the standardized normal distribution function.

$$P_f = P\{(R - S) \leq 0\} = P(M \leq 0) = \varphi\left\{\frac{0 - (\mu_R - \mu_s)}{\sqrt{\sigma_R^{\,2} + \sigma_s^{\,2}}}\right\} = \Phi\left(-\beta\right) \tag{10}$$

The basic principles and mathematical relationships used in the design and analysis of prestressed concrete flexural members are unique. It should be apparent that in order to review a member as described here, the dimensions of the concrete section, the properties of the materials, the amount and eccentricity of the prestressing steel, the amount of non-prestressed reinforcement, as well as the amount of web reinforcement must be known [22]. The design of a member consist of selecting and proportioning a concrete section in which the stresses in the concrete do not exceed the permissible values under any condition of loading and prestressing, design also includes the determination of the amount and eccentricity of the prestressing force that is required for the specific section. Although, the design of a flexural member is normally a trial and error procedure, thus in considering the case study, a concrete section was assumed as well as the eccentricity and prestressing force required to confine the concrete stresses within the allowable limits under the load resisting conditions.

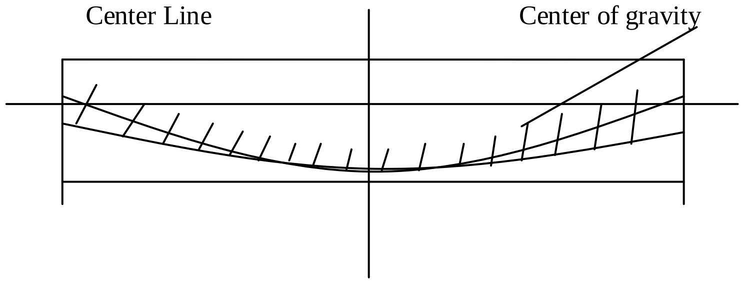

Figure 3: Schematic diagram showing locus of prestressing force.

The locus of the prestress force is the area in which the prestress force must be applied in order to satisfy initial and final stress requirements. Eccentricity is the distance from the centroid of the prestressing steel to the center of gravity of the beam section, and its variation defines the shape of the profile of the centroid of steel. The eccentricity may be defined by a parabola or a combination of a parabola and a fourth degree curve. The eccentricity varies along the beam, and this variation influences the indeterminate moments [23].

The Figure 3 below represents a simply supported prestressed concrete beam. The variation of eccentricity which also defines the position of the centroid of the steel is presented by a parabola symmetric at the midspan and assumed to be the origin while Figure 4 shows the cross-section of the beam.

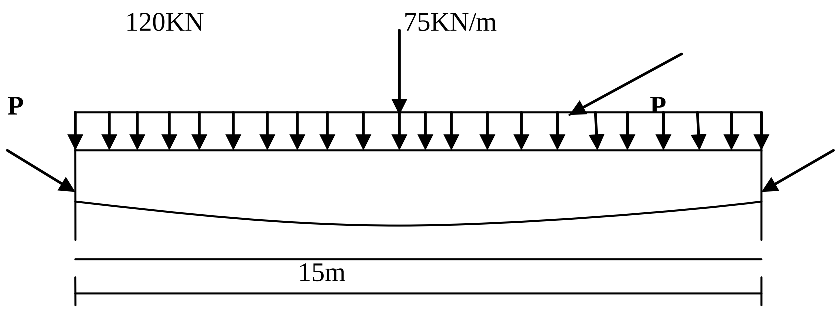

Figure 3: Simply supported post-tensioned bridge beam showing centroid of prestress steel.

The provision of ultimate design can be used to proportion a section with a rigorous control of both strength and ductility. The provisions of working stress design can then be used to check the stresses at transfer, and the service loads in the section so designed. A rational design of the section is considered simpler by ultimate design than by service-load design [24]. The procedure for the simple parabola was used in the determination of the ultimate moment of resistance.

