



Modelling Volatility of Stock Returns: Is GARCH(1,1) Enough?

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Abstract

In this paper, we apply the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of different lag order to model volatility of stock returns on Uganda Securities Exchange (USE). We use the Quasi Maximum Likelihood Estimation (QMLE) method to estimate the models. Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are used to select the best GARCH(p,q) model. From the empirical results, it has been found that USE returns are non-normal, positively skewed and stationary. Overall, GARCH(1,1) outperformed the other GARCH(p,q) models in modeling volatility of USE returns.

Keywords: Volatility; Modelling; stock returns; GARCH(1,1).

1. Introduction

In the financial sector, the knowledge on volatility of any financial time series is an important aspect for risk management. According to [1], volatility is defined as a statistical measure of the dispersion of returns for a given security or market index and it can either be measured using the standard deviation or variance between returns from that same security or market index.

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There is observed considerable uncertainty and volatility both in the emerging and mature stock markets. Great concern is about the fluctuating returns of their investments due to the market risk and variation in the market price speculation as well as the unstable business performance, [2]. In the real world of financial markets, investors and financial analysts are generally more interested in the profit or loss of the stock over a period of time that is; the increase or decrease in the price, than in the price self.

Modelling volatility in financial markets is important because it sheds further light on the data generating process of the returns, [3] and the riskiness associated with the asset since volatility is related to risk, [4]. Due to the usefulness of volatility, various models have been developed since Engel's paper of 1982.

Engle (1982), [5] studied on ARCH and Bollerslev (1986), [6] on GARCH models, and revealed that, these models were designed to deal with the assumption of non-stationarity found in real life financial data. He further pointed out that these models have become widespread tools for dealing with time series heteroscedasticity. The ARCH and GARCH models treat heteroscedasticity as a variance to be modelled. The goal of such models is to provide a volatility measure like a standard deviation that can be used in financial decisions concerning risk analysis, portfolio selection and derivative pricing.

The assumption that variance is constant through time is statistically inefficient and inconsistent, [8]. In real life, financial data for instance stock market returns data, variance changes with time (a phenomenon termed as heteroscedasticity), hence there is need for studying models which accommodate this possible variation in variance. Many studies have suggested that volatility of returns in stock markets world over can be modelled and forecasted using the GARCH type models.

Financial time series usually exhibit stylized characteristics. Firstly, it was observed by [9], that financial returns displayed volatility clustering meaning that large changes in the price of an asset are often followed by other large changes, and small changes are often followed by other small changes. Secondly, [10] demonstrated that financial data exhibit leptokurtosis meaning that the distribution of the returns is fat-tailed. Finally, [11] introduced the leverage effect meaning that volatility is higher after negative shocks than after positive shocks of the same magnitude. A good volatility model, then, must be able to capture and reflect these stylized facts, [12].

The main objective of this study is to examine whether GARCH(1,1) model is the best at volatility modeling. The rest of the paper is organized as follows; section 2 looks at the methodology, section 3 discusses data analysis and results, section 4 discusses conclusions and recommendations.

2. Methodology

2.1 GARCH(p,q) Model

The Autoregressive Conditional Heteroscedasticity (ARCH) model by Engle (1982) [5] and its generalization, GARCH by Bollerslev (1986) [6] are the major and widely used methodologies in modeling and forecasting volatility of financial time series. The standard GARCH (p, q) model expresses the variance at time, t as:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2. \quad (1)$$

σ_t^2 is the conditional variance, ε_t the residual returns, defined as; $\varepsilon_t = \sigma_t z_t$ and $z_t \sim N(0,1)$ i.e are standardized residual returns. ω , α_i and β_j are the parameters to be estimated. In order for the variance to be positive the necessary condition is that $\omega > 0$, $\alpha_i \geq 0$ (for $i = 1, \dots, q$) and $\beta_j \geq 0$ (for $j = 1, \dots, p$). For $p = 0$, equation (1) reduces to an ARCH(q) model and for $p = 0 = q$, equation(1) reduces to simply white noise. In this model, the conditional variance only depends on the magnitude, and not the sign, of the underlying asset. Large ARCH coefficients, α_i imply that volatility reacts significantly to market movements while large GARCH coefficients, β_j indicate that shocks are persistent, [13].

GARCH models can be estimated using Maximum Likelihood (ML) and Quasi Maximum Likelihood (QML) approaches. ML assumes and maximizes a density function for the parameters that are conditional on a set of sample outcomes. [6] Bollerslev, propose a QML technique that adjusts for small deviations from normality. Under this study, the models will be estimated using QML.

