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Dynamical Evolution of the Coffee Berry Borer, Hypothenemus Hampei

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Abstract

In this paper, we propose a mathematical delay model that can describe the dynamics of the *coffee berry borer*, *Hypothenemus hampei*. We propose a model include different delays which are important for the life cycle of *Hypothenemus hampei*. The obtained model is completely analysed and we present a numerical simulation in order to validate the theoretical results.

Keywords: Hypothenemus hampei; coffee berry; mathematical models; delays; stability.

1. Introduction

The existence of the first use of coffee beans would it have been described in certain texts of the Old Testament in the passage of (Samuel 17/28): ”There was some wheat, barley, flour and grilled grain ”? Only the author of this sentence holds for always the true meaning of the grilled grain. However, it is the pioneers Razes and Avicenne, respectively, at IX^{th} et XI^{th} century that brought back the first written mentions on coffee. This drink remains relatively confined to Arabia and, after Persia, Syria, Turkey, the public discovers this new flavor. The cultivation of *coffea Spp*. On a global scale occupies about 10 million hectares, of which 42% in South America (Brazil, Colombia, Venezuela, Ecuador), 14% in Central America (Mexico, Guatemala, Dominican Republic), 34% in Africa (Cameroon, Cte d’Ivoire, Ethiopia, Angola, Congo) and Indonesia. The rest of the production is divided between the Philippines, Madagascar, India, Cuba and Haiti. Nearly 90 species of this plant are now inventoried: Among these species less than a dozen have been cultivated in the past, to leave only two of them today: Coffea arabica and Coffea canephora.

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The species C. arabica accounts for 64 to 75% of world production, while the species emph C. canephora (more commonly ”robusta”) represents 26 to 35% of the production [7]. The coffee (*Coffea canephora L.*) robusta variety remains the most widespread species in Africa and represents about 36% of world coffee production [7]. Worldwide production was 152.4 million bags of 60 kg in February 2015/2016, while it was 142.2 million bags of 60 kg in the same month of the previous year (2014/2015), a progression of 7%. It is estimated that 100 million coffee growers around the world who depend, entirely or partially, coffee growing. Millions more earn a living by transporting and / or processing coffee. The coffee has an economic dimension of prime importance, plays a key role for world trade and that of many developing countries. As such, he is at the forefront of agricultural commodities marketed globally cereals and the second largest market traded worldwide, just after oil. In Cameroon, coffee production occupies an important place among the main cash crops and plays a singular role in structuring landscapes and productive systems, spatial relationships between city and campaign, and intervenes in the improvement of the GDP (Gross Domestic Product) of the nation. Two coffee species are cultivated: The species *C. arabica* and the species *C. canephora*. Cameroon occupies the 19th rank worldwide and the first in Central Africa [1, 4] with an estimated production of 0.397750 million bags of 60 kg during the 2014/2015 season, a decrease of approximately 27% compared to the 2013/2014 production; a very weak performance, compared to 2.2 million bags of 60 kg recorded in 1986 for a rank of 12th world ranking of producing countries. However, during the years 2015/2016, an increase in the national production of coffee of the order of 3 to 8% was recorded. compared to 2014/2015. Despite the joint efforts of the cocoa-coffee inter-branch and the public authorities, with a view to raising domestic coffee production, this sector in Cameroon continues to decline. According to statistics from the National Office of Cocoa and Coffee (NOCC) [6, 7], which were officially revealed at the launch of the 2017-2018 coffee campaign, on April 4, 2018 in Yaounde-Cameroon, the 2016-2017 production peaked at 20,270 tons , down almost 20% from 24,500 tonnes in the 2015-2016 season. This volume of production is especially the 2th the poorest performance achieved by Cameroonian producers in the past five seasons, after 16,142 tonnes in 2012-2013, qualified by the local actors of the sector worst campaign ”last 50 years” [1, 3]. Reputed delicious and with a particular flavor, the Cameroonian coffee is very appreciated by the professionals of the taste. As proof, five Cameroonian brands were crowned on April 4, 2018 in Yaounde, at the award ceremony of the roasted coffees at the origin, organized every year in France by the Agricultural Commodities Development Agency (ACDA). The major production areas of Cameroonian coffee are the following regions: Adamaoua, Center, East, Littoral, North-West, West, South, South-West. This sector mobilizes some 650,000 coffee growers, of whom more than 92% are small farmers. 60% to 70% of these farmers have, depending on the region, less than 2 ha of exploitable area [2, 3]. In addition to coffee growers, more than 50,000 other stakeholders are involved in the value chain, including nurserymen, traders, transporters and processors. The main questions stem directly from the crisis of production coffee growing. The objectives set for this study were enriched by readings, discussions and field visits in order to formulate the following questions:

− What are the reasons for the decline of Cameroonian coffee growing?

