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## Road Design Types, Expected Queue Size and its Relief Times

L. I. Igbinosun<sup>a\*</sup>, V.O. Ezugwu<sup>b</sup>

<sup>a,b</sup> Department of Mathematics and Statistics

University of Uyo, P.M.B. 1017, Uyo, Akwa Ibom State, Nigeria

<sup>a</sup>Email: [luckyigbinosun@uniuyo.edu.ng](mailto:luckyigbinosun@uniuyo.edu.ng)

### Abstract

Most road networks are designed horizontally on the same plane and this often leads to intersections and conflicts. Vehicular traffic at a road junction and roundabout is examined. Queue theory models are discussed and applied to two road junction types, A T-junction and a roundabout. Expected road traffic build-up and relaxation times are presented. The time that must elapse for the system to return to free-flow state was calculated and results show that it takes longer time for traffic congestion to relax compared to its build up time. Road traffic build-up is higher in the T-junction than the roundabout.

**Keywords:** Vehicular traffic queues; Road intersection; Roundabout; Relaxation and build-up times.

**2010 Mathematics subject classification:** 60K25, 60K30, 90B06, 90B20.

### 1. Introduction

Road transportation has assumed a complex dimension in almost every city. It is evident that road users struggle for space rightly or wrongly, to the disadvantage of others. Although additional roads may be needed, there is need to develop proper road network utilization urgently. The horizontal design of most road network on the same plane leads to intersections and often conflicts [1].

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\* Corresponding author.

E-mail address: [luckyigbinosun@uniuyo.edu.ng](mailto:luckyigbinosun@uniuyo.edu.ng).

Considerable achievements have been made to reduce traffic congestions in recent times, but there has also been a steady increase of vehicle ownership. Road traffic queues and delays resulting from intersection conflicts has become a common phenomenon in Nigeria towns and cities [2].

Though some state governments had made various efforts to sanitize the roads by employing agent to maintain free flow of vehicular traffic, there have been cases of high handedness and responsibilities [3]. Also improper use of road network has often led to road accidents and loss of lives and useful man-hour [4].

In this paper, we focus on road traffic build up and relaxation time in a road network in Uyo metropolis. We seek answers to the following questions: when the road traffic intensity is higher than one, what happens to the queue length and by how much? Also, how long does it take the traffic build-up to relax to its free- flow state? This paper is structured as follows: section 2 give a brief review of relevant literature on road traffic flow and its associated congestion problems. Section 3 gives the model analysis, application and results. Discursion and conclusion is presented in section 4.

## **2. Literature Review**

A number of studies have been carried out to identify causes of road traffic congestion. Most of the studies obtained their data using questionnaire and personal observation. In Nigeria, some of the towns and cities covered include Abuja [5]; Akure [6]; Benin City [7,1, 8]; Lagos [9, 10, 11]; Enugu City [12]; Ilorin [13] Warri [14]. Some of the causes identified by these authors include: street trading, inadequate public mass transit buses, poor road traffic control, poor road maintenance, traffic light failure, absence of traffic wardens, too many cars on the roads, poorly maintained vehicles, poor road design, disregard for road traffic rules, flooding, poor drainage system, poor waste management, bad drivers behaviour, and absence of parking lots, see ([15, 9, 12, and 1]). Summary of the possible causes of road traffic congestion was given in a list by [16], and asserted that poor driving habit ranked highest (approximately 82%) in Nigeria.

Traffic control measures to deal with the problem of congestion demand an understanding of the nature of the build up and the factors that cause it. However, detecting developing congestion as early as possible is necessary so that recovering measures can be taken to reduce its impact [17]. The nature of the road network and nature of congestion determines the options to be taken. However, whatever combination of such options, present day traffic control methods show clear limitations, especially when traffic reach critical levels.