2.2 Model selection

In financial modelling, one of the main challenges is to select a suitable model from a candidate family to characterize the underlying data. The choice of a good model in the application of time series analysis is crucial; the total process cannot be automated since the context is all important and there is never a perfect or unique model. Model selection criteria provide useful tools in this regard and assesses whether a fitted model offers an optimal balance between goodness-of-fit and parsimony. Ideally, a criteria will identify candidate models that are either too simplistic to accommodate the data or unnecessarily complex. The most common model selection criteria are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). A desirable model is one that minimizes the AIC or the BIC.

$$AIC = -2(\text{Loglikelihood}) + 2k \tag{2}$$

$$BIC = -2\ln(\text{likelihood}) + k\ln N, \tag{3}$$

where N is the number of observations or equivalently, the sample size and k denotes the number of parameters. BIC penalizes more complex models (those with many parameters) relative to simpler models. This definition permits multiple models to be compared at once; the model with the highest posterior probability is the one that minimizes BIC.

Under this study, we consider four different GARCH(p,q) models; GARCH(1,1), GARCH(1,2), GARCH(1,3), and GARCH(1,4).

3. Data analysis and results

3.1 Data

We data from the daily closing prices of the USE All share index for a period of starting from January 2005 to December 2013. Let P_t and P_{t-1} denote the closing market index of USE at the current(t) and previous day (t-1), respectively. The USE All Share returns (log returns or continuously compounded returns) at any time are given by:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \quad (4)$$

3.2 Basic statistics of USE return series

Before we employ the GARCH model to our data under consideration, it is necessary that we study its statistical properties. These include among others; excesskurtosis, skewness, and Jarque- Bera statistic for normality.

Table 1: Descriptive statistics of USE return series

Statistic	Value
Maximum	0.4766000
Minimum	-0.4844000
Median	0.0002228
Mean	0.0009705
Standard deviation	0.03649952
Excess Kurtosis	99.9358
Skewness	0.3190972
Jarque-Bera	630969.748
JB probability	<2e-18
No. of observations	1425

Table1 above shows the descriptive statistics for the USE return series. The mean is close to zero as is expected for a time series. The return series are positively skewed an indication that the USE returns are non-symmetric. There is a high positive value of excess kurtosis indicating that the underlying distribution of the returns is leptokurtic or heavy tailed. The return series is non-normal according to the JB test which rejects normality at the 1% level.

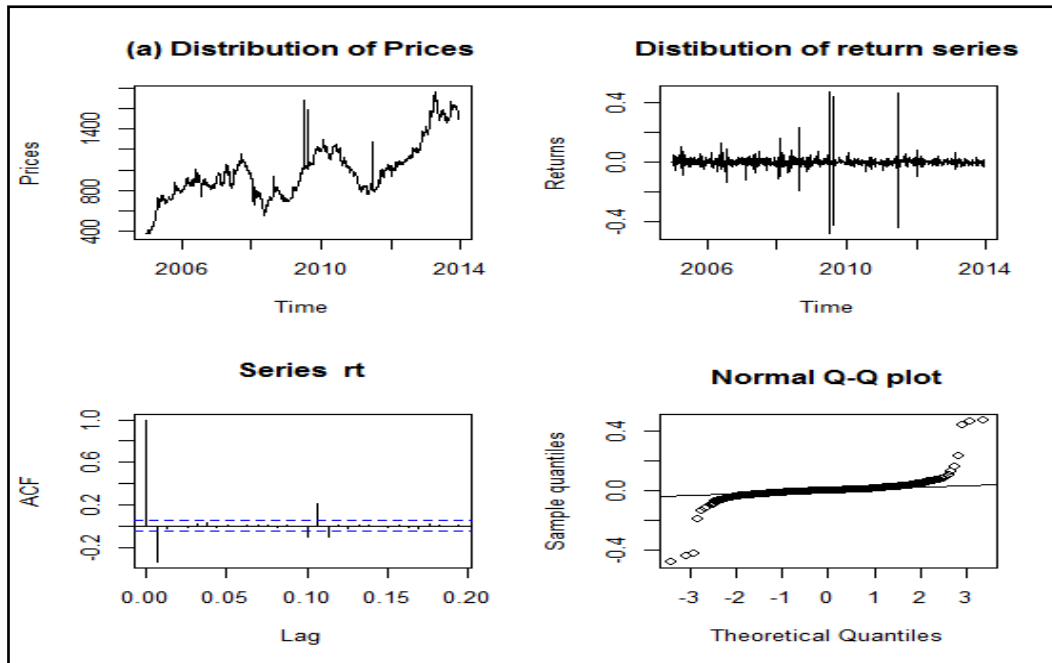


Figure1: Distribution of USE All share Index and return series.