− What are the different types of coffee producers and their prospects for evolution?

− What can be the future of coffee production in Cameroon?

Although the expansion of coffee growing is a recent phenomenon, linked to the colonial history of European nations, the culture of Today coffee constitutes an important factor of stability because of the jobs it generates. On the economic plan; in because of the annual turnover that it generates estimated between 7 and $ 12 billion according to world prices she's considered a vital source of income for the economy of Many countries; Health importance; caffeine acts on the nervous system by improving mental performance, attention, alertness and reflexes. And according to a very recent study September 2010 and published on March 10, 2011 in the journal "Stroke", the Amercian Heart Association newspaper), coffee consumption participate in the reduction of stroke (stroke cerebral). A large study conducted on more than 83,000 Japanese followed for 13 years shows the interest of this popular drink for protect your brain and limit the risk of these disorders. In addition to the health importance of coffee, the dangers are to be has more than 6 cups of daily and regular coffee: Too many coffee can impair our sleep cycle as it decreases quality of our rest, which presents risks of fatigue for our organism, which eventually lead to many other problems The *H. hampei* population especially are able to cause enormous damages on coffee trees. Female *H.hampei* also use coffee stem, pods and twig to lay their eggs. The development of *H.hampei* population on these parts of coffee trees causes damage which can lead to the destruction of coffee trees after many years. The damage on pods can lead to fruit abortion when it occurs at cherelle stage. But on a mature fruits, damage are generally less harmful as they can be harvested and the beans can be sold. Extensive feeding on fan branches results in the degradation of the canopy usually of more or less discrete groups of trees up to around one hundred in number referred to as *H. hampei* pockets generally located in the sunniest areas of plantation [3]. We want to predict the evolution of this damages. The biological studies give us the data about the development and the ecology of the mirid. The mathematical modelling of H. hampei population can help us to use biological data in order to predict the evolution of the *H. hampei* population. Our aim in this present paper is therefore to propose the *H. hampei* evolution dynamic models following the above objective and achieve a complete theoretical analysis of these models. We shall illustrate the theoretical results through numericals simulations.

1.1. Formulation of the model(s) system include delay(s)

According to the bibliographic data collected [10, 12] the development cycle of *H.hampei Ferr* can be summarized by the figure 1. This scheme only introduces individuals from the first cluster of eggs. The durations reported are only an order of magnitude. Indeed, the speed of development of the different stages is very variable depending on the thermal conditions. These time are very important and we want to take into account of this time to model the evolution of *H. hampei*. Then, we construct the model of evolution of *H. hampei* include delays. These delays are different time which are necessary to pass of one stage to another. The life cycle of *H. hampei* is given by figure 1.

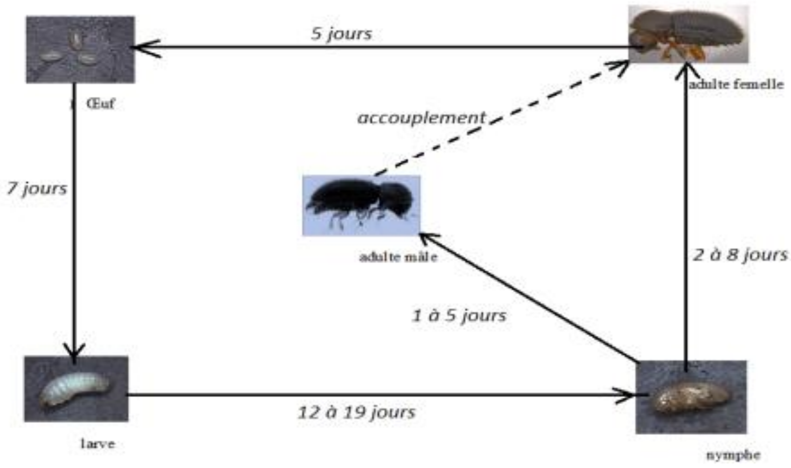

Figure 1: Life cycle of *H. hampei*.

