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Optimal Strategy Analysis of Two-Phase, N-Policy of M/EK/1 Queuing System with System Break -Downs and Balking

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Abstract

This Paper deals with the economic behaviour of the M/E_k/1 queue with server start-up, two-phases of compulsory service, and server breakdowns during both batch and individual services with balking under N-policy. The first phase of service is a batch service to all existing customers in the queue and the second phase of service is to each customer in the batch in ’ k’ independent and identically distributed exponential phases. Arriving customers may balk with a certain probability and may depart without getting service due to impatience. For this model the probability generating functions for the number of customers present in the system at various states of the server are derived and obtained the closed-form expressions for various performance measures of interest. Further a total expected cost model is formulated to determine the optimal threshold of N at a minimum cost. Finally, numerical examples are given.

Keywords: Vacation, Start-up, Server Breakdowns, Balking, Cost model.

1. Introduction

We consider an N policy M/Ek/1 two-Phase queuing system in which the server is typically subject to unpredictable breakdowns. Queuing systems in which the server provides to each customer two phases of heterogeneous service in succession, have been proved very useful to model computer networks, production lines and telecommunication systems where messages are processed in two stages by a single server.

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Such kinds of systems have firstly been discussed by Krishna and Lee. As related literature we should mention some papers [1,7,12] arising from distributed system control where all customers receives batch mode service in the first phase followed by individual service in the second phase.

The concept of the *N* policy was first introduced by Yadin and Naor [13]. Past work regarding queuing systems under the *N* policy may be divided into two categories: (i) cases with server vacations, and (ii) cases with server breakdowns.

The queuing model with server vacations (server absences) has been well studied in the past three decades and successfully applied in many areas such as manufacturing/service and computer/communication network systems. Queuing systems with server vacations have attracted much attention from numerous researchers since the paper was presented by Levy and Yechiali [9]. Server vacations are useful for the system where the server wants to utilize his idle time for different purposes. An excellent survey of queuing systems with server vacations can be found in papers by Doshi and Takagi [3,4,5,11].

Models with customers' impatience in queues have been studied by various authors [2,10] in the past, where the source of impatience was either a long wait already experienced in the queue, or a long wait anticipated by a customer upon arrival. There is an extensive amount of literature based on this kind of model .Haight[6] first considered an M/M/1 queue with balking.

However, to the best of our knowledge, for two –phase queuing systems with N-Policy, server breakdowns, there is no literature which takes customers' impatience into consideration. This motivates us to study a two-phase queuing system with N-policy, server start-up, breakdowns and balking. Thus, in this present paper, we consider two-phase M/ E_k/1 queuing system with server Start-up, N-Policy, unreliable server and Balking where customers become impatient when the server is unavailable.

The article is organized as follows. A full description of the model is given in Section. 2. The steady-state analysis of the system state probabilities is performed through the generating functions in Section. 3 while some, very useful for the analysis, results on the expected number of customers in different states are given in Section. 4. In Section. 5 the characteristic features of the system are investigated. Optimal control policy is explained in section.6, while, in Section. 7, numerical results are obtained and used to compare system performance under various changes of the parameters through sensitivity analysis. Finally in Section 8.conclusions and further scope of study are presented.

The main objectives of the analysis carried out in this paper for the optimal control policy are:

i. To establish the steady state equations and obtain the steady state probability distribution of the number of customers in the system in each state.
ii. To derive expressions for the expected number of customers in the system when the server is in vacation, in startup, in batch service (working and broken conditions) and in individual service (working and broken conditions) respectively.

iii. To formulate the total expected cost functions for the system, and determine the optimal value of the control parameter N.

iv. To carry out sensitivity analysis on the optimal value of N and the minimum expected cost for various system parameters through numerical experiments.

2. The system and assumptions

We consider the $M/E_K/1$ queuing system with server startup, two phases of service, unreliable server and customers' impatience, where the unreliable server operates under N-policy with the following assumptions:

1. Customers are assumed to arrive according to Poisson process with mean arrival rate λ and join the batch queue. Customers will get the service in the order in which they arrive.
2. The service is in two phases. The first phase of service is batch service to all customers waiting in the queue. On completion of batch service, the server immediately proceeds to the second phase to serve all customers in the batch individually. Batch service time is assumed to follow exponential distribution with mean $1/\beta$ which is independent of batch size. Individual service is in k independent and identically distributed exponential phases each with mean$1/k\mu$. On completion of individual service, the server returns to the batch queue to serve the customers who have arrived. If the customers are waiting, the server starts the batch service followed by individual service to each customer in the batch. If no customer is waiting the server takes a vacation.
3. Whenever the system becomes empty, the server is turned off. As soon as the total number of arrivals in the queue reaches or exceeds the pre-determined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. The server needs a startup time which follows an exponential distribution with mean $1/\theta$. As soon as the server finishes startup, it starts serving the first phase of waiting customers.
4. The customers who arrive during the batch service are also allowed to join the batch queue which is in service.
5. The breakdowns are generated by an exogenous Poisson process with rates ξ_1 for the first phase of service and α_1 for the second phase of service. When the server fails it is immediately repaired at a repair rateξ_2 in first phase and α_2 in second phase, where the repair times are exponentially distributed. After repair the server immediately resumes the concerned service.
6. A customer may balk from the queue station due to impatience with probability of b_0 when the server is in vacation or may balk with a probability of $b_1$when the server is in service by seeing the length of the queue.

3. Steady-State analysis

In steady – state the following notations are used.

$p_{0,i,0}$ = The probability that there are i customers in the batch queue when the server is on vacation, where i= k, 2k, 3k,... (N-1)k.

$p_{1,i,0}$ = The probability that there are i service phases in the batch queue and the server is doing pre-service (startup work), where i=Nk, (N+1) k, (N+2)k,…

$p_{2,i,0}$ = The probability that there are i service phases in the batch which is in first phase of service, i = k, 2k, 3k,…

$p_{3,i,0}$ = The probability that there are i service phases in the batch which is in first phase of service, but the server is found to be broken down, i = k, 2k, 3k,…

$p_{4,i,j}$ = The probability that there are i service phases in the batch queue and j phases in the individual queue when the server is in individual service, i = 0, k, 2k,… and j = 1, 2, 3,…

$p_{5,i,j}$ = The probability that there are i service phases in the batch queue and j phases in the individual queue when the server is in individual service , but found to be broken down , i = 0,k,2k,… and j = 1,2,3,…