In intersections and for many networks, the problem is to determine the nature of arrivals, departures and delays from measurements of traffic flows made automatically and (or) continually at the entrance and exits to the network. In the analysis of traffic flow at signalized intersections, [18] emphasized that “a better understanding of the interaction between demand (i.e. arrival pattern) and supply (i.e. signal indications and types) at traffic signals is a prerequisite to the formulation of optimal signal control strategies”. Several models including fluid theory approach, steady-state queuing approach were discussed with their various advantages and limitations. In studying or modeling traffic flow at controlled intersections, average delay per vehicle, the number of vehicles stopped, the number of vehicles in the queue, expected delay and the average queue length are among the

important performance measures that can be derived. The authors in [19] investigated the distribution of queue length and delay at a major/minor road intersection without traffic control. Also, [19] argued that the system could be studied using the  $m/m/1$  queuing model and also used simulation to develop models for predicting the distribution of queue length and delay from the parameter of the major /minor road junction. In this study, we use the  $m/m/1$  queuing model and the renewal theory argument when the system is normal (unsaturated) and congested (saturated) respectively. [20] studied travel time in Southampton, the mechanistic and neural network approach were described for estimating journey times on non-signalized roads using loop detectors. Data on journey times can be used to identify abnormal congestion on the road so that effective road transport management strategy can be deployed. Our findings on causes of congestion suggests that congestion usually results when (a) the roads system is unable to accommodate traffic at an adequate speed (b) conflicts among the different types of traffic (cars, trucks, buses, motorcycles' or pedestrians), (c) Improper utilization of traffic controls, (d) Peculiar human behavior (e) Frequent breakdown of second hand vehicles and accidents etc.

There have been several efforts in the management of road traffic congestion in Nigeria. The authors in [21] listed some road traffic management techniques to include: road junction improvement, grade separation using bridges or tunnels, reversible lanes, preferential treatment for Higher Occupancy Vehicles (HOV), separate lanes for specific user groups, traffic calming measures, and improved traffic signs. Other authors including [22, 23, 24, 25] have also studied road traffic management in Nigeria. Their findings and recommendations are similar to that of [21]. Due to the predominance of road traffic congestion on our roads resulting from increase in vehicle ownership, methodologies for reducing traffic on the roads have their various limitations arising from some unrealistic assumptions made. [26] argue that road traffic congestion is likely to increase if nothing is done about it. Hence there is need to continue to study road traffic problem.

In spite of the remedies however, traffic congestion has remained in our major cities. It is only honest to state that disasters, man-made disasters, periodically and tragically wake us all up to the abysmal level of governance in most under-developed and developing countries. It is high time we stopped this wilful bleeding and wastages of human lives and man-hour. It takes no special magic to do so; just shunning the pervasive lawlessness on our roads. Not only is safety improved by reducing street intersections, but also the economy is affected both in time and money.

From the review it is evident that road congestion is not restricted to Nigeria alone. However, many places of the world are using computer aided gadgets to obtain knowledge about the phenomenon. Such knowledge is being used to develop efficient road traffic management system. Many causes of road congestion have been outlined in literature and remedies proposed ([1, 24, 5]). However, strategies for combating vehicular traffic congestion have been down played by the policy managers.

### **3. Model Analysis, Application and Results**

The condition for attaining steady state in a queuing system is that the traffic intensity, ( $\rho$ ) is less than 1, ([27, 28]). In general when  $\rho < 1$  at a road junction, then the junction is said to be under saturated (non-peak period

or free-flow) On the other hand, when  $\rho > 1$ , the system is said to be over-saturated (peak period). However, there is no name for the system in literature when  $\rho = 1$ .

The  $m/m/1$  model is used to approximate the queue length and waiting time during the non-peak period. The mean service rate,  $\mu$ , is estimated following the works of [19] and [1]. From [19], simulation of the  $m/m/1$  model, the relationship between the demand flow variation  $\lambda_q(t)$  in the non- priority stream and the flow variations,  $\lambda_Q(t)$  in the priority stream is associated with junction type and traffic origin-destination patterns in the network.

$$k = \frac{\lambda_q(t)}{\lambda_Q(t)} \tag{1}$$

Where  $k$  is the turning ratio.  $\lambda_q(t) = k\lambda_Q(t)$

$k$  is commonly 1 for round - about and 0.5 for major/minor road junctions.