In order to elaborate the model include delays, we make the followings assumptions:

- H_1 The population of larvae can not grow indefinitely: It is regulated by a limit capacity of K_L. This limiting capacity depends on the availability of the resource.
- H_2 There is a time needed τ_1 between female spawning and egg hatching. The larvae that integrate the larval compartment at time t come from the eggs laid by the females $t - \tau_1$ days ago and that would have survived. A proportion $e^{-\tau_1 d_e}$ of the eggs will survive and evolve into larvae. The de parameter represents the natural mortality of the eggs.
- H_3 A larva emerges instantly t if it has integrated the larval compartment τ_2 days before and if it has survived during this time. τ_2 represents the larval development time, $e^{-\tau_2 d_L}$ the proportion of larvae that will survive and become nymphs and d_L natural larval mortality.
- H_4 It takes a time of τ_3 days for a proportion of α of nymphs to emerge as adult males and a time of τ_4 days for the remaining proportion of nymphs $(1 - \alpha)$ to emerge. In female adults. $e^{-\tau_3 d_N}$ and $e^{-\tau_4 d_N}$ represent respectively the proportion of nymphs that survived during these τ_3 and τ_4 days. Through this hypothesis, the *H. hampei* development is represented by the following delays differential equations system:

Through this hypothesis, the *H. hampei* development is represented by the following delays differential equations system:

$$\begin{cases} \dot{L}(t) &= bF(t-\tau_1)\left(1-\frac{L(t)}{K_L}\right)\exp(-\tau_1 d_e) - \beta L(t-\tau_2)\exp(-\tau_2 d_L) - d_L L(t), \\ \dot{N}(t) &= \beta L(t-\tau_2)\exp(-\tau_2 d_L) - qN(t-\tau_3)\exp(-\tau_3 d_N) - (d_N + d_1 N(t))N(t), \\ \dot{M}(t) &= q\alpha N(t-\tau_3)\exp(-\tau_3 d_N) - d_M M(t), \\ \dot{F}(t) &= (1-\alpha)qN(t-\tau_4)\exp(-\tau_4 d_N) - (d_F + d_2 F(t))F(t). \end{cases} \tag{1}$$

with the initial condition $\varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ is defined in the Banach space

$$\mathcal{C}_1 = \{\varphi \in \mathcal{C}([-\tau; 0], \mathbb{R}^2_+)\}, \tag{2}$$

where

$$\tau = \tau_1 + \tau_2 + \tau_3 + \tau_4, \quad \varphi_1 = L(\theta), \quad \varphi_2 = F_1(\theta), \quad \varphi_3 = F_2(\theta), \quad \varphi_4 = M(\theta)$$

with $L(\theta) > 0, \quad F_1(\theta) > 0, \quad F_2(\theta) > 0, \quad M(\theta) > 0, \quad \theta \in [-\tau, 0]$ are given functions.

Now, we consider the model system with $\tau_1 = \tau_3 = 0$ and $\tau_2 > 0$. The dynamics of model system (2) is completely determined if we know the dynamics of larvae (L(t)) and the dynamic of mature female (F_2(t)). Then, we will study the system includes only these two equations by considering $\tau_1 = \tau_3 = 0$ and $\tau_2 > 0$.

1.2. Model system with one delay

The model system with one delay is given by equation (2).

$$\begin{cases} \dot{L}(t) &= bF(t)\left(1-\frac{L(t)}{K_L}\right) - \beta L(t-\tau)\exp(-\tau d_L) - d_L L(t), \\ \dot{N}(t) &= \beta L(t-\tau)\exp(-\tau d_L) - (q + d_N + d_1 N(t))N(t), \\ \dot{M}(t) &= q\alpha N(t) - d_M M(t), \\ \dot{F}(t) &= (1-\alpha)qN(t) - (d_F + d_2 F(t))F(t). \end{cases} \tag{3}$$

with, $X(0) = \varphi = (\varphi_1, \varphi_3)$, where $\varphi \in \mathcal{C}([-\tau_2; 0], \mathbb{R}^2_+)$ is a positive initial condition.