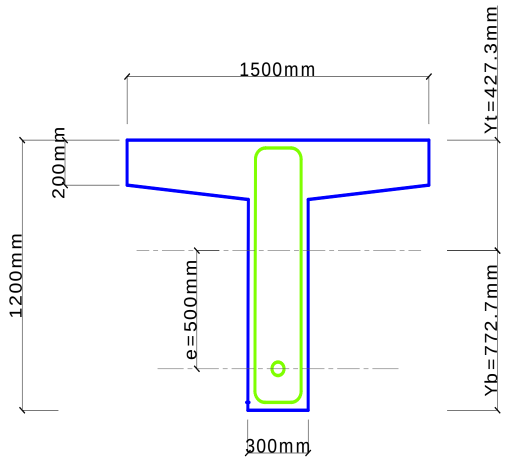

Figure 4: Cross section through bridge beam.

4. Analysis

4.1 MODE I: Moment Equation

Resultant stress *R*, induced due to concrete properties:

$$R = \left(\frac{P}{A} + \frac{P_e}{Z_b}\right) \tag{11}$$

But, $\frac{M}{Z} = \sigma$; therefore, M = σZ

$$M_R = \left\{\frac{P}{A} + \frac{P_e}{Z_b}\right\} Z_b \tag{12}$$

Ultimate moment applied due to UDL:

$$M_A = \left(\frac{Wl^2}{8} + \frac{Pl}{4}\right) \tag{13}$$

Limit state equation for flexure is thus,

$$G(x) = M_R - M_A \tag{14}$$

M_U = Moment due to ultimate applied uniformly distributed loads,

M_R = Moment due to resistance of beam,

$$G(x) = \left\{\frac{P}{A} + \frac{P_e}{Z_b}\right\} Z_b - \left\{\frac{Wl^2}{8} + \frac{Pl}{4}\right\} \tag{15}$$

where: P_e = Effective prestressing force applied (kN); A = Cross section area (mm^2); e = eccentricity (mm); Z_b = Bottom sectional modulus of elasticity (mm^3); w = Design load at ultimate limit state (KN/m), L = Span of beam (m),

Example: Consider a prestressed concrete simply supported beam

Design Parameters assumed: P = 4488KN; e = 500mm; A = 660,000mm^2; Z_b = 1.04E8mm^3; W = 75KN/m; L = 15m.

$$G(x) = \left\{\frac{4488E3}{660E3} + \frac{4488E3 \; X \; 500}{1.04E8}\right\} 1.04E8 - \left\{\frac{75 \, X \, 15^2}{8} + \frac{120 \; X \; 15}{4}\right\}$$

G(x) = 2951– 2559 = 392KNm

4.2 Computation of Safety Index

The First Order Reliability Method (FORM) coded in FORM5 [25], is employed in the computation, making use of the tabulated data in Table 1 and the relevant limit state functions.

Table 1: Parameters of the Stochastic Model

S/No	VARIABLES	DISTRIBUTION TYPE	COEFFICIENT OF VARIATION	EXPECTED VALUE, (EX)	STANDARD DEVIATION (SX)
1	Tendon eccentricity, e (mm)	Normal	0.10	500	50
2	Beam span, L(m)	Log-Normal	0.05	15	0.75
3	Effective prestressing force P_e (KN)	Log-Normal	0.05	4488	224.4
4	Bottom sectional modulus Z_b (mm^3)	Normal	0.01	100000000	1000000

4.3 Results and Discussion

A FORTRAN program was developed for strength equations at limit state conditions and using FORM5 as suggested by Gollwitzer, [25]; from which results were obtained using the limit states in in Equation (9) in conjunction with the iterated parameters for design. The results are presented in Figures 5 to 16.