Accordingly,

$$\mu(t) = \mu_0 - \sum_i a_i \lambda_{Q_i}(t) \tag{2}$$

where  $\mu_0$  and  $a_i$  depend on junction type and geometry.

$$\mu(t) = \mu_0 - \lambda_{Q_i}(t) \sum_i a_i \alpha_i \tag{3}$$

$i \in [1,4]$  for T-junction.

Typically,  $\alpha_1 \approx \alpha_3 = 0.45$ ,  $\alpha_2 \approx \alpha_4 = 0.05$  and  $\mu_0 \approx 10$  veh/min for left turn on-to the major road,

$a_1 \cong a_3 \cong 0.3$ ,  $a_2 \cong 0.1$ ,  $a_4 \cong 0.4$  Therefore, equation (2) becomes

$$\mu(t) = 10 - 0.3 \lambda_Q(t) \text{ veh/min} \tag{4}$$

Personal observation of some road junction and the flow of traffic on some major roads were undertaken in Uyo metropolis. We collected data on arrival and number of vehicles at a road junction. The road junction used for the study is the Ikot Ekpene- Ikpa road junction; it is one of the major and busy intersections in Uyo metropolis. The junction considered have been redesigned as a roundabout in 2012, it was a T-junction originally. We consider both cases of design and report the road traffic behaviour at both scenarios. As at the time when it was a T-junction, the Ikpa road junction was not signalised, therefore, motorists used the gap acceptance mechanism to access the Ikot Ekpene road from the Ikpa road end.

The arrival of vehicles at the minor road junction was observed from 7.00am to 9.00am (peak period) and 11.00am to 1 pm (non-peak period). Road traffic flow data were collected for both cases when the road was a T-junction and when it was changed to a roundabout. Using equations (1 and 4), Table 1 shows the mean arrival

rate,  $\lambda$  and the estimated mean service rate,  $\mu$ , during both periods. The traffic intensity  $\rho$  during the non-peak period was found to be 0.54 while it was 1.54 during the peak period.

**Table 1:** Mean arrival rate and estimated mean service rate during the peak and non-peak periods when the road was a T-junction.

| Queue parameter              | Peak period | Non-peak period |
|------------------------------|-------------|-----------------|
| Arrival/min ( $\lambda$ )    | 8.00        | 4               |
| Service rate/min ( $\mu$ )   | 5.2         | 7.6             |
| Traffic intensity ( $\rho$ ) | 1.54        | 0.53            |

**Table 2:** Mean arrival rate and estimated mean service rate during the peak and non-peak periods when the road is a roundabout.

| Queue parameter              | Peak period | Non-peak period |
|------------------------------|-------------|-----------------|
| Arrival/min ( $\lambda$ )    | 12.00       | 5               |
| Service rate/min ( $\mu$ )   | 10.00       | 9.00            |
| Traffic intensity ( $\rho$ ) | 1.20        | 0.56            |

Using  $m/m/1$  model at steady state, if  $L_S$  is the expected number of vehicles in the system,  $W_S$  and  $W_q$  are expected waiting time in the system and queuing time respectively, then

$$L_S = \frac{\rho}{(1-\rho)} \tag{5}$$

$$W_S = \frac{L_S}{\lambda} = \frac{1}{\mu(1-\rho)} \tag{6}$$

$$W_q = W_S - \frac{1}{\mu} = \frac{\rho}{\mu(1-\rho)} \tag{7}$$

$L_q$  = expected number of vehicles in the queue

$$L_q = L_S - \frac{\lambda}{\mu}, \tag{8}$$

$$\text{Variance} = \frac{\rho}{(1-\rho)^2} \tag{9}$$

From the relationships of equations (5)-(9), the measures of performance for the non-peak period are shown in Table 3. Clearly, it shows that the road network is without congestion.