1.2.1 Existence and uniqueness

The problem of Cauchy relative to (3) writes:

$$\begin{cases} \dot{X}(t) = G(X_t) \\ X(t_0) = \varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) \end{cases} \tag{4}$$

$\forall\varphi \in$ C (This is the set of continuous functions of $[-\tau, 0]$ dans $\mathbb{R}^n$), the standard of φ is defined by

$$\|\varphi\| = \sup_{\theta \in [-\tau, 0]} \mid \varphi(\theta) \mid$$

We define $G(X_t)$ by:

$$G(X_t) = \begin{pmatrix} bF(t)\left(1-\frac{L(t)}{K_L}\right) - \beta L(t-\tau)\exp(-\tau d_L) - d_L L(t) \\ \beta L(t-\tau)\exp(-\tau d_L) - (q + d_N + d_1 N(t))N(t) \\ q\alpha N(t) - d_M M(t) \\ (1-\alpha)qN(t) - (d_F + d_2 F(t))F(t) \end{pmatrix} \quad \textit{and}$$

$$G(\varphi) = \begin{pmatrix} b\left(1-\dfrac{\varphi_1(0)}{K_L}\right)\varphi_4(0) - \beta\varphi_1(-\tau)\exp(-\tau d_L) - d_L\varphi_1(0) \\ \beta\varphi_1(-\tau)\exp(-\tau d_L) - (q + d_N + d_1\varphi_2(0))\varphi_2(0) \\ q\alpha\varphi_2(0) - d_M\varphi_3(0) \\ (1-\alpha)q\varphi_2(0) - (d_F + d_2\varphi_4(0))\varphi_4(0) \end{pmatrix}$$

G is locally Lipschitzian, according to the cauchy theorem, we deduce the existence and uniqueness of maximum solution to the problems of Cauchy relative to (3). **Proposition 1** The positive cone $(R_+^*)^4$ is positively invariant for the system (3), that is to say for everything

$\phi \in C([-\tau, 0], (R_+^*)^4)$, the only maximal solution noted (I, (L, N, M, F)), checked: $\forall$ t $\in$ I, (L(t), N(t), M(t), F(t)) $\in (R_+^*)^4$.

Evidence

Suppose there is t $\in$ I such as (L(t), N(t), M(t), F(t)) $\notin (R_+^*)^4$ and ask T = inf{t $\in$ I , (L(t), N(t), M(t), F(t)) $\notin (R_+^*)^4$ } We have T > 0 since $\phi(0) > 0$, L(T)N(T)M(T)F(T) = 0 and for all $-\tau \leq t < T$, L(T) > 0, N(T) > 0, M(T) > 0, F (T) > 0.

1. Suppose L(t) = 0: According to the system (3), we hat:

$$L(T) = \exp(-d_L T)(L_0 - \int_{-\tau}^{0} \exp(-d_L(-u))\beta F(u)du)$$
$$+ \int_{T-\tau}^{T} \exp(-d_L(T-s))\beta F(s)ds + \int_{0}^{T-\tau} \exp(-d_L(T-u))(1-\beta)F(u)du$$

or

$$L_0 - \int_{-\tau}^{0} \exp(-d_L(-u))\beta F(u)du \geq 0$$

and

$$\int_{T-\tau}^{T} \exp(-d_L(T-s))\beta F(s)ds > 0 \; and \; \int_{0}^{T-\tau} \exp(-d_L(T-u))(1-\beta)F(u)du \geq 0,$$

Because for everything $t \in [-\tau, T[$, $F(t) > 0$, so $L(T) > 0$. Which contradicts the hypothesis.

2. Suppose N(T) = 0: From the system (3), on the one hand:

$$N'(T) = \lim_{t \to T \; and \; t<T} \frac{N(t) - N(T)}{t - T} \leq 0,$$

And on the other hand

$$\dot{N}(T) = \beta L(T-\tau)\exp(-\tau d_L) > 0.$$

There is a contradiction since for all $t \in [-\tau, T[$,

$$0 < N(t) < \frac{\beta K_L}{(q + d_N)} \; et \; L(t) > 0$$

.

3. Suppose $M(T) = 0$:
According to the system (3), And on the other hand:

$$M'(T) = \lim_{t \to T \; and \; t<T} \frac{M(t) - M(T)}{t - T} \leq 0,$$

And on the other hand

$$\dot{M}(T) = q\alpha N(T) \geq 0,$$

so $M'(T) = 0$ and consequently $N(T) = 0$, which is impossible.

4. Suppose $F(T) = 0$:
   According to the system (3), And on the other hand

$$F'(T) = \lim_{t \to T \text{ and } t<T} \frac{F(t) - F(T)}{t - T} \leq 0,$$

   And on the other hand

$$\dot{F}(T) = q(1-\alpha)N(T) \geq 0.$$

   So $F'(T) = 0$ and consequently $N(T) = 0$, which is impossible.

