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Generalized Dynamic Inversion Based Aircraft Lateral Control

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Abstract

This paper illustrates how the Generalized Dynamic Inversion (GDI) is used to control aircraft lateral motion. To implement the GDI control law, the yaw channel constraint dynamics are first constructed and then inverted using Moore-Penrose Generalized Inverse (MPGI). Consequently, the auxiliary component of this control law is affine in a null control vector, which is designed to guarantee asymptotic aircraft stability. A significant benefit of GDI control is the additional design flexibility afforded by its two independent control actions. Extensive simulations have been conducted to prove the efficacy of the proposed method.

Keywords: Null control vector; Projection matrix; Generalized dynamic inversion; Lateral control; Virtual constraint dynamics.

1. Introduction

Precision in tracking and maneuverability of sophisticated aircraft are essential to the system's performance and reliability. When a pilot applies force along one axis, the aircraft responds by moving in three dimensions because of the aircraft's intrinsically coupled dynamics. Real-world aircraft flight is characterized by a close relationship between and significant coupling of the lateral and directional modes, which results in yaw and roll rates because of roll rate and sideslip, respectively. Various methods are offered in the research in an effort to develop a decoupling control system for aircraft. In [1,2,3], the approach of eigen structure was introduced to decouple the motion of aircraft. In [4,5], the idea of using inverted dynamics to decouple aircraft motion was presented.

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Utilizing the concept of dynamic inversion depicted some drawbacks, such as removal of important nonlinearities, as well as it demonstrated significant control effort [6]. The GDI approach discussed in this work is a method, which relies on the non-square inversion concept to result in prescribed control design process. Instead of inverting the entire system's dynamics, this approach inverts the dynamics encapsulated by the defined set of constraints. As a result, this method simplifies traditional dynamic inversion by eliminating the need to invert the entire system's dynamics and prevent the elimination of beneficial nonlinear effects. Moore-Penrose Generalized Inverse (MPGI), as shown in [7, 8], and the corresponding nullspace parameterization, as seen in [9], are used to invert the dynamic constraints. An under-determined system of equations can have an infinite number of solutions, which can be characterized by utilizing Greville formula. Numerous aeronautical and robotic applications have implemented GDI control method. See [10,11,12,13,14,15,16,17,18,19].

The literature reviewed in this paper summarizes the problems that face the design of robust control systems and suggests numerous remedies, some of which are costly to adopt. As a result, research is being conducted in this area to improve performance, particularly the asymptotic convergence of the states to their desired attitudes. Therefore, in this paper; we propose GDI control law structure for dynamic systems stabilization. Our method improves the performance of aircraft lateral dynamics by using GDI approach. Within the architecture of GDI, the Greville formula provides for two basic collaborating controllers: one which imposes the required constraints and another which enables an extra degree of design flexibility. This additional level of flexibility enables the incorporation of several design techniques inside GDI. The usage of a null projection matrix ensures that the auxiliary component operates on the constraint matrix's null space, whereas the particular part operates on the range space of the constraint matrix's transpose. The non-interference of control actions is ensured by the orthogonality of two control subspaces, and hence both actions strive forward into a single aim. Constraint dynamics incorporate the performance criteria and then are reversed using the Moore-Penrose generalized inverse to produce the dynamic constraints, i.e., the particular part is responsible for enforcing the desired system behavior. Another control action is carried out by the control law's auxiliary component, which is carefully designed to guarantee asymptotic stability. Therefore, the key points of the study can be summarized as follows:

- Creates the dynamic constraints as well as establishes the GDI control law's fundamental form;
- Develop a null control vector that updates the closed-loop system;

The remainder of this work is structured as follows: section 2 exhibits the formulation of the Linear Time-Invariant (LTI) model for lateral dynamics of aircraft . In section 3, we focus on the develop of GDI control law. In section 4, the application to aircraft lateral dynamics is presented. Conclusions and thoughts are drawn in the final section.