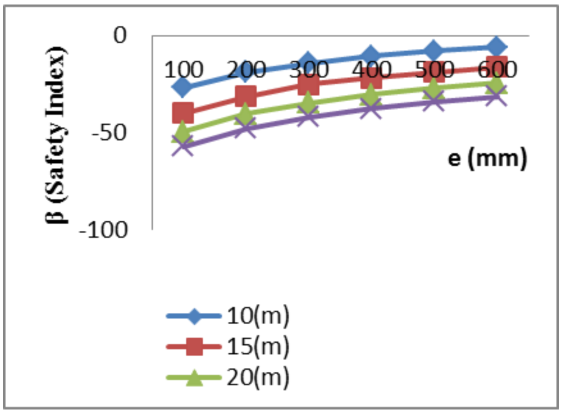

Fig 5: β Values at P_E = 1000KN and $Z_B = 1.0 \times 10^8$ mm^3

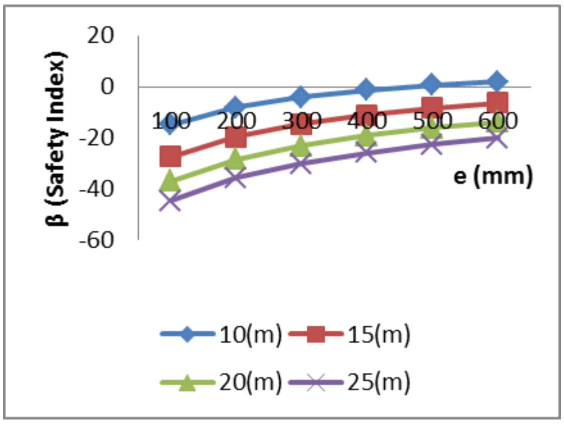

Fig 8: β Values at P_E = 2000KN and $Z_B = 1.0 \times 10^8$ mm^3

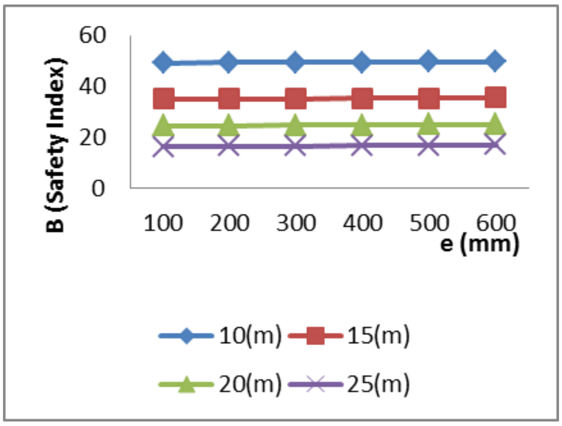

Fig 6: β Values at P_E = 1000KN and $Z_B = 1.0 \times 10^{10}$ mm^3

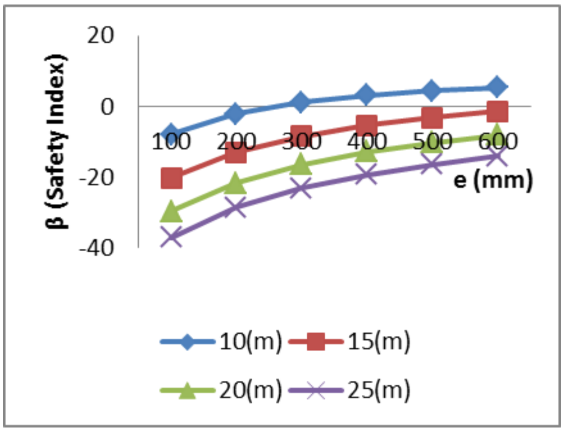

Fig 9: β Values at P_E = 3000KN and $Z_B = 1.0 \times 10^8$ mm^3

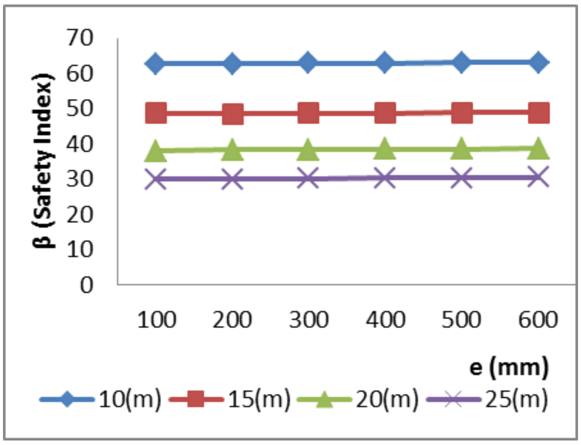

Fig 7: β Values at P_E = 2000KN and $Z_B = 1.0 \times 10^{10}$ mm^3

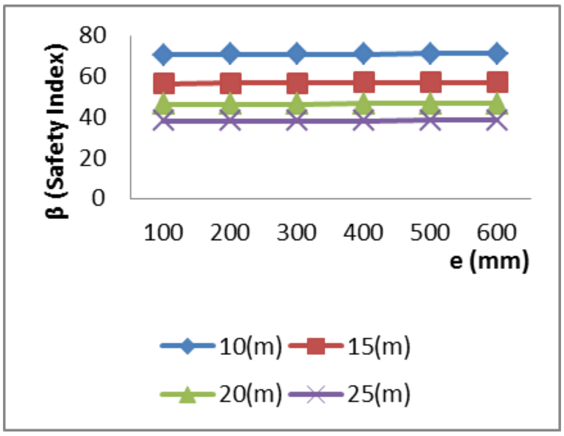

Fig 10: β Values at P_E = 3000KN and $Z_B = 1.0 \times 10^{10}$ mm^3

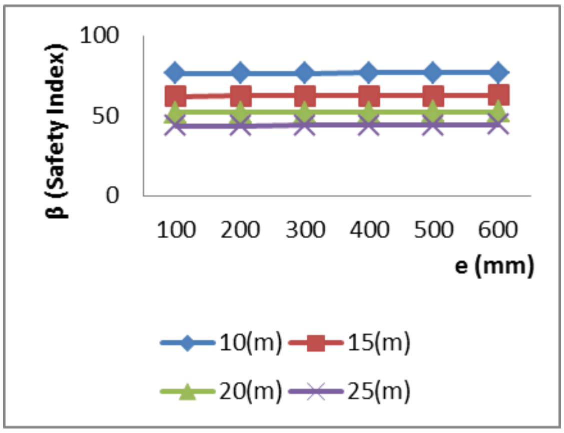

Fig 11: β Values at P_E = 4000KN and Z_B = 1.0 x 10^{10} mm^3

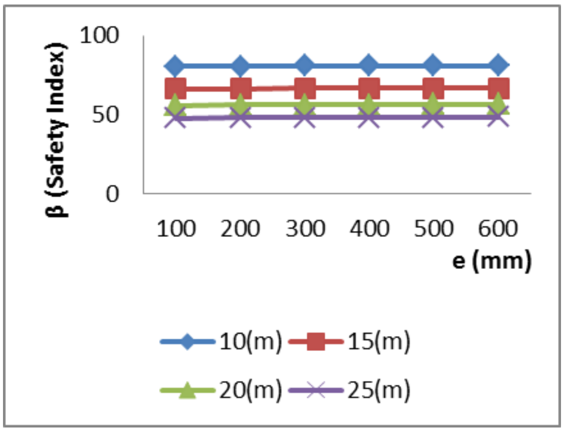

Fig 14: β Values at P_E = 5000KN and Z_B = 1.0 x 10^{10} mm^3