5. Suppose $L(T) = K_L$:
   So we have on the one hand

$$\dot{L}(T) = -\beta L(T-\tau)\exp(-\tau d_L) - d_L K_L < 0$$

   And on the other hand

$$L'(T) = \lim_{t \to T \text{ and } t<T} \frac{L(t) - L(T)}{t - T} \geq 0,$$

   which is absurd.

6. Suppose $N(T) = \dfrac{\beta K_L}{(q + d_N)}$:
   So we have on the one hand

$$\dot{N}(T) \leq \beta L(t) - (q + d_N)N(t) < 0$$

   And on the other hand

$$N'(T) = \lim_{t \to T \text{ and } t<T} \frac{N(t) - N(T)}{t - T} \geq 0,$$

   which is absurd.

7. Suppose $M(T) = \dfrac{q\alpha\beta K_L}{d_M(q + d_N)}$:
   So we have on the one hand

$$\dot{M}(T) = q\alpha N(T) - d_M M(T) < 0$$

   And on the other hand

$$N'(T) = \lim_{t \to T \text{ and } t<T} \frac{N(t) - N(T)}{t - T} \geq 0,$$

   which is absurd.

1.3 Balance points

Proposition 2 The base net reproduction rate for the model (3) is given by:

$$\mathcal{R}_1 = \frac{\beta\, q\,(1-\alpha)\, b\, e^{-\tau\, d_L}}{(\beta + d_L)(q + d_N)d_F}$$

Proposition 3 (i) The delay system (3) always has the point trivial equilibrium X_0^* =(0, 0, 0, 0).

(ii) When R1 > 1 , that is when $0 < \tau < \tau^* = \frac{1}{d_L}\ \ln(R_0)$, this trivial equilibrium point coexists with a unique equilibrium positive called coexistence equilibrium noted $X^* = (L^*, N^*, M^*, F^*)$ where F^* is the solution of the equation

$$a_3 F^3 + a_2 F^2 + a_1 F + a_0 = 0 \tag{6}$$

and N^*, M^* *and* F^* *are given by:*

$$N^* = \frac{d_F + d_2 F^*}{(1-\alpha)q} F^*;\ \ L^* = \frac{N^*}{\beta}(d_N + q + d_1 N^*);\ \ M^* = \frac{q\alpha}{d_M} N^*$$

with

$$a_0 = -\sigma \frac{(q + d_N)d_F}{((1-\alpha)q)^2} + b, \quad a_1 = \frac{-\sigma}{((1-\alpha)q)^2}((q + d_N)(1-\alpha)q + d_1 d_F)$$
$$a_2 = \frac{-\sigma}{((1-\alpha)q)^2} d_1 d_2 d_F, \quad a_3 = \frac{-\sigma}{((1-\alpha)q)^2} d_1 d_2^2, \quad \sigma = 1 + \exp(\tau d_L)\left(1 + \frac{d_L}{\beta}\right)$$

1.3.1 Stability of trivial equilibrium point X_0 = (0, 0, 0, 0)

Local stability is obtained by studying the equation system characteristic (3) linearized around the point equilibrium (0, 0, 0, 0). Consider for this the matrices following:

$$J_0 = \begin{pmatrix} -d_L & 0 & 0 & b \\ 0 & -(d_N + q) & 0 & 0 \\ 0 & q\alpha & -d_M & 0 \\ 0 & (1-\alpha)q & 0 & -d_F \end{pmatrix}, \quad J_\tau = \begin{pmatrix} -\beta \exp(-\tau d_L) & 0 & 0 & 0 \\ \beta \exp(-\tau d_L) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial associated with the trivial equilibrium point is given by:

$$\begin{aligned} \triangle(\lambda) &= \det(\lambda I_4 - J_0 - \exp(-\tau\lambda) J_\tau) \\ &= \begin{vmatrix} \lambda + d_L + \beta \exp(-\tau(d_L + \lambda)) & 0 & 0 & -b \\ -\beta \exp(-\tau(d_L + \lambda)) & (d_N + q) + \lambda & 0 & 0 \\ 0 & -q\alpha & d_M + \lambda & 0 \\ 0 & -(1-\alpha)q & 0 & d_F + \lambda \end{vmatrix} \\ &= (d_M + \lambda) P(\lambda) \end{aligned}$$