**Table 3:** measures of performance during non-peak period

| Parameter/measure of performance.                 | (T-junction) | Roundabout |
|---|--------------|------------|
| Expected no of vehicles in the queue, $L_q$ (veh) | 0.60         | 0.71       |
| Expected no of vehicle in the system, $L_S$ (veh) | 1.13         | 1.27       |
| Expected waiting time in the system, $W_S$ (min)  | 0.28         | 0.25       |
| Expected waiting time in the queue, $W_q$ (min)   | 0.15         | 0.14       |

When  $\rho > 1$  the steady state queuing results are not applicable, so we use renewal theory argument to analyze the system [27]. When the junction is congested,  $(\rho = \frac{\lambda}{\mu} > 1)$ , and at any time in the interval  $(0, T)$ , the probability that the server is free is very small. We apply the renewal theory argument, [27]. It is reasonable to let the queue size at time  $t$ ,  $t \in (0, T)$ , denoted by  $N_t$  be

$$N_t = N_0 + \lambda t - \mu t \tag{10}$$

$N_0$  = number of vehicles in the queue initially.

The expected queuing time,  $W_q$ , say of a vehicle at time  $t$ ,  $t \in (0, T)$ , is the sum of  $N_t$  service times.

Therefore,

$$E[W] = \mu E[Nt] \tag{11}$$

Equation (11) becomes  $\frac{N_0}{\mu} + (\rho - 1)t$

From Tables 1 and 2 with  $N_0 = 2$  (number of vehicles in the queue initially), then

equation (10) becomes

$$N_t = 2 + 2.8t, \text{ (when it was a T-junction)} \tag{12}$$

$$N_t = 2 + 2.0t \text{ (for a roundabout)} \tag{13}$$

It is reasonable that  $\lambda$  can be replaced by  $\lambda t$  in equation (10), and arrival rate can increase during peak period, called non-stationary arrival process [29]. It is possible for the service rate to be reduced to zero at such congestion [1]. The relaxation time,  $T$ , can be determined from the relation,

$$T = \frac{N_0 - \bar{N}}{\mu(1-\rho)}, \quad \rho < 1 \tag{14}$$

At this point,  $N_0 \approx N_t$ .

Equations (12), (13) and (14) are used to generate the traffic build-up time and relaxation time for a T-junction and a roundabout respectively.

**Table 4:** Queue building up time from 2 Vehicles to  $N_t$

| Time (min) | $N_t$ (veh)(T-junction) | $N_t$ (veh)(Roundabout) |
|------------|-------------------------|-------------------------|
| 5          | 16                      | 12                      |
| 10         | 30                      | 22                      |
| 15         | 44                      | 32                      |
| 20         | 58                      | 42                      |
| 25         | 72                      | 52                      |
| 30         | 86                      | 62                      |
| 35         | 100                     | 72                      |
| 40         | 114                     | 82                      |
| 45         | 128                     | 92                      |
| 50         | 142                     | 102                     |
| 55         | 156                     | 112                     |
| 60         | 170                     | 122                     |

If the queue length is  $N_t$ , arrival rate drops to 4 and service rate increases to 7.6 in the case of the T-junction while arrival rate drops to 5 and the service rate increases to 9.0 in the case of the roundabout respectively. Then the queue relaxation time from  $N_t$  to 0 is shown in table 4.

Figures 1, 2 and 3 show the queue build up and relaxation times respectively. Graphs show that queue length increases with time with minimal service thus leading to congestion. From table 5, it shows that it takes longer time for traffic to relax compared to build up time, for example, it will take approximately 3mins for the first 10 vehicles (assuming no more vehicles joining the 170 vehicles in the queue) to leave the queue while the last vehicle will leave the queue in 339minutes approximately.