The steady-state equations governing the system size probabilities are as follows:

$$\lambda b_0 p_{0,0,0} = \mu k p_{4,0,1} \text{ .} \tag{1}$$

$$\lambda b_0 p_{0,i,0} = \lambda b_0 p_{0,i-k,0} \text{ ; } k \le i \le (N-1)k. \tag{2}$$

$$(\lambda b_1 + \theta) p_{1,Nk,0} = \lambda b_0 p_{0,N-k,0}. \tag{3}$$

$$(\lambda b_1 + \theta) p_{1,i,0} = \lambda b_1 p_{1,i-k,0}; i > Nk. \tag{4}$$

$$(\lambda b_1 + \beta + \xi_1) p_{2,i,0} = \lambda b_1 p_{2,i-k,0} + \mu k \, p_{4,i,1} + \xi_2 p_{3,i,0} \text{ ; } k \le i \le (N-1)k \text{ .} \tag{5}$$

$$(\lambda b_1 + \beta + \xi_1) p_{2,i,0} = \lambda b_1 p_{2,i-k,0} + \mu k \, p_{4,i,1} + \xi_2 p_{3,i,0} + \theta p_{1,i,0}; i = Nk, (N+1)k, \dots \tag{6}$$

$$(\lambda b_1 + \xi_2) p_{3,i,0} = \lambda b_1 p_{3,i-k,0} + \xi_1 p_{2,i,0} \text{ ; } i = k, 2k, \dots \tag{7}$$

$$(\lambda b_1 + \alpha_1 + \mu k) p_{4,0,j} = \mu k p_{4,0,j+1} + \beta p_{2,j,0} + \alpha_2 p_{5,0,j}; j \ge 1. \tag{8}$$

$$(\lambda b_1 + \alpha_1 + \mu k) p_{4,i,j} = \mu k p_{4,i,j+1} + \lambda b_1 p_{4,i-k,j} + \alpha_2 p_{5,i,j}; i = k, 2k, \dots, j \ge 1. \tag{9}$$

$$(\lambda b_1 + \alpha_2) p_{5,0,j} = \alpha_1 p_{4,0,j}; j \ge 1. \tag{10}$$

$$(\lambda b_1 + \alpha_2) p_{5,i,j} = \alpha_1 p_{4,i,j} + \lambda b_1 p_{5,i-k,j}; i - k, 2k, \dots, j \ge 1. \tag{11}$$

To obtain the analytical closed expression of$p_{0,0,0}$, the technique of probability generating function can be successfully applied as detailed below.

$G_0(z) = \sum_{i=0}^{(N-1)k} p_{0,i,0} z^i$, $G_1(z) = \sum_{i=Nk}^{\infty} p_{1,i,0} z^i$,

$G_2(z) = \sum_{i=k}^{\infty} p_{2,i,0} z^i$, $G_3(z) = \sum_{i=k}^{\infty} p_{3,i,0} z^i$,

$G_4(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} p_{4,i,j} z^i y^j$, $G_5(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} p_{5,i,j} z^i y^j$

and $R_j(z) = \sum_{i=0}^{\infty} p_{4,i,j} z^i$

Multiplication of equation (2) by z^i and adding over i ($1 \leq i \leq N-1$) gives $G_0(Z) = \frac{(1-z^{Nk})}{(1-z^k)} p_{0,0,0}$.(12)

Multiplication of equations (3) and (4) by z^i and adding over i ($i \geq N$) gives

$$G_1(Z) = \frac{(\lambda b_0 z^{Nk})}{(\lambda b_1(1-z^k)+\theta)} p_{0,0,0} \ . \qquad (13)$$

Multiplication of equations (5) and (6) by z^i and adding over i ($i \geq 1$) gives

$$(\lambda b_1(1-z^k) + \beta + \xi_1)\, G_2(Z) = \xi_2 G_3(z) + \mu k R_1(z) + \theta G_1(z) - \lambda b_0 p_{0,0,0}. \qquad (14)$$

Multiplication of equation (7) by z^i and adding over i ($i \geq 1$) gives

$$(\lambda b_1(1-z^k) + \xi_2)\, G_3(Z) = \xi_1 G_2(z). \quad (15)$$

Multiplication of equations (8) and (9) by $z^i y^j$ and adding over corresponding values of i and j gives

$$\big((\lambda b_1(1-z^k) + \alpha_1)y - \mu(1-y)\big) G_4(z,y) = \big(\alpha_2 G_5(z,y) + \beta G_3(y) - \mu k R_1(z)\big) y. \ (16)$$

Multiplication of equations (10) and (11) by $z^i y^j$ and adding over Corresponding values of i and j gives

$$(\lambda b_1(1-z^k) + \alpha_2) G_5(z,y) = \alpha_1 G_4(z,y). \quad (17)$$

The total probability generating function G (z, y) is given by

$$G(z,y) = G_0(z) + G_1(z) + G_2(z) + G_3(z) + G_4(z,y) + G_5(z,y). \quad (18)$$

The normalizing condition is

$$G(1,1) = G_0(1) + G_1(1) + G_2(1) + G_3(1) + G_4(1,1) + G_5(1,1) = 1. \quad (19)$$

From equations (12) to (19)

$$G_0(1) = N p_{0,0,0}, \qquad (20)$$

$$G_1(1) = \left(\frac{\lambda b_0}{\theta}\right) p_{0,0,0}, \quad (21)$$

$$G_2(1) = \left(\frac{\mu k}{\beta}\right) R_1(1), \quad (22)$$

$$G_3(1) = \left(\frac{\xi_1}{\xi_2}\right) G_2(1), \quad (23)$$

$$G_4(1,1) = \left(\frac{\alpha_2\left(\beta G_2'(1) - \mu k R_1'(1)\right)}{(\mu\alpha_2 - \lambda b_1(\alpha_1+\alpha_2))k}\right)$$

let $t_1 = \left(\mu\alpha_2 - \lambda b_1(\alpha_1 + \alpha_2)\right)$, then

$$G_4(1,1)=\left[\frac{\left(\lambda b_1 \mu k^2(\xi_1+\xi_2)\right)R_1(1)+\beta\xi_2 k\left(\frac{\lambda b_0(\lambda b_1+N\theta)}{\theta}\right)p_{0,0,0}}{t_1 k\beta\xi_2}\right]\alpha_2. \quad (24)$$

$$G_5(1,1)=\left(\frac{\alpha_1}{\alpha_2}\right) G_4(1,1). \quad (25)$$

The normalizing condition (19) gives,

$$R_1(1) = \frac{\left(\left(t_1\left(1 - p_{0,0,0}\left(\frac{\lambda b_0}{\theta} + N\right)\right) + (\alpha_1 + \alpha_2)\frac{\lambda b_0(\lambda b_1 + N\theta)}{\theta}\right)\beta\xi_2\right)}{(\mu k(\xi_1 + \xi_2)t_1 + (\alpha_1 + \alpha_2)\lambda b_1 k^2)}.$$

Substituting the value of R_1 (1) from (22) to (25) gives G_2 (1), G_3 (1), G_4 (1,1)and G_5 (1,1).