So

$$P(\lambda) = \lambda^3 + \alpha_1\lambda^2 + \alpha_2 \exp(-\tau\lambda)\lambda^2 + \alpha_3\lambda + \alpha_4\lambda \exp(-\tau\lambda) + \alpha_5 \exp(-\tau\lambda) + \alpha_6$$

where

$$\alpha_1 = d_F + d_N + d_L + q, \quad \alpha_2 = \beta \exp(-\tau d_L),$$
$$\alpha_3 = (q + d_N + d_F)d_L + d_F(d_N + q), \quad \alpha_4 = \beta \exp(-\tau d_L)(d_N + d_F + q),$$
$$\alpha_5 = \beta \exp(-\tau d_L)(d_F(d_N + q) - b\, q\,(1-\alpha)), \quad \alpha_6 = d_L\, d_F\,(q + d_N).$$

The polynomial $P(\lambda)$ can be rewritten in the form

$$P(\lambda) = P_1(\lambda) + Q(\lambda)\exp(-\tau\lambda) \tag{7}$$

or

$$P_1(\lambda) = \lambda^3 + \alpha_1\lambda^2 + \alpha_3\lambda + \alpha_6 \quad et \quad Q(\lambda) = \alpha_2\lambda^2 + \alpha_4\lambda + \alpha_5$$

We seek to verify the hypotheses of the theorem of the stability:

(1) Suppose an imaginary root ix common to $P_1(\lambda)$ and $Q(\lambda)$, where x, y $\in \mathbb{R}^*$, then we have the relation:

$$P_1(ix) = Q(ix) = 0 \Longrightarrow \begin{cases} -i\,x^3 - \alpha_1\,x^2 + i\,\alpha_3\,x + \alpha_6 = 0, \\ -\alpha_2\,x^2 + i\,\alpha_4\,x + \alpha_5 = 0. \end{cases}$$

$$\Longrightarrow \begin{cases} -\alpha_1\,x^2 + \alpha_6 = 0, \\ -x^3 + \alpha_3\,x = 0, \\ -\alpha_2\,x^2 + \alpha_5 = 0 \\ \alpha_4\,x = 0. \end{cases}$$

This system does not admit solutions. So $P_1(\lambda)$ and Q(λ) do not have an imaginary root common.

(2) $\overline{P_1(-iy)} = iy^3 - \alpha_1 y^2 + i\alpha_3 y = P_1(iy)$ and $\overline{Q(-iy)} = Q(iy)$.

(3) $P_1(0) - Q(0) = \alpha_5 \neq 0$.

(4) $F(y) = |P_1(iy)|^2 - |Q(iy)|^2 = y^6 + b_1 y^4 + b_2 y^2 + b_3$ where:

$$b_1 = \alpha_1^2 - \alpha_2^2 - 2\alpha_3, \quad b_2 = \alpha_3^2 - \alpha_4^2 + 2\,\alpha_2\,\alpha_5 - 2\alpha_1\,\alpha_6, \quad b_3 = \alpha_6^2 - \alpha_5^2.$$

By putting $Y = y^2$, we arrive at:

$$F(Y) = Y^3 + b_1 Y^2 + b_2 Y + b_3$$

The rule of Descartes signs that we summarize in the table next, allows us to discuss the existence of the possible solutions of this equation. F(Y) has at most a finite number of real zeros. We then have the following result on the asymptotic stability of the trivial equilibrium point

Table 1: Sign table of Descartes.

b_3	b_2	b_1	Number of solutions
$-$	$-$	$-$	0 solution
$-$	$-$	$+$	1 solution
$-$	$+$	$-$	2 solutions
$-$	$+$	$+$	1 solution
$+$	$+$	$+$	0 solution
$+$	$+$	$-$	1 solution
$+$	$-$	$+$	2 solutions
$+$	$-$	$-$	1 solution

Lemma 1 If $b_1 < 0$, $b_2 < 0$ and $b_3 < 0$ or$b_1 > 0$, $b_2 > 0$ and $b_3 > 0$ then the equation F(Y) = 0 has no positive solution. In this case, if the trivial equilibrium point is locally asymptotically stable when tau = 0, it remains locally asymptotically stable whatever the delay value.

If at least one of the coefficients b_1, b_2 and b_3 has a sign opposite to the others, the equation F(Y)=0 has at least one positive real root. In this case, if the trivial equilibrium point is unstable when tau = 0, it remains so regardless of the value of the delay. Now, we will set the values of the parameters to do the numerical simulations to validate the theoretical results obtained. The values of the parameters used are recorded in the following table: For parameter values (value 1), the coefficients b_1, b_2 and b_3 are all positive. By applying lemma (1), since the trivial equilibrium point is locally asymptotically stable for the model without delay, it remains locally asymptotically stable for the model with delay. The figure illustrates this result because in this figure 2.1 (a), all the trajectories converge towards the point of equilibrium $X_0 = (0, 0, 0, 0)$. For values 2 in the parameter table, the coefficients b_2 and b_3 are negative while the coefficient b1 is negative. The equation F (Y) admits 0 or 2 roots, and in this case the change in stability of the equilibrium can be observed. Figure 2.1 (b) presents trajectories that converge locally towards a coexistence equilibrium. The trivial equilibrium is therefore unstable.