1.1. Lateral Dynamics of Aircraft

The LTI model of the transport aircraft is

$$\dot{x} = Ax + Bu \tag{1}$$

The LTI model of (1) can be expressed as in [20], which is approximated as

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\emptyset} \\ \dot{p} \\ \dot{\varphi} \end{bmatrix} = 10^{-2} \begin{bmatrix} -10 & -100 & 11.5 & 0 & 0 \\ 40.9 & -24.5 & 0 & -4 & 0 \\ 0 & 0 & 0 & 100 & 0 \\ -160.4 & 28.5 & 0 & -109.3 & 0 \\ 0 & 100 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \emptyset \\ p \\ \varphi \end{bmatrix} + 10^{-2} \begin{bmatrix} 0 & 1.8 \\ -0.2 & -24.4 \\ 0 & 0 \\ 32.2 & 8.7 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{2}$$

where sideslip angle, yaw rate, roll angle, roll rate, and yaw rate are the accessible state vector ($x \in \mathbb{R}^5$) that represented respectively by $\beta, r, \phi, p, and\ \psi$. The control vector ($u \in \mathbb{R}^2$) $is\ represented$ by aileron and rudder deflections.

1.2. GDI Control

To begin, we will provide a basic overview of GDI control. Two provisions comprise the GDI control law. The first type of controller is the particular controller, which imposes the specified constraints; the second type of controller is the auxiliary controller, which offers a degree of design freedom. This additional degree of freedom enables the incorporation of alternative design approaches within GDI. The usage of a null projection matrix ensures that the auxiliary controller operates on the constraint matrix's nullspace, while the particular controller operates on the range space of the constraint matrix's transpose. The orthogonality of two control subspaces assures that control actions do not interfere with one another, ensuring that both actions operate toward a same objective.

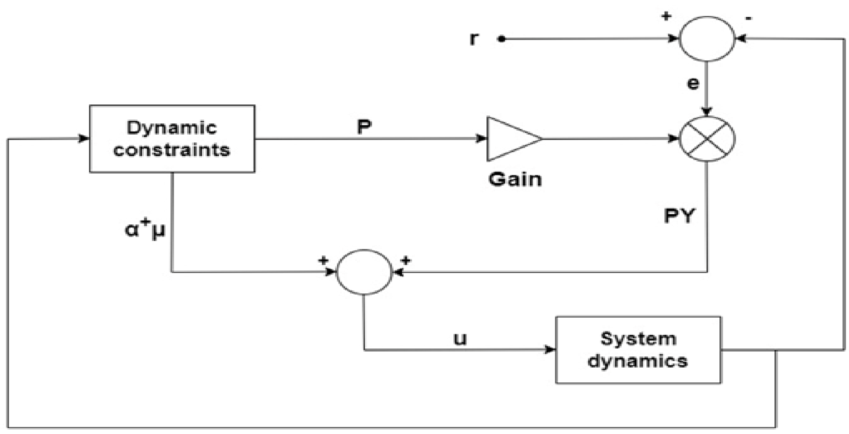

Figure 1: Block diagram of GDI control.

The GDI control law is constructed by formulating a set of constraints that fully capture the control objectives, then inverting those constraints utilizing the MPGI-based Greville approach. To stabilize the yaw axis dynamics, a differential equation with a differential order equal to the relative degree of the state function is defined as part of GDI control, which results in

$$\ddot{\psi} + c_1\dot{\psi} + c_2\psi = 0 \tag{3}$$

Alternatively, equation (3) can be expressed as:

$$\dot{r} + c_1 r + c_2 \psi = 0 \tag{4}$$

Where c_1 and c_2 are positive constants that must be selected to achieve asymptotic stability of (3). Now, substituting the corresponding $\dot{r}$ values from (2) in (4) transforms the differential form into the following algebraic expression.