Probability that the server is neither in batch service nor in individual service is given by

$$G_0(1) + G_1(1) = 1 - \left(\frac{\lambda b_1}{\beta}\left(1 + \frac{\xi_1}{\xi_2}\right) + \frac{\lambda b_1}{\mu}\left(1 + \frac{\alpha_1}{\alpha_2}\right)\right).$$

This gives $p_{0,0,0} = (1-\rho)\frac{\theta}{(\lambda b_0 + N\theta)}$ (26)

Where $\rho = \left(\frac{\lambda b_1}{\beta}\left(1 + \frac{\xi_1}{\xi_2}\right) + \frac{\lambda b_1}{\mu}\left(1 + \frac{\alpha_1}{\alpha_2}\right)\right)$ is the utilizing factor of the system.

From Equation (26) we have ρ < 1, which is the necessary and sufficient condition under which steady state solution exits.

Under steady state conditions, let $p_0, p_1, p_2, p_3, p_4,$ *and* p_5 be the probabilities that the server is in vacation, startup, in batch service, in batch service with break down, in individual service and in individual service with breakdown states respectively. Then,

$$p_0 = G_0(1), \quad (27)$$

$$p_1 = G_1(1), \quad (28)$$

$$p_2 = G_2(1), \quad (29)$$

$$p_3 = G_3(1), \quad (30)$$

$$p_4 = G_4(1,1), \quad (31)$$

$$p_5 = G_5(1,1). \quad (32)$$

4. Expected number of customers at different states of the server

Using the probability generating functions expected number of customers in the system at different states are presented below. Let L_0, L_1, L_2, L_3, L_4 and L_5 be the expected number of customers in the system when the server is in idle, startup, batch service, break down in batch service, individual service and break down in individual states respectively. Then

$$L_0 = \sum_{i=0}^{(N-1)k} i\, p_{0,i,0} = G_0^{'}(1) = \frac{Nk(N-1)}{2} p_{0,0,0} \ . \quad (33)$$

$$L_1 = \sum_{i=N}^{\infty} i\, p_{1,i,0} = G_1^{'}(1) = \frac{\lambda b_0 k(\lambda b_1 + N\theta)}{\theta^2} p_{0,0,0}. \quad (34)$$

$$L_2 = \sum_{i=k}^{\infty} i\, p_{2,i,0} = G_2^{'}(1)$$

$$= \left(\frac{\lambda b_1(\xi_1+\xi_2)G_2(1)+\theta\xi_2 G_1^{'}(1)}{t_1\beta\xi_2}\right)\mu\alpha_2 \ . \quad (35)$$

$$L_3 = \sum_{i=k1}^{\infty} i\, p_{3,i,0} = G_3^{'}(1) = \frac{\xi_1(G_2^{'}(1)\xi_2 + \lambda b_1 k G_2(1))}{\xi_2^2} \ . \quad (36)$$

$$L_4 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty}(i+j)\, p_{4,i,j} = G_4^{'}(1,1)$$

$$= \frac{\left[\alpha_2\left(\beta G_2^{''}(1) - \mu k R_1^{''}(1)\right) + 2(\alpha_2 - \lambda b_1 k)\left(\beta G_2^{'}(1) - \mu k R_1^{'}(1)\right)\right]}{2t_1}$$

$$- \frac{2\lambda b_1 k(\alpha_1 + \mu k - \lambda b_1 k) + \lambda b_1 k \alpha_1(k-1) + \lambda b_1 k \alpha_2(k+1)}{2t_1} G_4(1,1). \quad (37)$$

$$L_5 = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty}(i+j)\, p_{5,i,j} = G_5^{'}(1,1) = \frac{\alpha_1}{\alpha_2} L_4 + \frac{\lambda b_1 k}{\alpha_2} G_5(1,1). \quad (38)$$

The expected number of customers in the system is given by

$$L(N) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5. \quad (39)$$

5. Characteristic features of the system

In this section, we obtain the expected system length when the server is in different states. Let E_0, E_1, E_2, E_3, E_4 and E_5denote the expected length of vacation period, startup period, batch service period , batch service breakdown period, individual service period, and breakdown period during individual service respectively. Then the expected length of a busy cycle is given by

$$E_c = E_0 + E_1 + E_2 + E_3 + E_4 + E_{5.}$$

The long run fractions of time the server is in different states are as follows:

$$\frac{E_0}{E_c} = p_0, \quad (40)$$

$$\frac{E_1}{E_c} = p_{1,} \quad (41)$$

$$\frac{E_2}{E_c} = p_2\ , \quad (42)$$

$$\frac{E_3}{E_c} = p_3, \quad (43)$$

$$\frac{E_4}{E_c} = p_{4,} \quad (44)$$

$$\frac{E_5}{E_c} = p_5. \quad (45)$$

Expected length of vacation period is given by

$$E_0 = \frac{N}{\lambda b_0}\ . \quad (46)$$

Hence,

$$E_c = \frac{1}{(\lambda b_0 p_{0,0,0})} (47)$$

6. Optimal control policy

In this section, we determine the optimal value of N that minimizes the long run average cost of two- phase $M/E_k/1$, N-policy queue with server start-up, break downs and balking. To determine the optimal value of N we consider the following linear cost structure.

Let T (N) be the average cost per unit of time, then

$$T(N) = C_h L(N) + C_o\left(\frac{E_2}{E_c} + \frac{E_4}{E_c}\right) + C_m\left(\frac{E_S}{E_c}\right) + C_{b1}\left(\frac{E_3}{E_c}\right) + C_{b2}\left(\frac{E_5}{E_c}\right) + C_s\left(\frac{1}{E_c}\right)$$

$$+C_b\big(\lambda(1-b_0)p_0+\lambda(1-b_1)(p_1+p_2+p_3+p_4+p_5)\big) - C_r\left(\frac{E_0}{E_c}\right). \qquad (48)$$

Where

C_h= Holding cost per unit time for each customer present in the system,

C_o = Cost per unit time for keeping the server on and in operation,

C_m = Startup cost per unit time,

C_s = Setup cost per cycle,

C_{b1}= Break down cost per unit time for the unavailable server in batch service mode,

C_{b2}= Break down cost per unit time for the unavailable server in individual service mode,

C_b = Cost per unit time when a customer balks,

C_r = Reward per unit time as the server is doing secondary work in vacation.

For the determination of the optimal operating N-policy, minimize T (N) in equation (48).

An approximate value of the optimal threshold N* can be found by solving the equation $\left.\frac{dT_1(N)}{dN}\right|_{N=N^*}=0$

(49)

7. Sensitivity analysis

In order to verify the efficiency of our analytical results, we perform numerical experiment by using MATLAB.

The variations of different parameters (both monetary and non-monetary) on the optimal threshold N*, mean number of jobs in the system and minimum expected cost are shown.Parameters for which the model is relatively sensitive would require more attention of researchers, as compared to the parameters for which the model is relatively insensitive or less sensitive.