From Tables 4 and 5, we observe that it take approximately an hour for 170 vehicles to be in the queue while it takes about 5 hours for the 170<sup>th</sup> vehicle to leave the queue for a T-junction.

**Table 5:** Relaxation time for the T-junction

| $\bar{N}$ (veh) | Time(min) | Cumulative time (min) |
|-----------------|-----------|-----------------------|
| 170             | 0         | 0                     |
| 160             | 2.80112   | 2.80112               |
| 150             | 5.602241  | 8.40336               |
| 140             | 8.403361  | 16.80672              |
| 130             | 11.20448  | 28.0112               |
| 120             | 14.0056   | 42.01681              |
| 110             | 16.80672  | 58.82353              |
| 100             | 19.60784  | 78.43137              |
| 90              | 22.40896  | 100.8403              |
| 80              | 25.21008  | 126.0504              |
| 70              | 28.0112   | 154.0616              |
| 50              | 33.61345  | 218.4874              |
| 30              | 39.21569  | 294.1176              |
| 10              | 44.81793  | 338.9356              |

Here, we suppose that there are 170 vehicles in the queue at time  $t=0$  ( $N_0$ ) for T-junction.

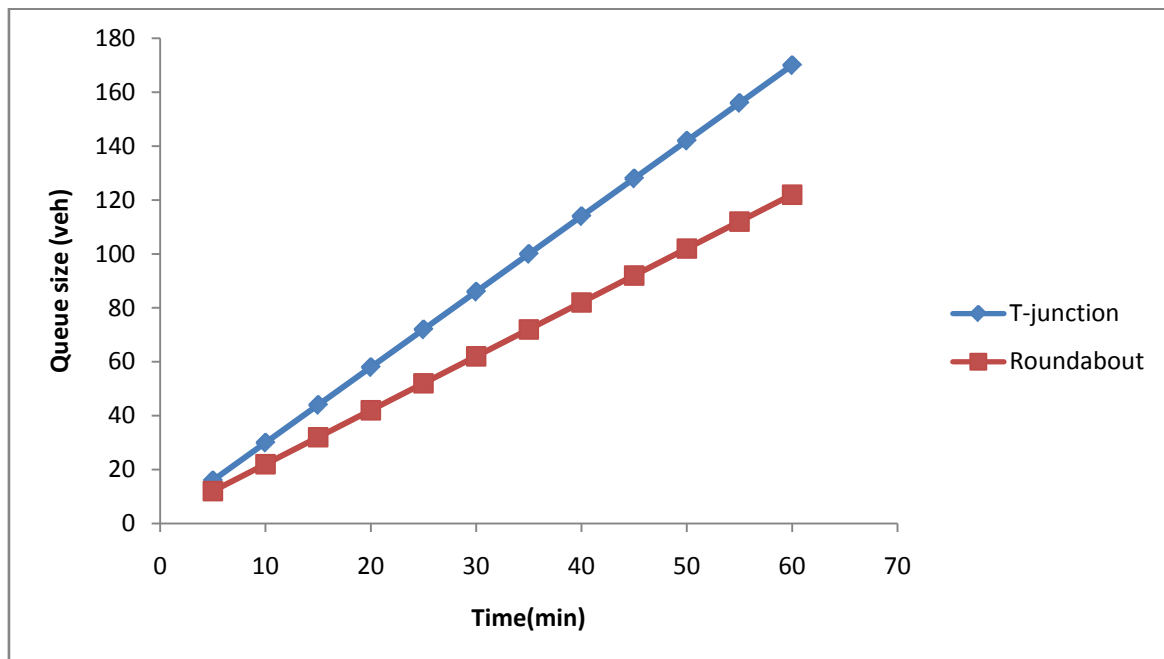
**Table 6:** Relaxation time for the roundabout

| $\bar{N}$ (veh) | Time(min) | Cumulative time (min) |
|-----------------|-----------|-----------------------|
| 122             | 0         | 0                     |
| 120             | 0.505051  | 0.505051              |
| 110             | 3.030303  | 3.535354              |
| 100             | 5.555556  | 9.090909              |