From Figure1 above, the stock prices are non stationary while the return series are stationary with a mean return of zero. There is also evidenced volatility clustering in the return series which is similar to other studied stock exchanges. The quantile-quantile (Q-Q) plot shows that the USE return series are non-normal supporting the high positive excess kurtosis and the Jarque-Bera test indicated in Table 1. There is little evidence of serial correlation in the return series according to the autocorrelation function (ACF).

Before we apply the GARCH models to the USE data, it is always required that we test if the USE return series are stationary and test if ARCH effects are present in the residual return series. To do this, we use Augmented Dickey Fuller (ADF) , [14] to test for stationarity and the ARCH-LM test, [5] to test for ARCH effects. Below is the table indicating the above tests. For the ADF tests, we reject the null hypothesis of a unit root if the test statistic is less than the critical values for the levels of significance. For the ARCH-LM test, we reject the null hypothesis that there are no ARCH effects in the residual returns if the probability is less than the significance level. Note that the Critical values are taken from MacKinnon(1996),[15]

Table 2: ADF and ARCH_LM test results.

Series	ADF statistic	Critical Values		
		1%	5%	10%
Price Index	-2.2484(11)	-3.43	-2.86	-2.57
Returns	-10.1458(11)	-3.43	-2.86	-2.57
ARCH-LM test statistic		Prob. chi-square(12)		
128.9103		< 2.2e-16		

From Table 2 above, we accept the null hypothesis of a unit root for the price index; meaning that the price index is non-stationary for all the significance levels while we reject the null hypothesis for the return series meaning that the USE return series are stationary. The ARCH-LM test statistic indicates that there are ARCH effects present in the residual returns.

3.3 Empirical Results

We consider four GARCH(p,q) models; GARCH(1,1), GARCH(1,2), GARCH(1,3), and GARCH(2,1), estimate the parameters and compare their performance. The results are showed in the table below.

Table 3: Estimation results of the GARCH (p,q) models

Model	(1,1)	(1,2)	(1,3)	(1,4)
μ	-5.36e-4	-5.34e-4	-5.39e-4	-5.47e-4
ω	7.73e-6	7.72e-6	7.72e-6	7.69e-6
α	6.14e-1*	6.13e-1	6.13e-1**	6.14e-1**
β_1	7.63e-1***	7.63e-1	7.63e-1	7.63e-1
β_2	-	1.0e-8	1.0e-8	1.e-8
β_3	-	-	1.0e-8	1.0e-8
β_4	-	-	-	1.0e-8
$(\alpha + \beta)$	1.377	1.376	1.376	1.377
ARCH-LM	0.4075928	0.407486	0.407340	0.408595
Prob	0.9999999	0.999999	0.999999	0.999999
AIC	-4.3378	-4.3362	-4.3346	-4.3338
BIC	-4.3231	-4.3177	-4.3125	-4.3079

Note: *** indicates 1%, ** 5% and * 10% significance levels.

From Table 3 above, it can be seen that the mean return is insignificantly different from zero for all the models. All the parameters; ω , α_i and β_j have the expected signs. For GARCH(1,1), α is significant at the 10% level while β is significant at the 1% level. For the GARCH(1,2), all the parameters are non-significant while for GARCH(1,3) and GARCH(1,4), except for α , the rest of the parameters are not significant. For all the models, the persistence; $\alpha + \beta > 1$ indicating that the process is explosive.

The ARCH LM statistics indicate that there are no ARCH effects remaining in the residuals of the USE return series. Overall, GARCH(1,1) outperformed the other models as observed from the smallest AIC and BIC values.

4. Conclusion and Recommendations

The volatility of USE returns has been modelled for a period of 04/01/2005 to 18/12/2013 using different GARCH(p,q) models; GARCH(1,1), GARCH(1,2), GARCH(1,3) and GARCH(1,4).

Basing on the empirical results obtained, we can conclude the following: Firstly, it was found that the USE returns are not normally distributed and ARCH effects were found to be present in the residual return series which supports the use of the above models. Secondly, the USE return series also exhibit volatility clustering and leptokurtosis as seen from the high excess kurtosis values. Thirdly, the persistence, $\alpha + \beta > 1$ for all the models meaning that volatility of USE stock returns is an explosive process. Over all, GARCH(1,1) performed best in modeling volatility of USE stock returns.

It is recommended that Integrated GARCH is used to better explain the volatility process of USE returns. It is also recommended that asymmetric GARCH models are also used to test for the presence of leverage effects in the USE returns.

Acknowledgements

The authors would like to thank the African Union commission for funding this research study.

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