We perform the sensitivity analysis by fixing

Non –monetary parameters as

$\lambda=0.5, \mu=8, \alpha_1=0.2, \alpha_2=3.0, \xi_1=0.2, \xi_2=0.3, \theta=6, \beta=12$, $b_0=0.4$, $b_1=0.2$,k=2 and

Monetary parameters as

C_r=15, C_{b1}=50,C_{b2}=75,C_b=15,C_m=200,C_h=5 and C_s=1000;

7.1. Effect of variation in the non-monetary parameters

(i)Variation in λ : For specified range of values of λ the optimal threshold N*, the mean number of customers in the system L(N*) and minimum expected cost T(N*) are presented in figure 1.

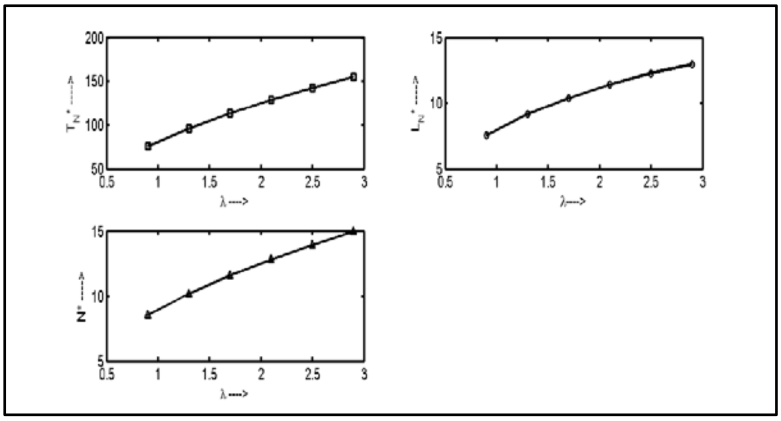

Figure1: Effect of **λ** on N*, expected system length and minimum expected cost

It is observed from Figure1 that with increase in the values of λ,

a) N* is increasing.

b) Mean number of customers in the system is increasing.

c) Minimum expected cost is increasing.

(ii)Variation in μ: For specified range of values of **μ** the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T (N*) are presented in figure 2.

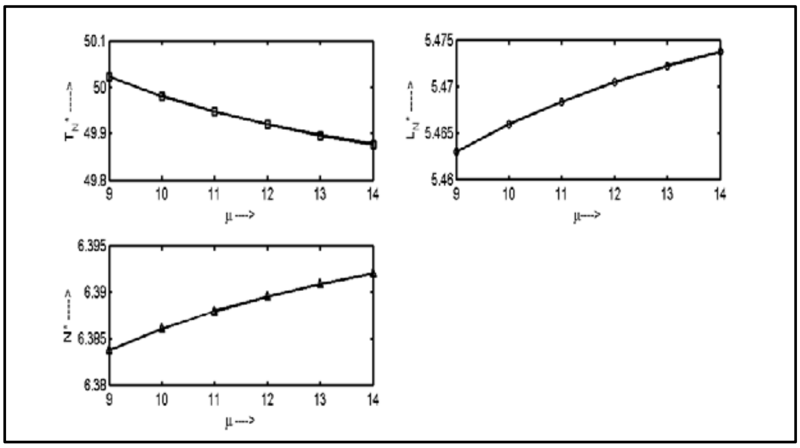

Figure2: Effect of **μ** on N*, expected system length and minimum expected cost

It is observed from figure 2 that with increase in the values of **μ**,

a) N* is increasing.

b) Mean number of customers in the system is increasing.

c) Minimum expected cost is decreasing.

(iii) Variation in α_1 : For specified range of values of α_1 the optimal threshold N*, the mean number of customers in the system L(N*) and minimum expected cost T(N*) are presented in figure 3.

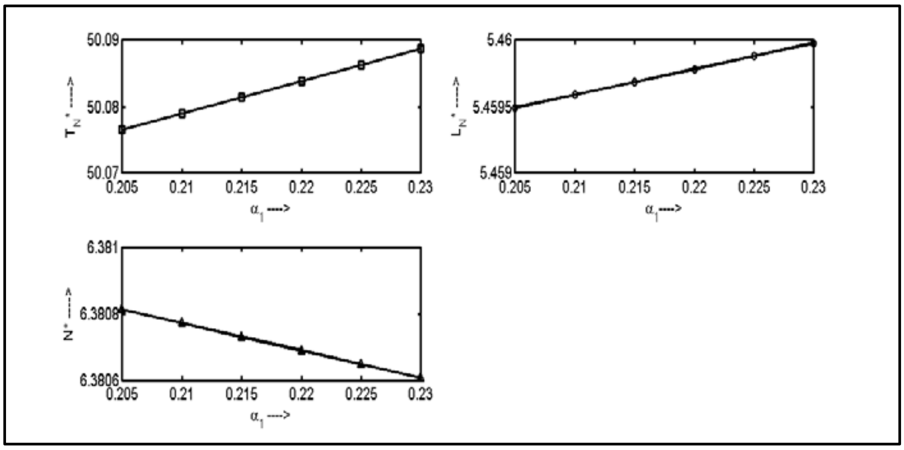

Figure 3: Effect of α_1 on N*, expected system length and minimum expected cost

It is observed from figure 3 that with increase in the values of α_1,

a)N* is decreasing.

b) Mean number of customers in the system is insensitive.

c) Minimum expected cost is increasing.

(iv) Variation in α_2 : For specified range of values of α_2 the optimal threshold N*, the mean number of customers in the system L(N*) and minimum expected cost T(N*) are presented in figure 4.

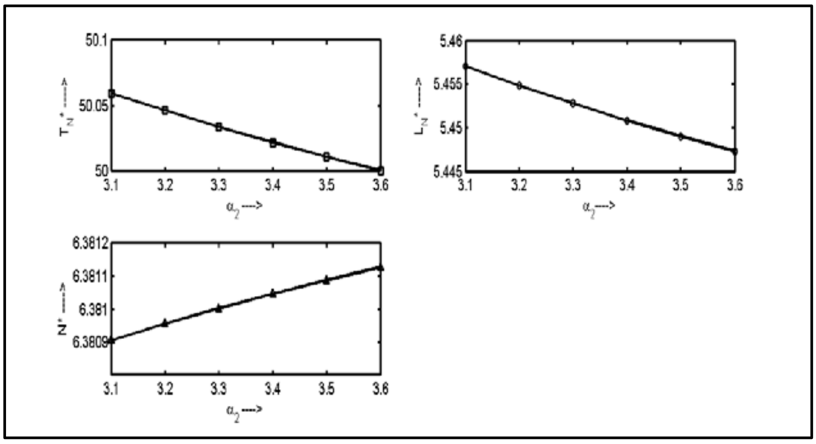

Figure 4: Effect of α_2 on N*, expected system length and minimum expected cost

It is observed from figure 4 that with increase in the values of α_2,

a) N* is increasing.

b) Mean number of customers in the system is decreasing.

c) Minimum expected cost is decreasing.