Table 2: Values of the parameters used for the numerical simulation of the trivial equilibrium point of the delay. model.

Settings	Value 1 ($\mathcal{R}_1 \leq 1$)	Value 2 ($\mathcal{R}_1 > 1$)
b	3	3
α	0.4	0.4
K_L	500	500
d_{1N}	0.0007	0.0007
d_{2F}	0.0001	0.0001
d_L	0.2	0.0001
d_N	0.002	0.002
d_M	0.05	0.25
d_F	0.05	0.25
q	1/5	1/5
β	1/15	1/15
$\mathcal{R}$	0.7201	6.9188
b_1	0.0833	0.0990
b_2	0.0018	-0.0010
b_3	2.7316 e-06	-4.1315 e-04

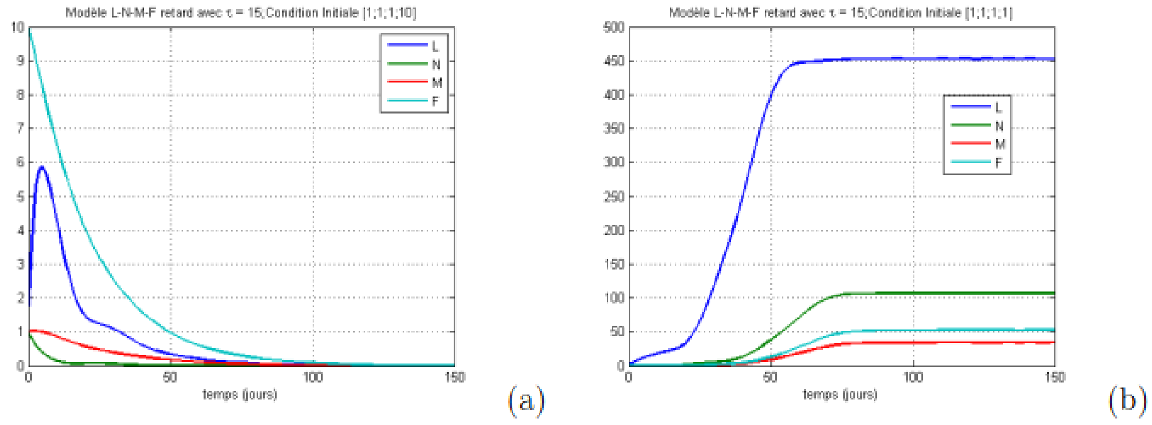

Figure 2: Local stability of equilibrium points for the delay model.

1.3.2 Stability of the equilibrium point of coexistence X^*

The local stability of the equilibrium point of coexistence is obtained by studying the characteristic equation of the system (ref unretar) linearized around the point of equilibrium (L^*, N^*, M^*, F^*). Consider the following matrices:

$$J_0 = \begin{pmatrix} -d_L - \dfrac{b\,F^*}{K_L} & 0 & 0 & b(1 - \dfrac{L^*}{K_L}) \\ 0 & -(d_N + q + 2d_1N^*) & 0 & 0 \\ 0 & q\alpha & -d_M & 0 \\ 0 & (1-\alpha)q & 0 & -(d_F + 2d_2F^*) \end{pmatrix},$$

$$J_\tau = \begin{pmatrix} -\beta\exp(-\tau d_L) & 0 & 0 & 0 \\ \beta\exp(-\tau d_L) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial associated with the non-equilibrium point trivial is given by:

$$\begin{aligned} \triangle(\lambda) &= \det(\lambda I_4 - J_0 - \exp(-\tau\lambda)J_\tau) \\ &= \begin{vmatrix} \lambda + d_L + \dfrac{b\,F^*}{K_C} + \beta\exp(-\tau(d_L+\lambda)) & 0 & 0 & -b\left(1 - \dfrac{L^*}{K_L}\right) \\ -\beta\exp(-\tau(d_L+\lambda)) & (d_N + q + 2d_1N^*) + \lambda & 0 & 0 \\ 0 & -q\alpha & d_M + \lambda & 0 \\ 0 & -(1-\alpha)q & 0 & d_F + \lambda + 2d_2F^* \end{vmatrix} \\ &= (d_M + \lambda)P(\lambda) \end{aligned}$$