$$\mathcal{H}u = \mathcal{B}\mathrm{X} \tag{5}$$

Where the control coefficient $\mathcal{H} \in \mathbb{R}^{1\times m}$ is

$$\mathcal{H} = I_5 AB \tag{6}$$

Where I_5 is the s_5^{th} row of the identity matrix$I_{5\times5}$. The control load function β is obtained as

$$\beta_1 = -I_5(A^2 + c_1 A + c_2 I_{5\times5}) \tag{7}$$

Due to under-determined system of (5), the number of solutions will be infinite. Therefore, it can be parameterized by utilizing Greville approach that yields

$$u = \mathcal{H}_1^+ \beta_1 + \rho_1 u_1 \tag{8}$$

Where u_1 is the corresponding null control vector that must be designed later to achieve asymptotic stability for roll channel. $\mathcal{H}_1^+ \in \mathbb{R}^2$ is the MPGI of $\mathcal{H}_1$ and is given by:

$$\mathcal{H}_1^+ = \mathcal{H}_1^T / \mathcal{H}_1 \mathcal{H}_1^T \tag{9}$$

And $\rho_1 \in \mathbb{R}^{2\times2}$ is the projection matrix on the nullspace of $\mathcal{H}_1$and is given by:

$$\rho_1 = I_{2\times2} - \mathcal{H}_1^+ \mathcal{H}_1 \tag{10}$$

Now substituting (8) in (1), the updated closed-loop system is

$$\dot{x} = Ax + B(\mathcal{H}_1^+ \beta_1 + \rho_1 u_1) \tag{11}$$

Or,

$$\dot{x} = A_{cl_1} + B\mathcal{H}_1^+ \beta_1 \tag{12}$$

And the control input matrix B_{cl_1}yields,

$$B_{cl_1} = B\rho_1 \tag{13}$$

Because of its orthogonality, the null control vector u_1 in the GDI complement works on the nullspace of the constraint dynamics, hence introducing an additional degree of freedom. Consequently, a second time-varying constraint dynamics is proposed for stabilizing the roll dynamics, leading to

$$\dot{p} + c_3 p + c_4 \phi = 0 \tag{14}$$

By substituting the $\dot{p}$ values from (2) in (15), the differential form is transformed into algebraic system that obtained as

$$\mathcal{H}_2 u_1 = \beta_2 x \tag{15}$$

Where the control coefficient $\mathcal{H}_2$ is

$$\mathcal{H}_2 = \begin{bmatrix} B_{cl1_{(4,1)}} & B_{cl1_{(4,2)}} \end{bmatrix} \tag{16}$$

And the control load function β_2 is obtained as

$$\beta_2 = -A_{cl(4,2)} - \left(A_{cl(4,2)} + c_4\right) - \left(A_{cl(4,4)} + c_3\right) - A_{cl(4,5)} \tag{17}$$

Asymptotic convergence of the error dynamics in the roll channel is guaranteed by the GDI control law, which given as

$$u_1 = \mathcal{H}_2^{+} \beta_2 + \rho_2 u_2 \tag{18}$$

Where u_2 the corresponding is null control vector, $\mathcal{H}_2^{+} \in \mathbb{R}^2$ is the MPGI of $\mathcal{H}_2$ and is given by:

$$\mathcal{H}_2^{+} = \mathcal{H}_2^{T} / \mathcal{H}_2 \mathcal{H}_2^{T} \tag{19}$$

And $\rho_2 \in \mathbb{R}^{2\times 2}$ is the projection matrix on the nullspace of $\mathcal{H}_2$ and is given by:

$$\rho_2 = I_{2\times 2} - \mathcal{H}_2^{+} \mathcal{H}_2 \tag{20}$$

Using the null control vector u_1 obtained by (18) in (12), resulting in

$$\dot{x} = A_{cl1} + B_{cl1}(\mathcal{H}_2^{+} \beta_2 + \rho_2 u_2) \tag{21}$$

Where the updated system matrix $A_{cl1}$is obtained as

$$A_{cl2} = A_{cl1} + B_{cl1} \mathcal{H}_2^{+} \beta_2 \tag{22}$$

And the control input matrix B_{cl2} yields

$$B_{cl2} = B_{cl1} \rho_2 = 0 \tag{23}$$

It can be deduced from (23) that the null control vector u_2 is incapable of further forcing the states to the required attitudes since the control load matrix B_{cl2} becomes zero, which will be proven numerically in simulation section.