(v)Variation in ξ_1 : For specified range of values of ξ_1 the optimal threshold N*, the mean number of customers in the system L(N*) and minimum expected cost T(N*) are presented in figure 5 .

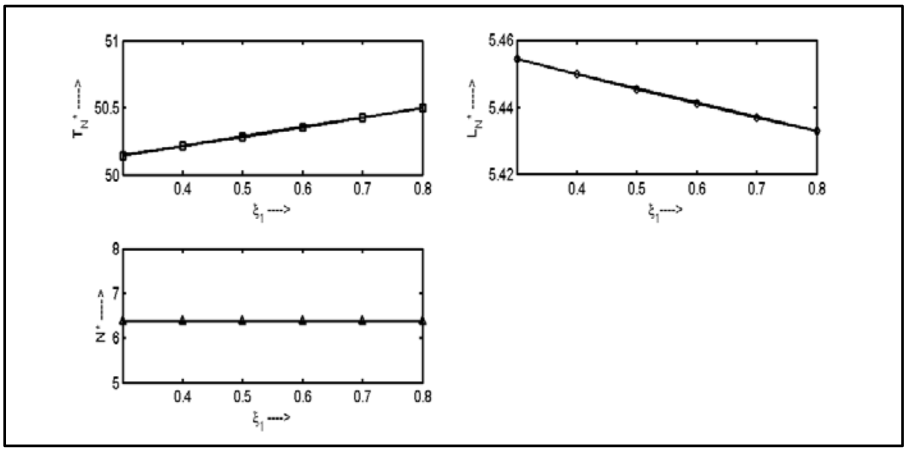

Figure 5: Effect of ξ_1 on N*, expected system length and minimum expected cost

It is observed from figure 5 that with increase in the values of ξ_1,

a) N* is insensitive.

b) Mean number of customers in the system is decreasing.

c) Minimum expected cost is increasing.

(vi)Variation in ξ2: For specified range of values of ξ_2 the optimal threshold N*, the mean number of customers in the system L(N*) and minimum expected cost T(N*) are presented in figure 6.

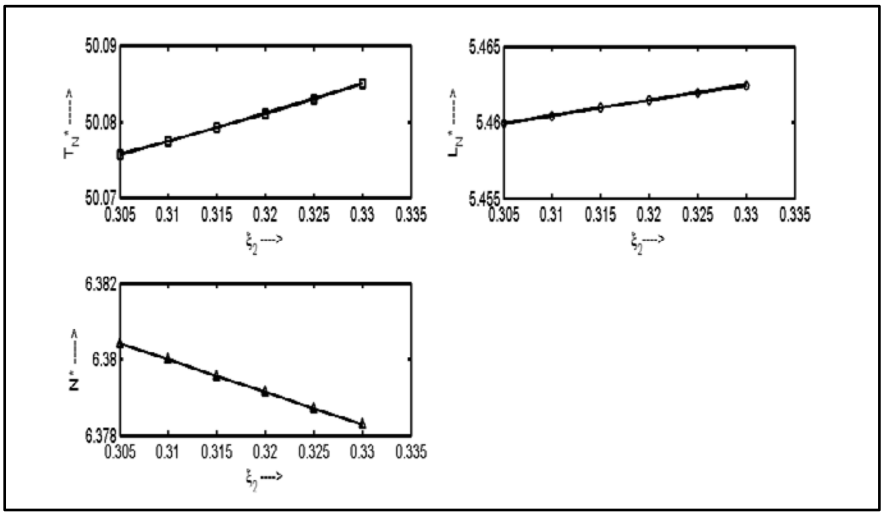

Figure 6: Effect of ξ_2 on N*, expected system length and minimum expected cost

It is observed from figure 6 that with increase in the values of ξ_2,

a)N* is decreasing.

b) Mean number of customers in the system is increasing.

c) Minimum expected cost is increasing.

(vii) Variation in θ: For specified range of values of **θ** the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 7.

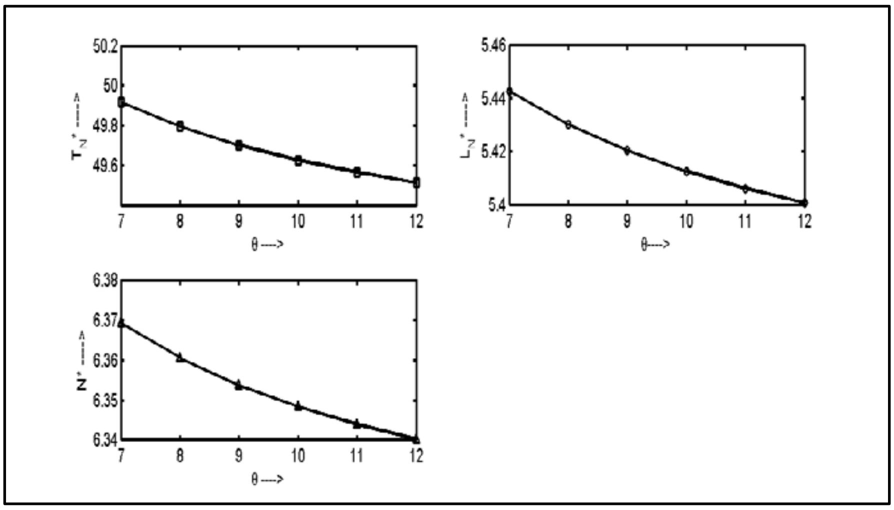

Figure 7: Effect of **θ** on N*, expected system length and minimum expected cost

It is observed from figure 7 that with increase in the values of θ,

a) N* is decreasing.

b) Mean number of customers in the system is decreasing.

c) Minimum expected cost is decreasing.

viii) Variation in β: For specified range of values of **β** the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 8 .

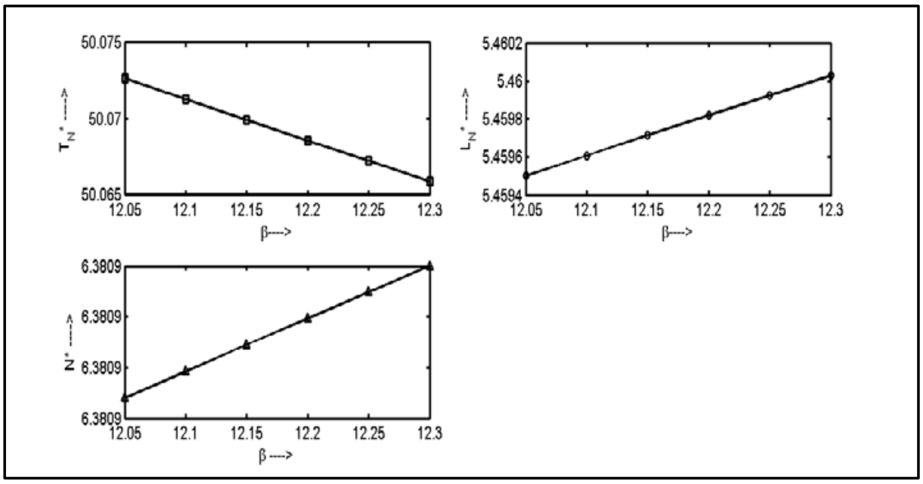

Figure 8: Effect of **β** on N*, expected system length and minimum expected cost

It is observed from figure 8 that with increase in the values of **β**,

a) N* is insensitive.

b) Mean number of customers in the system is increasing.

c) Minimum expected cost is decreasing.

ix) Variation in b_0 : For specified range of values of $\mathbf{b_0}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented figure 9 .

