International Journal of Sciences: Basic and Applied Research (IJSBAR)

ISSN 2307-4531
(Print & Online)

http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

An Epidemiological Model for Sexual Activities

B. A. Obeng [a*], W. Obeng-Denteh[a], G. O. Fosu[b], D. Yaro[a]

[a] *Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana*
[b] *Department of ICT and Mathematics, Presbyterian University College, Abetifi, Ghana*

Abstract

An epidemiological model depicting the dynamics of sexual activities among male lecturers is analysed in this study. The model is a mime of the Susceptible -Infected -Recovered (SIR) model. Illegal sexual relationship by male lectures is considered as a disease called LESEX for the purpose of this research. The model suggests that the recruitment of new male lecturers play a significant role in the reduction of LESEX. However, the basic reproductive number is not enough to predict whether or not LESEX will persist among male lecturers on university campus.

Keywords: Epidemiology; Lecturers; Sex; SIR model.

1. Introduction

Mathematical modelling of topics involving addiction and spread of disease has been carried out in many areas of studies due to its importance. It has become a predictive tool which helps to determine the existence and treatment strategies of certain diseases. It has become the need to develop useful models which will give practitioners and academia valuable predictions. Mathematical models for disease spread and addiction subjects, or more generally epidemics, typically are based on differential equations with an in-built threshold that determine the behaviour of the system. The importance of the threshold is to determine the existence of an epidemic based on the parameter values when exceeding the threshold. The epidemic may include, drug taken, alcoholism, obesity, prostitution and many others [1, 2, 4, 6, 7].

In this study, the term LESEX is considered as having sexual act when not married for any reason or having sexual act outside your marriage partner. This means having sex with one partner even when dating is classified as LESEX.

Research on sexual relationship by male has been carried out by many researchers in many fields of studies [3, 5, 8]. This work is to analyze illegal sexual relationship by male lecturers on campus using mathematical model. The model depict the existence and spread of LESEX as an epidemic disease on university.

This is illustrated with a compartmental diagram and a set of system of differential equations. The existence and the spread of disease LESEX among the male lecturers on campus has both short and long term negative consequence on campus environment and may cause academic indiscipline.

*Corresponding author.
E-mail address: obendu@yahoo.com

Academic indiscipline corrupts the quality of future leaders. One of the challenges affecting the academic field is LESEX on our campuses of studies. The model together with the scenario in question is a hypothetical case based on assumptions.

2. Model

The model captures the dynamics of the spread of the disease using SIR model. The individuals in the male lecturer population is divided into three compartments. The Susceptible, Infected and the Recovery. The group of male lecturers who have never engaged in any illegal sexual activity forms the susceptible population. Infected male lecturers (P) are those with the disease LESEX. Recovered (R) are those who have stopped illegal sex (minimum of 12 months). ωN-the number of male lecturers recruited into the system, hence ω, the entering and departure rate from campus environment. μ is the transmission rate (rate at which a colleague influences another to join illegal sexual activities), ε is the rate of recovery from LESEX, π the relapse rate from recovered class to LESEX, η is the rate at which the infected are recruited on campus. A male lecturer joins campus as a susceptible or infected and leave in any of the three states. It is considered that males recruited as lecturers in the school at rate $\omega(S+P+R)$ are automatically 'at risk' for becoming infected. The lecturer exits the system naturally out of each state at rates ωS, $(\omega+\eta)P$, and ωR. The model refers to the assumption that the lecturers within each compartment are similar with regards to their behaviours. The model assumes, constant population size, the rate at which LESEX is acquired is proportional to the product of susceptible and infective present, non infected includes never and recovered admitted, recovered can only relapse, transmission is among male lecturers but the effects of breaks and vacations are not considered.