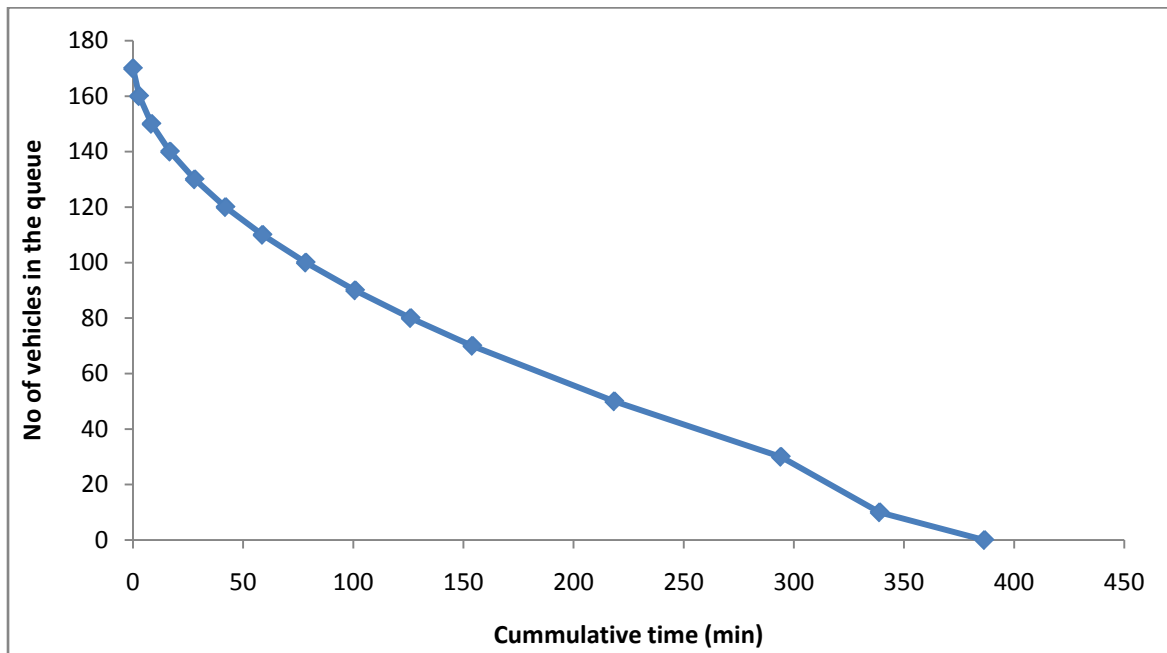


|    |          |          |
|----|----------|----------|
| 90 | 8.080808 | 17.17172 |
| 80 | 10.60606 | 27.77778 |
| 70 | 13.13131 | 40.90909 |
| 60 | 15.65657 | 56.56566 |
| 50 | 18.18182 | 74.74747 |
| 40 | 20.70707 | 95.45455 |
| 30 | 23.23232 | 118.6869 |
| 10 | 28.28283 | 146.9697 |
| 0  | 30.80808 | 177.7778 |