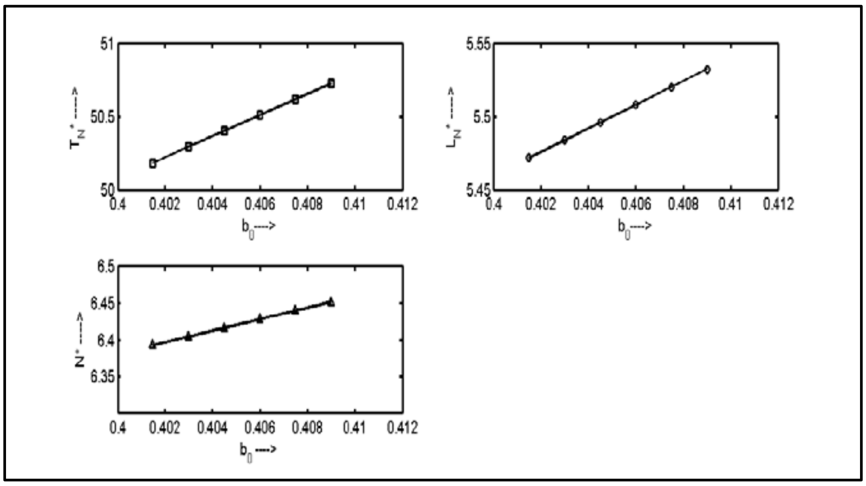

Figure 9: Effect of $\mathbf{b_0}$on N*, expected system length and minimum expected cost

It is observed from figure 9 that with increase in the values of $\mathbf{b_0}$,

a) N* is increasing.

b) Mean number of customers in the system is increasing.

c) Minimum expected cost is increasing.

x) Variation in $\mathbf{b_1}$: For specified range of values of $\mathbf{b_1}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 10 .

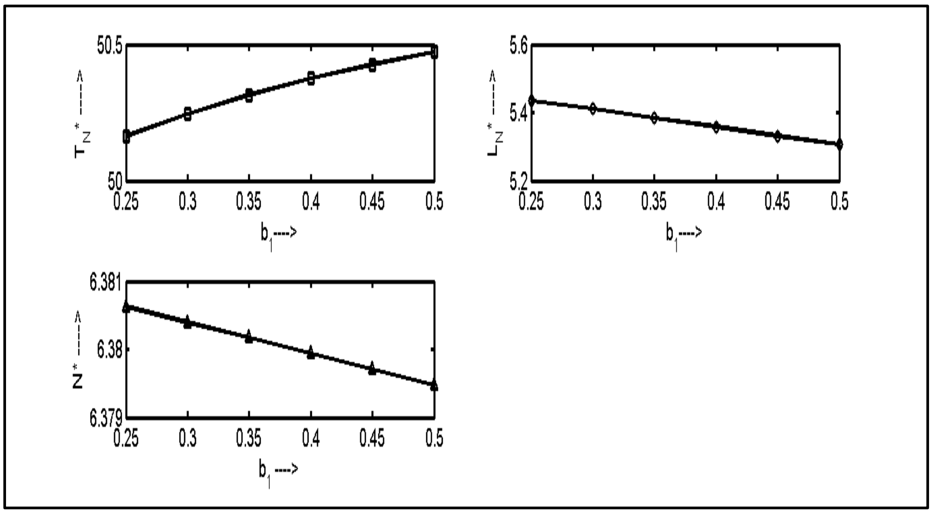

Figure 10: Effect of $\mathbf{b_1}$ on N*, expected system length and minimum expected cost

It is observed from figure 10 that with increase in the values of $\mathbf{b_1}$,

a) N* is decreasing.

b) Mean number of customers in the system is decreasing.

c) Minimum expected cost is increasing.

7.2. Effect of variation in the monetary parameters

xi)Variation in $\mathbf{C_r}$: For specified range of values of $\mathbf{C_r}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 11 .

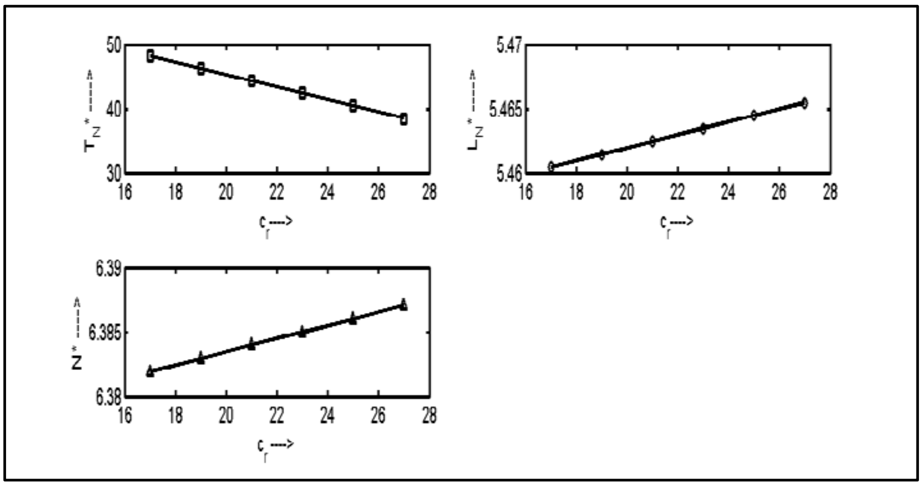

Figure 11: Effect of $\mathbf{C_r}$ on N*, expected system length and minimum expected cost

It is observed from figure 11 that with increase in the values of $\mathbf{C_r}$,

a) N* is almost insensitive.
b) Mean number of customers in the system is slightly increasing.
c) Minimum expected cost is decreasing.

Xii)Variation in $\mathbf{C_{b1}}$: For specified range of values of $\mathbf{c_{b1}}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 12 .