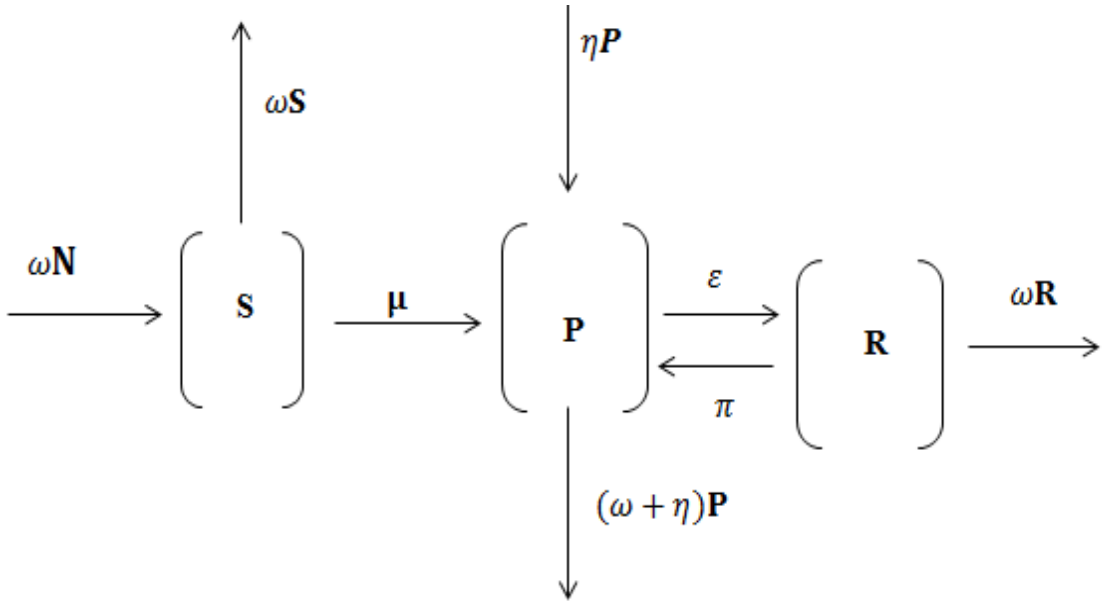

Fig. 1: Compartmental diagram for male lecturers

The dynamics are modelled using the following differential equations where S, P and R have been rescaled as proportions (see figure 1):

$$\begin{aligned}
\frac{ds}{dt} &= \omega - \mu sp - \omega s \\
\frac{dp}{dt} &= \eta p + \mu sp + \pi pr - \varepsilon p - (\omega+\eta)p \\
\frac{dr}{dt} &= \varepsilon p - \pi pr - \omega r
\end{aligned} \tag{1}$$

where $s+p+r=1$.

3. Model Analysis and Numerical Results

At the disease free equilibrium, the following values were obtained: $(s^*, p^*, r^*) = (1,0,0)$. By the local stability analysis which will determine whether the disease-LESEX will persist or not. The Jacobian matrix of the system is given as:

$$J(s,p,r) = \begin{bmatrix} -\mu p - \omega & -\mu s & 0 \\ \mu p & \mu s + \pi r - \varepsilon - \omega & \pi p \\ 0 & \varepsilon - \pi r & -\pi p - \omega \end{bmatrix}$$

the eigenvalues are: $\lambda_1 = -\omega$, $\lambda_2 = -\varepsilon$ and $\lambda_3 = \mu - \omega - \varepsilon$. The basic reproductive number R_0 is equal to $\frac{\mu}{\omega+\varepsilon-\mu}$. For unstable disease-free: $\mu > \frac{\omega+\varepsilon}{2}$. The disease-free state is stable when $\mu < \frac{\omega+\varepsilon}{2}$, meaning the disease LESEX will die out. From the relation of R_0 obtained, the transmission rate μ relative to the recovery rate ε and departure rate ω has a role in determining whether or not the disease LESEX on university campus among male lecturers exist. For the three eigenvalues being negatives means the disease-free equilibrium points is asymptotically unstable.

At the endemic equilibrium point we obtained: $(s^*, p^*, r^*) = (a,b,c)$ where,

$$a = \frac{\omega}{\omega + \mu p}$$

$$b = \frac{\sqrt{\mu^2(\varepsilon^2+\pi^2+\omega^2+2\omega\varepsilon+2\omega\pi-2\varepsilon\pi)+\pi^2(\omega^2-2\omega\mu)-2\omega\varepsilon\mu\pi-2\omega^2\mu\pi}}{2\mu\pi}$$

$$c = \frac{\varepsilon p}{\omega + \pi p}$$

Solving for the eigenvalues, we obtained the characteristic equation: $\lambda^3 - x\lambda^2 - y\lambda + z = 0$. Where

$$x = \frac{\omega}{\omega + \mu p}$$

$$y = \frac{\sqrt{\mu^2(\varepsilon^2+\pi^2+\omega^2+2\omega\varepsilon+2\omega\pi-2\varepsilon\pi)+\pi^2(\omega^2-2\omega\mu)-2\omega\varepsilon\mu\pi-2\omega^2\mu\pi}}{2\mu\pi}$$

$$z = \frac{\varepsilon p}{\omega + \pi p}$$

$$\lambda_1 = -z \quad \text{and} \quad \lambda^2 - x\lambda + d = 0$$

Hence $\lambda_{2,3} = \frac{T \pm \sqrt{T^2-4D}}{2} \Rightarrow T^2 - 4D < 0$. Thus λ_2, λ_3 have negative real parts. Therefore the system is stable at the endemic point. Thus the disease-LESEX is asymptotically stable, depicting its existence among the male lecturers.

A set of numerical values were chosen for the various parameters to perform the sensitivity analysis of the model: $\eta = 0.23, \varepsilon = 0.54, \pi = 0.45, \mu = 0.67, \omega = 0.38$. At the disease-free state:

$$R_0 = \frac{\mu}{\varepsilon + \omega - \mu} = \frac{0.67}{0.38 + 0.54 - 0.67} = 2.68$$

We varied the parameters in the model to determine their impact on the reproductive number.