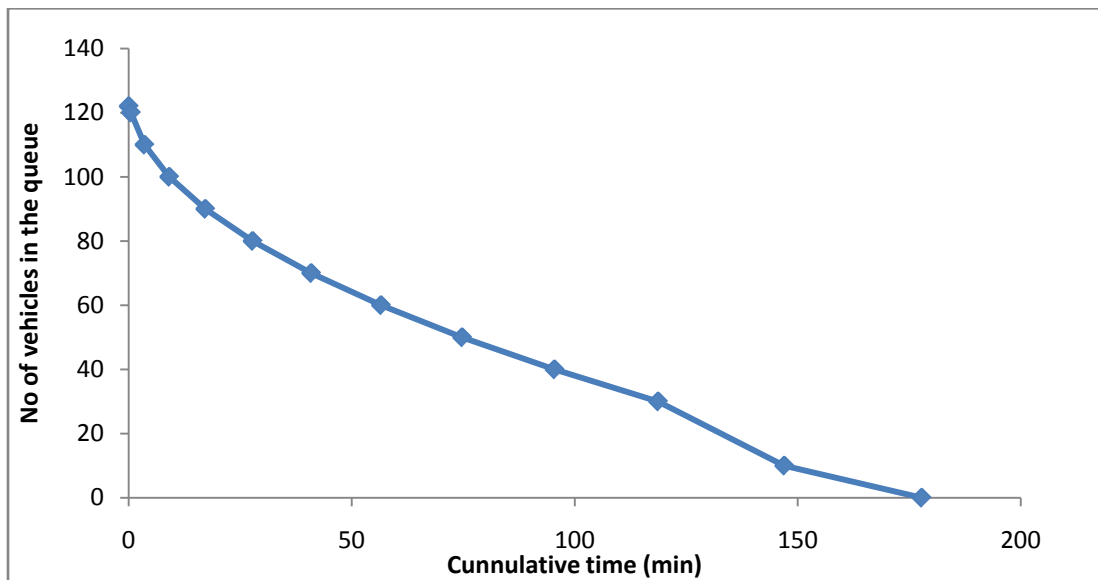
Suppose here that there are 122 vehicles in the queue at time  $t=0$  ( $N_0$ ) for the roundabout



**Fig 1:** Traffic build-up time.



**Fig 2:** Traffic Relaxation time for the T-junction.



**Figure 3:** Relaxation time for the roundabout

#### 4. Discussions and Conclusion.

From the analysis in this paper, during rush-periods the average vehicle queue size increases with time and service rate reduces thus leading to congestion. Because of low capacity utilization, the expected queuing time

of vehicles also increases. Investigation shows that at rush-hour period, the traffic intensity was 1.54 for the T-junction and 1.20 for the roundabout. The congestion shows an average queue size of  $2 + 2.28t$  and  $N_t = 2 + 2.0t$ ,  $t \in [0,60]$  for both cases respectively. For a one-hour rush period, the study revealed that an average of 170 vehicles would be in the queue waiting for service in the case of a T-junction while 122 vehicles will be in service when it was later changed to a roundabout. The times that must elapse for the system to return to normal (unsaturated) level have been presented. Results show that it takes longer time for traffic congestion to relax compared to its build up time. It is worth noting that the number of vehicles in the queue when the road was modified to a roundabout from a T-junction is lower. This is expectedly so because the dynamics of 'seek gap acceptance' and turning ratio for the T-junction has been made easier for the roundabout. Even though the data for the T-junction study was collected in 2010 while the data for the roundabout was collected in 2013; the number of vehicles using the road network has increased. The road traffic congestion level has reduced because of the improvement on the road. Because of the variation in dates, the roundabout witnessed a higher volume of vehicles because drivers naturally will be drawn more to a road that has been improved upon (this is also known as "Braess paradox" [30]).

Based on our observation and analysis of data collected, it is shown that improving the design of a road network can help in reducing road traffic congestion on our roads. To avoid the principle of Braess paradox, we propose that: road improvement should not be an isolated case, but adequate control measures should be adopted to ease the vehicular traffic flow on our major roads. Traffic lights should be installed at strategic road junctions, and there should be strict enforcement of traffic rules. Road widening and flyovers should be considered for emerging megacities. The effect of conflicts among road users on queue build up in Nigeria mega cities should be studied. Further work on comparing a T-junction and a roundabout should be studied to determine the level of vehicle relaxation times.

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