1.3. Numerical Simulations

It is assumed that the initial value of the state vector $[\beta\ \ r\ \ \phi\ \ p\ \ \psi]^T$ will be $[1\ \ 1\ \ 1\ \ 1\ \ 1]^T$ when performing simulation studies. Figure 2 depicts the open loop response of the aircraft's linear lateral dynamics, given by (1). The stability of the system's behavior is demonstrated by the simulation findings.

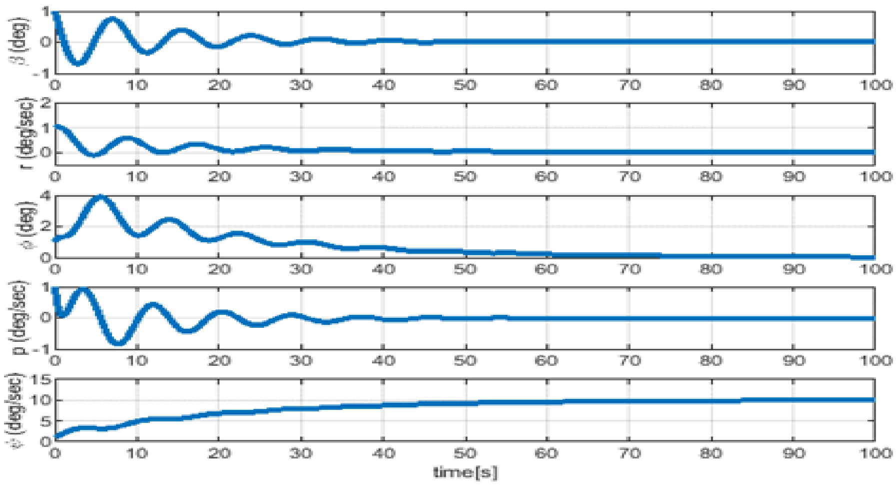

Figure 2: open-loop response.

A. Yaw-axis constraint:

Using a particular component of GDI control law, figure 3 guarantees constraint dynamics converge to zero asymptotically relying on the deviation function of the yaw channel. Initially, the state vector is assumed to be $[0\ \ 1\ \ 0\ \ 0\ \ 1]^T$ .Figure 3 depicts the closed-loop response when adopting the modification obtained in (8)}. The suggested GDI controller drives the system states toward zero. Additionally, it is shown that in figure 3 whereas the roll angle and its rate exhibition inherently unstable behavior. Figure 4 depicts the associated control deflections. Substituting $c_1 = 1.5$and $c_2 = 0.56$ and utilizing equations (13) and (14) we obtain

$$A_{cl1}= \begin{bmatrix} -0.0698 & -0.9074 & 0.1150 & -0.0030 & 0.0413 \\ 0 & -1.5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ -1.4538 & 0.7460 & 0 & -1.1077 & 0.2057 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

$$B_{cl1=}\begin{bmatrix} -0.0001 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.3213 & -0.0026 \\ 0 & 0 \end{bmatrix} \tag{25}$$

The GDI controller that enforces the yaw constraint is given as

$$K_{\psi} = \begin{bmatrix} 0.0137 & 0.0422 & 0 & -0.0013 & 0.0188 \\ 1.6761 & 5.1431 & 0 & -0.1639 & 2.2949 \end{bmatrix} \tag{26}$$