It reveals from Figure 2 that, R_0 is more sensitive to infectious rates μ than the recovery rate ε, and departure rate ω. As the rate μ is increase, the R_0 in also increases, showing the spread of lecturer LESEX on campus increase. Again, when recovery rate ε, or departure rate ω increase the R_0 decrease, but in this case the decrease is not that

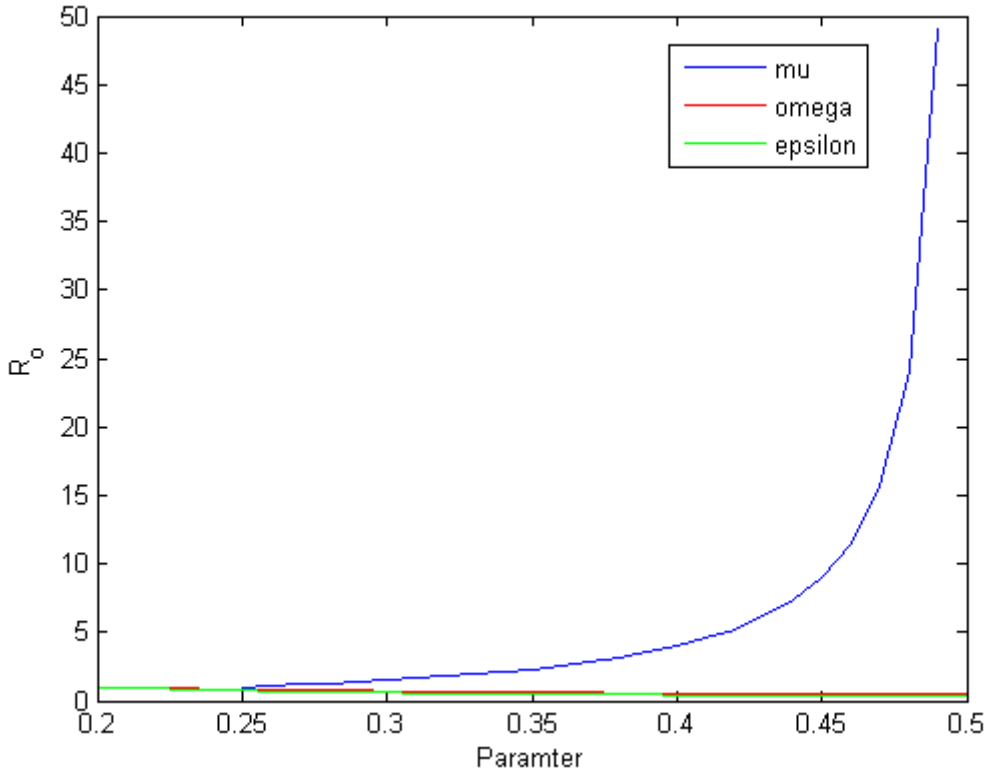

Fig. 2: Graph of R_0 for lecturers against changes in Parameters

much as compared to the change in μ against the R_0.

4. Conclusion

In this research, analysed the dynamics of the disease LESEX among male lecturers on university campus. We introduced an SIR mathematical model capturing the dynamics of the LESEX to describe the existence and spread of the disease by male lecturers. The stability analysis of the disease-free reveals asymptotically unstable equilibrium state. The eigenvalues of the endemic Jacobian matrix shows a stable endemic state. Everything in this research was purely based on assumption. Real data can also be used in the future to replicate the model.

References

[1] M. Correia Antonio, Fillepe C. Mena, and Ana J. Soares. An application of the SIR model to epidemics in portugal. 2011.

[2] Claver Pedzisai Bhunu. A mathematical analysis of alcoholism. *World Journal of Modelling and Simulation*, 8:124–134, 2012.

[3] Lily Davidoff, Karyn Sutton, Carlos Castillo-Chavez, Genevieve-Yvonne Toutain, Fabio Sanchez, and Christopher Kribs-Zaleta. Mathematical modeling of the sex worker industry as a supply and demand system. *Mathematical and Theoretical Biology Institute*, 2006.

[4] O. Diekmann and J. A. P. Heesterbeck. Mathematical epidemiology of infectious diseases, wiley, new york. 2000.

[5] Saroj Hardit. *Predicting Sexual Aggression among college men: The role of male peer groups and sexualized media*. PhD thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois, 2012.

[6] G. Hung and Y. Takeuchi. Global analysis on delay epidemiological dynamic models with nonlinear incidence. *Journal of mathematical Biology*, 63:125 –139, 2011.

[7] W. O. Kermack and A. G. McKendrick. Contributions to the mathematical theory of epidemics. *Proc. Roy. Soc. Ser A*, 115:700–721, 1927.

[8] Irmiyanti Kusumastuti. Association between knowledge, attitudes and sexual practices among unmarried indonesian young adults: A study from indonesian young adults reproductive health survey (iyarhs). *4th International Conference on Reproductive Health and Social Sciences Research*, 273:93 –104, 2007.