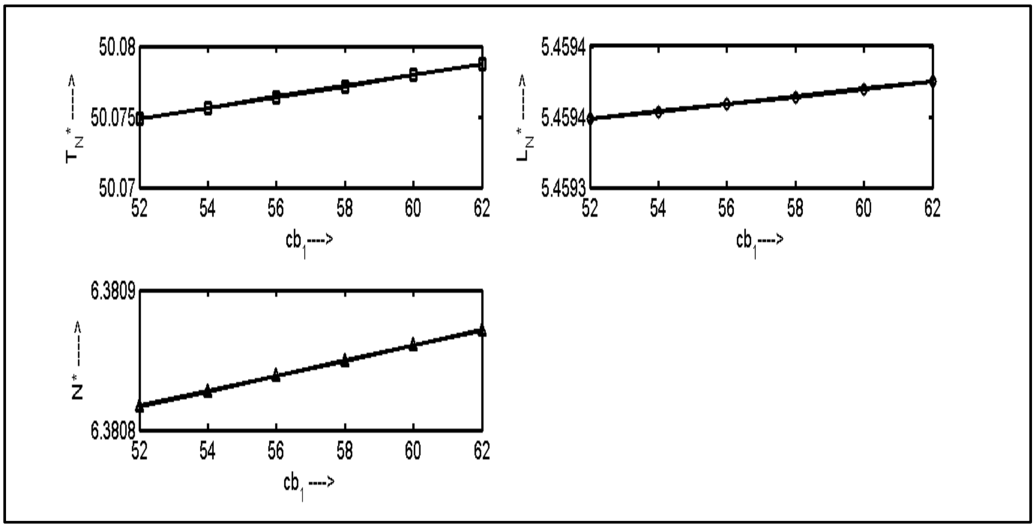

Figure 12: Effect of $\mathbf{C_{b1}}$ on N*, expected system length and minimum expected cost

It is observed from figure 12 that with increase in the values of $\mathbf{C_{b1}}$,

a) N* is almost insensitive.
b) Mean number of customers in the system is insensitive.

c) Minimum expected cost is insensitive.

Xiii) Variation in C_{b2}: For specified range of values of C_{b2} the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 13 .

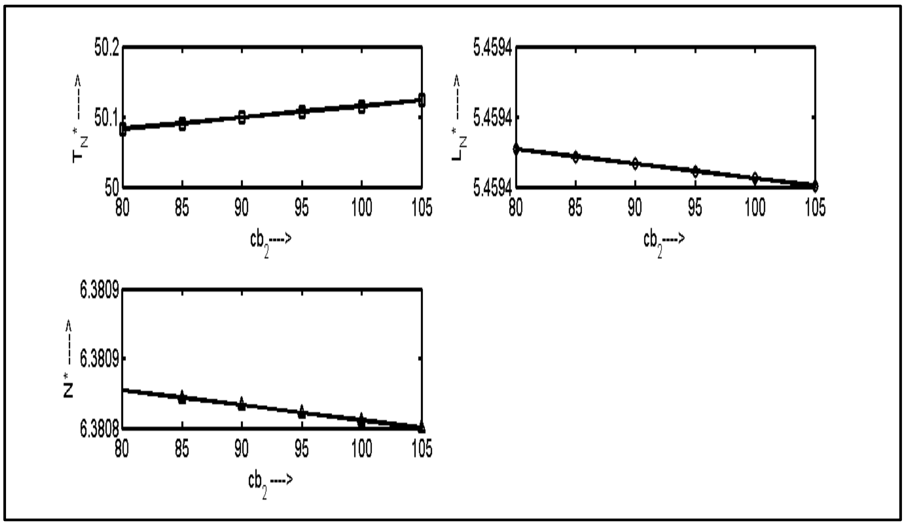

Figure 13: Effect of C_{b2} on N*, expected system length and minimum expected cost

It is observed from figure 13 that with increase in the values of C_{b2},

a) N* is decreasing.
b) Mean number of customers in the system is insensitive.
c) Minimum expected cost is slightly increasing.

Xiv) Variation in C_b : For specified range of values of C_b the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 14.

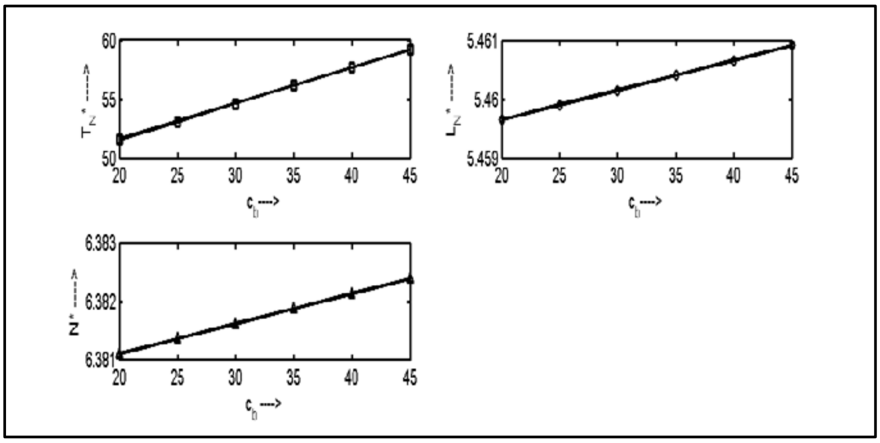

Figure 14: Effect of C_b on N*, expected system length and minimum expected cost

It is observed from figure 14 that with increase in the values of C_b,

a) N* is increasing.

b) Mean number of customers in the system is increasing.

c) Minimum expected cost is also increasing.

xv) Variation in $\mathbf{C_m}$: For specified range of values of $\mathbf{C_m}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 15 .

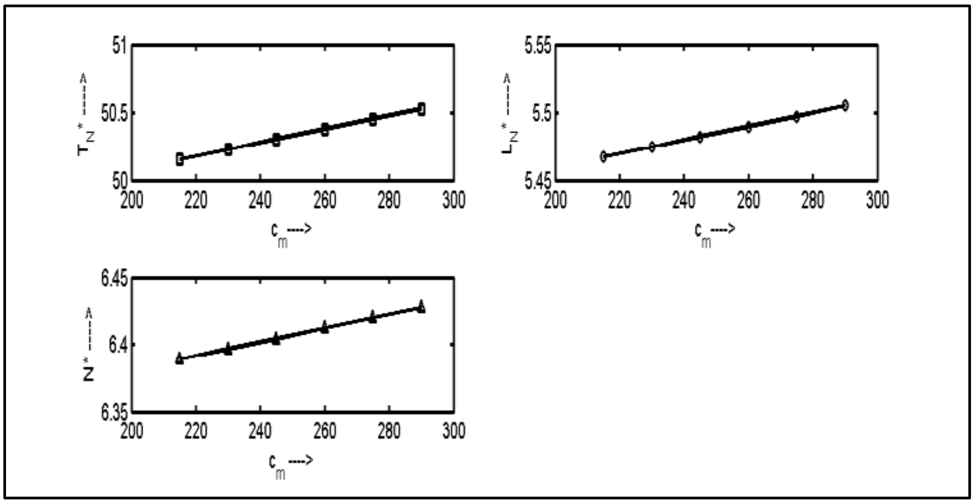

Figure 15: Effect of $\mathbf{C_m}$ on N*, expected system length and minimum expected cost

It is observed from figure 15 that with increase in the values of $\mathbf{C_m}$,

a) N* is increasing.

b) Mean number of customers in the system is increasing**.**

c) Minimum expected cost is increasing**.**

xvi)Variation in $\mathbf{C_o}$: For specified range of values of $\mathbf{C_o}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T (N*) are presented in figure 16.

