



Ground Bands Spectra Of ^{150}Sm , ^{152}Sm , ^{154}Gd and ^{192}Os nuclei

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Abstract:

Taking the effect of variation of moment of inertia ,besides to the effect of rotation – vibration interaction and the concept of nuclear softness ,one obtained a formula denoted RVS "Rotation ,Vibration ,Softness". The RVS model is used in calculating the energies of rotational ground bands of ^{150}Sm , ^{152}Sm , ^{154}Gd and ^{192}Os nuclei . The predicted results of the RVS are in close agreements with experimental data and other theoretical ones.

Keywords: rotational bands; variable moment of inertia (VMI) ; angular momentum; softness parameter (σ).

1. Introduction

From previous studies. It is confirmed that the Harris [1,2] two parameters ω^2 formula $E = \alpha\omega^2 + \beta\omega^4$ is better than the two parameter $I(I+1)$ expansion $E(I) = AI(I+1) + B[I(I+1)]^2$,where the first term represent the rotational part

$$A = \frac{\hbar^2}{2\theta_o}, \theta_o \text{ is momen of inertia and } I \text{ is nuclear spin follows the sequence } 0, 2, 4, 6, "$$

and the second term represents the rotation vibration interaction , the two parameters A and B can be fitted from experimental data.

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The authors in [3,4] proposed the deviation of observed data from two parameter $I(I+1)$ expansion, may be attributed due to the change of moment of inertia. In this work, one merges the effect of rotation vibration interaction with effect of variation of moment of inertia and the concept of nuclear softness to formulate an equation "RVS MODEL" which describing energies of states in rotation ground bands for ^{150}Sm , ^{152}Sm , ^{154}Gd and ^{192}Os nuclei. The predicted results of RVS model are compared with experimental data and other theoretical ones.

2. Method and Results

We know that the ground state bands of deformed nuclei are described by the formula [1,2]

$$E(I) = \frac{\hbar^2}{2\theta_0} I(I+1) \quad (1)$$

Introducing the effect of rotation – vibration interaction [3,4,5], we obtain

$$E(I) = \frac{\hbar^2}{2\theta_0} I(I+1) + B[I(I+1)]^2 \quad (2)$$

According to R.K. Gupta [6,7,8,9] which introduce the concept of variation of moment of inertia with angular momentum

$$\text{i.e., } \theta(I) = \theta_0(1 + o_1 I + o_2 I^2 + o_3 I^3 + \dots) \quad (3)$$

Where θ_0 is the moment of inertia at $I=0$, and σ_n is the softness parameter

$$\sigma_n = \frac{1}{n!} \left. \frac{\delta^n \theta(J)}{\delta J^n} \right|_{J=0} \dots \dots \quad (4)$$

$$n = 1, 2, 3, \dots$$

By simple manipulation we can write

$$E(I) = AI(I+1) + BI^2[I(I+1)] + C[I(I+1)]^2 \quad (5)$$

Where $A = \frac{\hbar^2}{2\theta_0}$, $B = A\sigma_1$ and C are fitting parameters

The An harmonic vibrator model (AVM) [5] is written as

$$E(J) = AJ + \frac{J(J-2)}{\theta(J)} + \frac{1}{2}C(\theta(J) - \theta_0)^2 \dots \dots (6)$$

By using the softness concept up to first order, the previous Eq (6) can be written in the form

$$E(J) = AJ + \frac{J(J-2)}{\theta_0(1 + \sigma_1 J)} + \frac{1}{2}C(\theta_0\sigma_1 J)^2 \dots \dots (7)$$

Using the experimental excitations energies $E(2)$, $E(4)$, $E(6)$ and $E(8)$ and the Equation (7) one can find the parameters σ_1 , θ_0 , A and C as follows:

$$\sigma_1 = \frac{\left[\frac{9E(2) - 9E(4) + 3E(6)}{3E(4) - E(2) + E(8) - 3E(6)} - 3 \right]}{24} \dots \dots (8)$$

$$\theta_0 = \frac{-48\sigma_1}{[3E(2) - 3E(4) + E(6)][1 + 4\sigma_1][1 + 6\sigma_1]} \dots \dots (9)$$

$$A = \frac{1}{24} \left[\frac{48}{\theta_0(1 + 48\sigma_1)} + 16E(2) - E(8) \right] \dots \dots (10)$$

$$C = \frac{1}{2} \left[\frac{E(2) - 2A}{(\theta_0\sigma_1)^2} \right] \dots \dots (11)$$

We calculated the energy levels of ground state bands for the chosen nuclei ,the compared predicted according to Eq (5) “RVS” model are compared with experimental data and VAMINS model [12] Eq(7).

3. Results and Discussion:

By using least square fitting ,and excitation energies of experimental data ,the the parameters A,B,and C RMS model Eq (5) are given as in Table (1) for chosen nuclei.