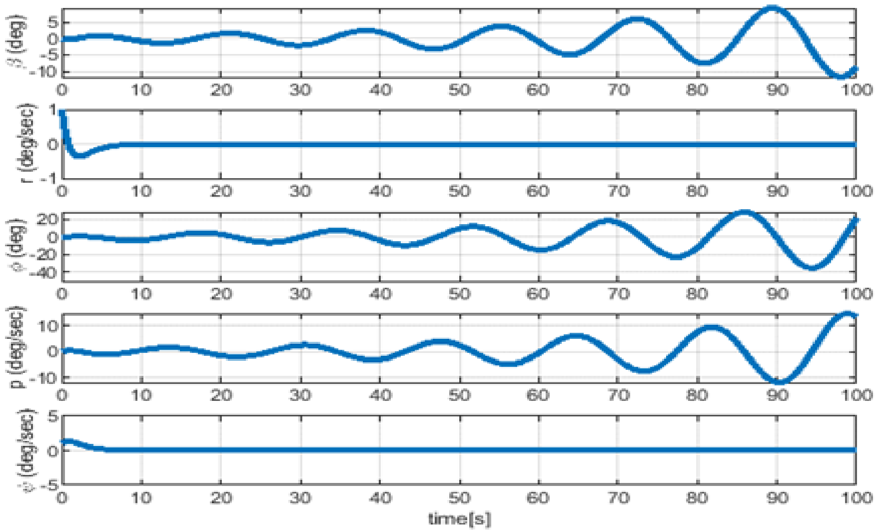

Figure 3: Yaw constraint states.

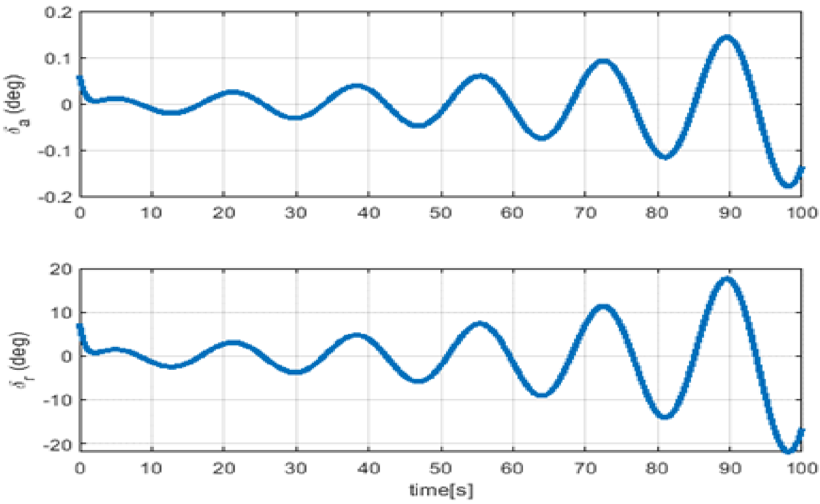

Figure 4: Yaw constraint control deflections.

B. Roll-axis constraint:

An additional constraint, derived from the error dynamics of the roll axis, is introduced via the design of a null control vector such that stabilizing the roll dynamics. Figure 5 depicts the closed-loop response including the presence of null control. Using the principle of a null control vector, we can observe that the behavior of all the states of the system tend to converge to zero asymptotically. Figure 6 depicts the associated control deflections. Substituting $c_3 = 2.2$and $c_4 = 1$ and utilizing equations (24) and (25) we obtain

$$A_{cl2=} \begin{bmatrix} -0.0703 & -0.9072 & 0.1150 & -0.0027 & 0.0414 \\ 0 & -1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -2.2 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

$$B_{cl2=} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (25)$$

The GDI controller that enforces the roll dynamics constraint is given as

$$K_{\phi} = \begin{bmatrix} 4.5244 & -2.3217 & -3.1122 & -3.3994 & -0.6380 \\ -0.0366 & 0.0188 & 0.0252 & 0.0275 & 0.0052 \end{bmatrix} \quad (26)$$