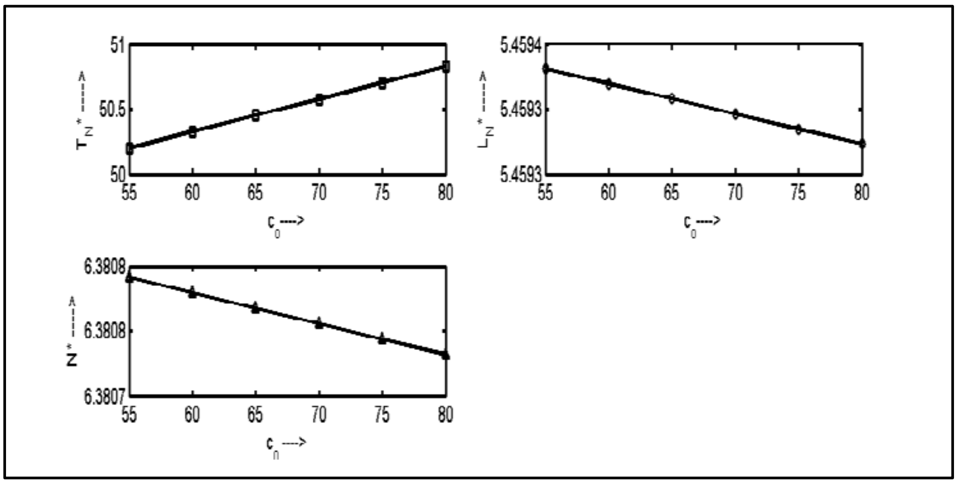

Figure 16: Effect of $\mathbf{c_o}$ on N*, expected system length and minimum expected cost

It is observed from figure 16 that with increase in the values of $\mathbf{C_o}$,

a) N* is almost insensitive.

b) Mean number of customers in the system is decreasing.

c) Minimum expected cost is increasing.

xvii)Variation in $\mathbf{C_h}$: For specified range of values of $\mathbf{C_h}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 17.

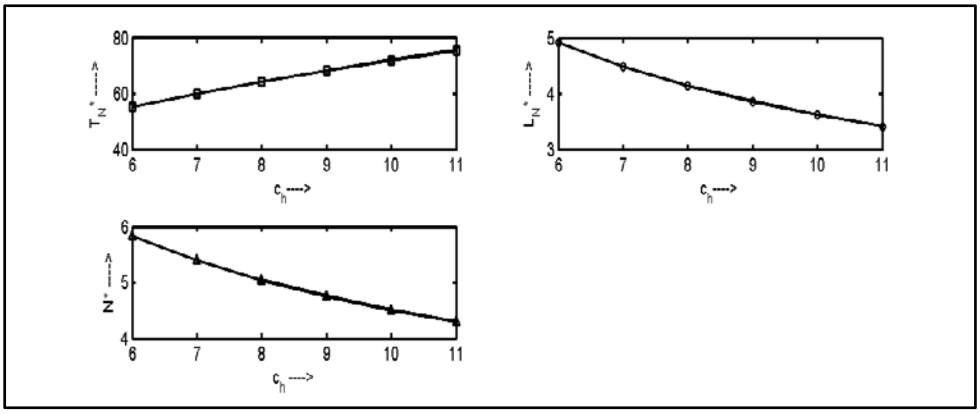

Figure 17: Effect of $\mathbf{C_h}$ on N*, expected system length and minimum expected cost

It is observed from figure 17 that with increase in the values of $\mathbf{C_h}$,

a) N* is decreasing.
b) Mean number of customers in the system is decreasing
c) Minimum expected cost is increasing.

Xviii) Variation in $\mathbf{C_s}$: For specified range of values of $\mathbf{C_s}$ the optimal threshold N*, the mean number of customers in the system L (N*) and minimum expected cost T(N*) are presented in figure 18.

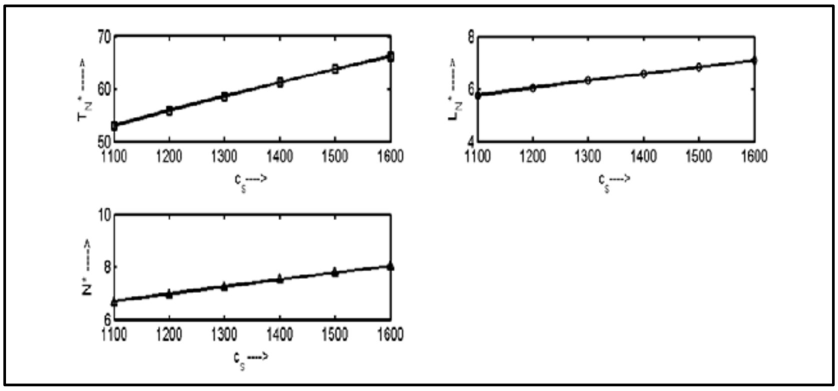

Figure 18: Effect of $\mathbf{C_s}$ on N*, expected system length and minimum expected cost

It is observed from figure 18 that with increase in the values of Cs,

a) N* is increasing.
b) Mean number of customers in the system is increasing.
c) Minimum expected cost is increasing.

8. Conclusions and further scope of study

We have analyzed the N-policy, two-phase $M/E_K/1$ queuing system with server startup, breakdowns and balking under exhaustive un-gated service. In this case,

(i) The steady state probability distribution of the number of customers in the system is obtained.
(ii) Some important system performance measures such as expected number of customers in the system when the server is in start- up, in vacation, at batch service, at individual service with breakdowns and balking in different states respectively and expected system length are derived.
(iii) The total expected cost function per unit time is formulated to determine the optimal value of the control parameter N that minimizes the total expected cost.
(iv) Sensitivity analysis between the optimal value of N, the specific values of system parameters and the cost elements is performed through numerical experiments.
(v) The numerical values will be useful in analyzing practical queuing systems and in making decisions.

The queuing systems studied in the present paper is of infinite capacity.

(a) Same study can be applied to finite capacity queuing systems.

(b) The same queuing systems can be studied by introducing the concepts of early startup and reneging.

(c) The queuing systems under study can be generalized by considering that the service time, breakdown time and repair time follow general distribution.

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