So

$$P(\lambda) = \lambda^3 + \alpha_1\lambda^2 + \alpha_2 \exp(-\tau\lambda)\lambda^2 + \alpha_3\lambda + \alpha_4\lambda \exp(-\tau\lambda) + \alpha_5 \exp(-\tau\lambda) + \alpha_6$$

where:

$$\alpha_1 = d_L + \frac{b\,F^*}{K_L} + q + d_N + d_F + 2\,d_1\,N^* + 2\,d_2\,F^*, \quad \alpha_2 = \beta\,\exp(-\tau d_L),$$

$$\alpha_3 = (d_F + 2\,d_2\,F^*)(q + d_N + 2\,d_1\mathbb{N}^*) + (q + d_N + d_F + 2\,d_1\,N^* + 2\,d_2\,F^*)\left(d_L + \frac{b\,F^*}{K_L}\right),$$

$$\alpha_4 = \beta\exp(-\tau d_L)(q + d_N + d_F + 2\,d_1 N^* + 2\,d_2 F^*),$$

$$\alpha_5 = \beta\,\exp(-\tau\,d_L)\left((d_F + 2\,d_2\,F^*)(q + d_N + 2\,d_1\,N^*) - b\,q\,(1-\alpha)\left(1 - \frac{L^*}{K_L}\right)\right),$$

$$\alpha_6 = (d_F + 2\,d_2\,F^*)(q + d_N + 2\,d_1\,N^*)\left(d_L + \frac{b\,F^*}{K_L}\right)$$

The polynomial P (λ) can be rewritten in the form P (λ) = P_1(λ) + Q(λ) exp(−τλ) where:

$$P_1(\lambda) = \lambda^3 + \alpha_1\lambda^2 + \alpha_3\lambda + \alpha_6 \quad et \quad Q(\lambda) = \alpha_2\lambda^2 + \alpha_4\lambda + \alpha_5$$

We seek to verify the hypotheses of the theorem of the stability:

Let

$$F(y) = |P_1(iy)|^2 - |Q(iy)|^2 = y^6 + b_1 y^4 + b_2 y^2 + b_3$$

Where:

$$b_1 = \alpha_1^2 - \alpha_2^2 - 2\alpha_3, \quad b_2 = \alpha_3^2 - \alpha_4^2 + 2\,\alpha_2\,\alpha_5 - 2\alpha_1\,\alpha_6, \quad b_3 = \alpha_6^2 - \alpha_5^2.$$

By putting $Y = y^2$, we arrive at:

$$F(Y) = Y^3 + b_1 Y^2 + b_2 Y + b_3$$

The rule of Descartes signs that we summarize in the table next, allows us to discuss the existence of the possible solutions of this equation. F(Y) has at most a finite number of real zeros. We then have the following result on the asymptotic stability of the equilibrium point of coexistence.

Table 3: Sign table of Descartes 2.

b_3	b_2	b_1	Number of solutions
$-$	$-$	$-$	0 solution
$-$	$-$	$+$	1 solution
$-$	$+$	$-$	2 solutions
$-$	$+$	$+$	1 solution
$+$	$+$	$+$	0 solution
$+$	$+$	$-$	1 solution
$+$	$-$	$+$	2 solutions
$+$	$-$	$-$	1 solution

Lemma 2 If $b_1 < 0$, $b_2 < 0$ and $b_3 < 0$ or $b_1 > 0$, $b_2 > 0$ and $b_3 > 0$ then the equation F(Y) = 0 has no positive solution. In this case, if the equilibrium point of coexistence is locally asymptotically stable when $\tau = 0$, it remains locally asymptotically stable regardless of the lag value.

If at least one of the coefficients b_1, b_2 and b_3 has a sign opposite to the others, the equation F(Y) = 0 has at least one positive real root. In this case, a finite number of stability changes appear and the considered system eventually becomes unstable.

2. Conclusion

In this paper we have formulated a model integrating a delay term related to the growth dynamics of H. hampei which couples the temporal components in order to identify the related factors life cycle or landscape influencing population dynamics. We have shown that there is always a threshold of existence of a coexistence equilibrium. We have also shown that the equilibria (trivial and coexistence) of the delay model change stability for some values of the parameters, whatever the value of the delay considered.

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