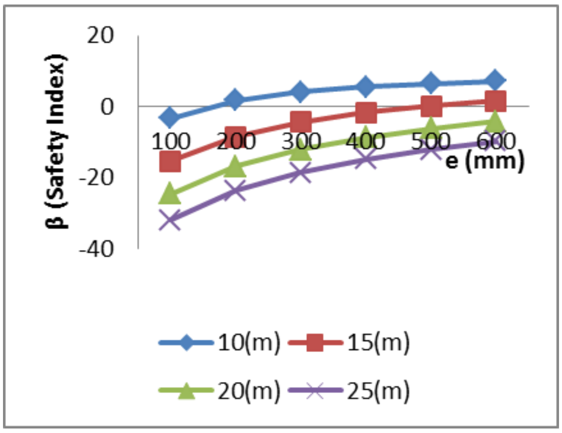

Fig 12: β Values at P_E = 4000KN and Z_B = 1.0 x 10^8 mm^3

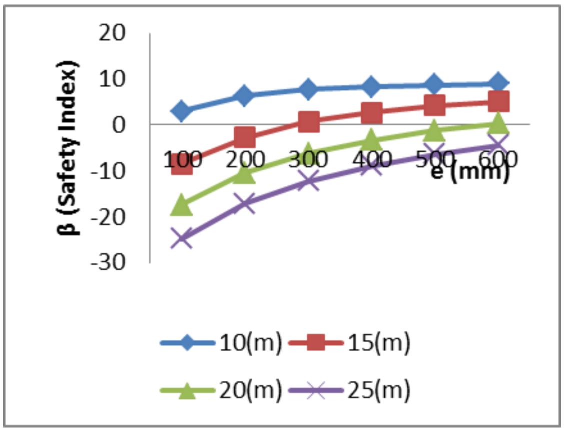

Fig 15: β Values at P_E = 6000KN and Z_B = 1.0 x 10^8 mm^3

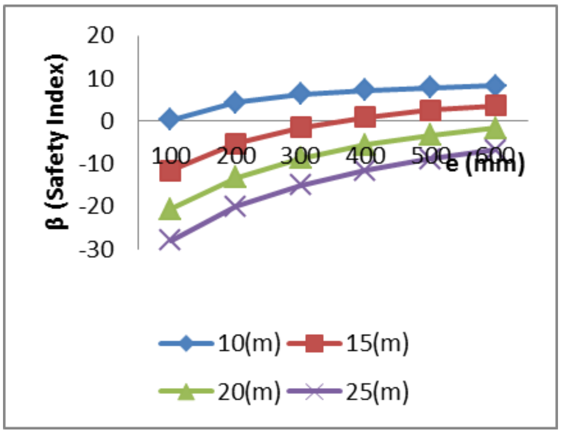

Fig 13: β Values at P_E = 5000KN and Z_B = 1.0 x 10^8 mm^3

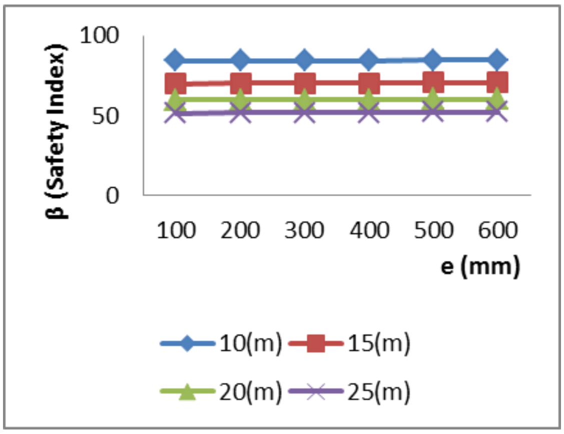

Fig 16: β Values at P_E = 6000KN and Z_B = 1.0 x 10^{10} mm^3

5. Conclusion

The safety of a simply supported prestressed concrete beam in service is dependent on the optimum eccentricity at which a given prestressed force is applied. Furthermore the study reveals as shown in Figures 5 to 16, that the ultimate moment of resistance the section can be expected to develop is dependent on the eccentricity of tendon profile as well as magnitude of the prestress force and the concrete bottom sectional modulus of the concrete at the bottom. Thus, the variation of parameters in the probabilistic evaluation using the First Order Reliability method and coded in FORM5 [25] indicates that, as the eccentricity and effective prestresssing force increases with bottom sectional modulus, the safety of the beam increases; but with smaller effective prestressing force, and lower eccentricity, the safety of the beam decreases drastically. It was observed that with increasing bottom sectional modulus and reduced eccentricity and effective prestress force the safety of the beam increases. Therefore, the safety of a prestressed concrete beam element depends greatly on the concrete bottom sectional modulus and the tendon eccentricity profile at which a given prestress force is applied.

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