Also using the experimental excitation energies E(2), E(4), E(6) & E(8) and Equations (8,9,10 &11), the parameters σ_1, θ_0, A and C are calculated. Using Eq (5) “RVS” model and the given parameters in Table(1) . By similar manar using “VAVMNS model “ and the given parameters σ_1, θ_0, A & C Table (2) ,we predicted the energies for chosen nuclei ^{150}Sm , ^{152}Sm , ^{154}Gd and ^{192}Os .which is listed in table (3). The

$$\text{deviation from experimental data } \tau = \frac{1}{N} \sum_{i=1}^N (E_{\text{cal}} - E_{\text{exp}})^2 .$$

The calculated results for the ground state rotational bands are given systematically in table 3. From this table we notice that the calculations are carried out for ^{150}Sm , ^{152}Sm , ^{154}Gd and ^{192}Os nuclei whose yrast bands are observed experimentally up to $J^\pi = 16^+$ for ^{150}Sm , up to $J^\pi = 14^+$ for ^{152}Sm , up to $J^\pi = 18^+$ for ^{154}Gd and $J^\pi = 12^+$ up to for ^{192}Os nuclei. And the energies calculated according to (RVS) model in comparison with experimental data [13] and the energies calculated by

VAVMNS model and VAVM model for the chosen nuclei.

As can be seen, the results are excellent for all nuclei, in the yrast majority nuclei, results of RVS are in close agreement with predicted by the other models compared with experimental data .Besides that the present model include three parameters while the other models contain four parameters

Table (1): The fitting parameters of RVS model as in Eq. (5).

Nucleus	Parameters		
	Ax10 ⁻²	Bx10 ⁻³	Cx10 ⁻⁴
¹⁵⁰ Sm	4.914	3.786-	1.004
¹⁵² Sm	2.134	-0.883	0.193
¹⁵⁴ Gd	2.116	-0.781	0.142
¹⁹² Os	3.602	-2.091	0.619

Table (2): The fitting parameters of VAVMNS model as in Eq. (7).

Nucleus	Parameters			
	σ_1	θ_0	A	$C \times 10^{-4}$
¹⁵⁰ Sm	0.261	60.024	0.157	0.412
¹⁵² Sm	0.068	67.091	0.054	3.490
¹⁵⁴ Gd	0.049	58.304	0.059	3.087
¹⁹² Os	0.068	52.231	0.091	9.564

Table (3): Experimental and Theoretical Energies in (Mev) of the Yrast bands of ¹⁵⁰Sm, ¹⁵²Sm, ¹⁵⁴Gd and ¹⁹²Os nuclei.

¹⁵⁰Sm Nucleus

Spin J^π	$E_{experimental}$ (Mev)	E_{RVS}	E_{VAVM} (Mev)	E_{VAVMNS} (Mev)
2 ⁺	0.340	0.253	0.340	0.340
4 ⁺	0.774	0.720	0.774	0.774
6 ⁺	1.279	1.287	1.279	1.279
8 ⁺	1.837	1.878	1.837	1.837
10 ⁺	2.432	2.456	2.417	2.443
12 ⁺	3.043	3.023	3.034	3.093
14 ⁺	3.646	3.618	4.676	3.787
16 ⁺	4.305	4.321	4.340	4.523
Mean Deviation		0.271	0.019	0.052

¹⁵²Sm Nucleus

Spin J^π	$E_{experimental}$ (Mev)	E_{RVS}	E_{VAVM} (Mev)	E_{VAVMNS} (Mev)
2 ⁺	0.122	0.118	0.122	0.122
4 ⁺	0.367	0.364	0.366	0.367

6 ⁺	0.707	0.708	0.698	0.707
8 ⁺	1.125	1.127	1.120	1.125
10 ⁺	1.609	1.609	1.591	1.610
12 ⁺	2.149	2.146	2.111	2.152
14 ⁺	2.736	2.737	2.671	2.747
Mean Deviation		0.014	0.056	0.001
¹⁵⁴Gd Nucleus				
Spin J^π	$E_{experimental}$ (Mev)	E_{RVS}	E_{VAVM} (Mev)	E_{VAVMNS} (Mev)
2 ⁺	0.123	0.118	0.123	0.123
4 ⁺	0.371	0.366	0.371	0.371
6 ⁺	0.718	0.717	0.718	0.718
8 ⁺	1.145	1.147	1.140	1.145
10 ⁺	1.637	1.640	1.622	1.638
12 ⁺	2.185	2.185	2.154	2.187
14 ⁺	2.778	2.774	2.730	2.784
16 ⁺	3.405	3.407	3.343	3.422
18 ⁺	4.017	4.023	3.992	4.098
Mean Deviation		0.015	0.026	0.008
¹⁹²Os Nucleus				
Spin J^π	$E_{experimental}$ (Mev)	E_{RVS}	E_{VAVM} (Mev)	E_{VAVMNS} (Mev)
2 ⁺	0.206	0.193	0.206	0.206
4 ⁺	0.580	0.578	0.580	0.580
6 ⁺	1.089	1.095	1.089	1.089
8 ⁺	1.708	1.711	1.700	1.708

10 ⁺	2.411	2.412	2.393	2.424
12 ⁺	3.212	3.213	3.154	3.224
Mean Deviation		0.032	0.031	0.002

The present study can also be useful in study the third term of equation (5) i.e. potential energy term with spin of the nucleons and with ground state of moment of inertia.

4. Conclusion

The present model Eq. (5) is practically fit to predict the ground state rotational bands of deformed even-even nuclei, and can also be applied to nuclei where the energies of levels are experimentally available. It includes three parameters which are determined straight forward using linear least squares fitting.

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