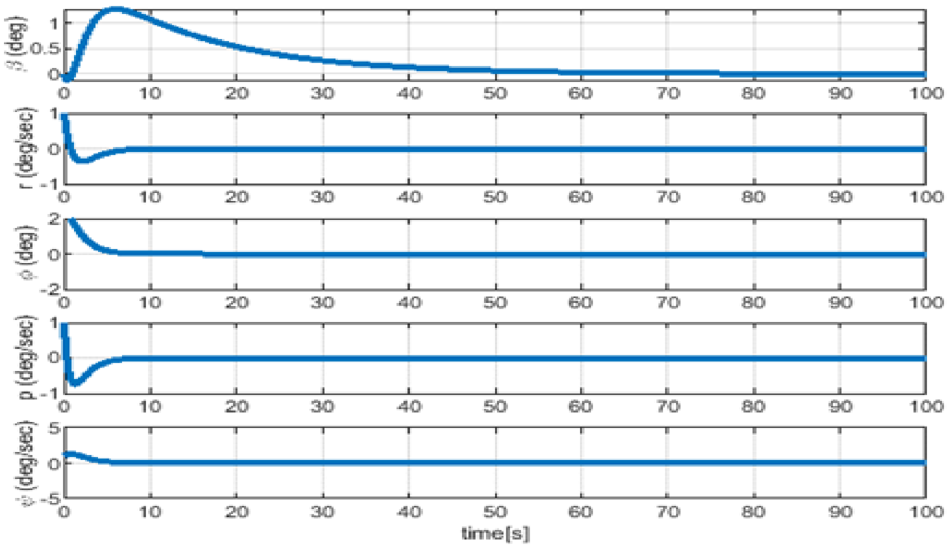

Figure 5: Roll constraint states.

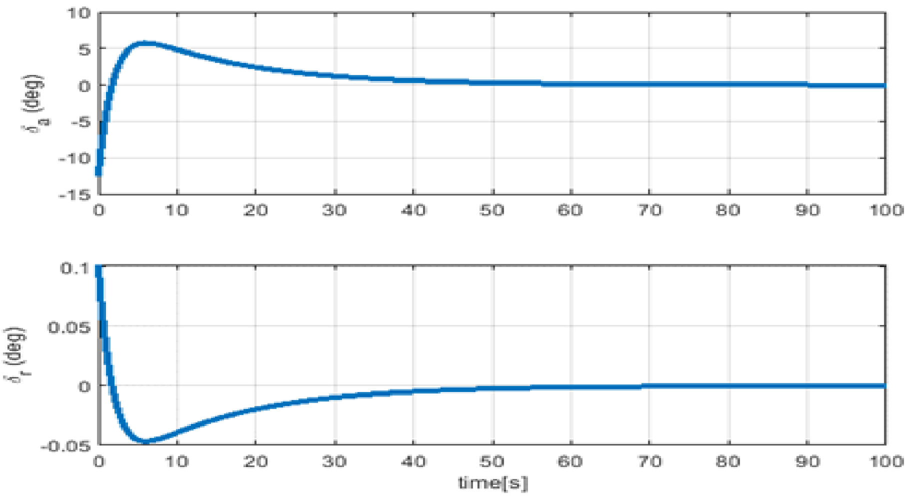

Figure 6: Roll constraint control deflections.

1.4. Conclusion

The GDI control system is properly developed to regulate the system's output to desired control objectives and to perform decoupling between the roll and yaw channel states. The LTI model of the aircraft is subjected to the GDI control law. In order to ensure that the yaw-axis dynamic is asymptotically stable, the particular part of the Greville formula is used, whilst the nullifying part is used to achieve asymptotically stable for roll channel. Numerical simulations prove the suggested control method succeeds, providing empirical evidence for its viability. The closed-loop results confirm the GDI control law's efficient functioning, inspiring the authors to implement it to boost the efficiency of adaptive systems.

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