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MHD Flow of Fractionalized Jeffrey Fluid with Newtonian Heating and Thermal Radiation Over a Vertical Plate

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Abstract

The objective of this paper is to analyze the influence of heat absorption/generation and mass diffusion on magnetohydrodynamics(MHD) Jeffrey fluid flow over a perpendicular plate moving exponentially immersed in a porous media. The Newtonian heating condition are udes for the fluid motion. The impact of thermal radiation is used in the energy equation. The two types of magnetic field have been evaluated. The main purpose of present work is to acquire the analytical solution with the help of Atangana-Baleanu (AB), Caputo, and Caputo-Fabrizio fractional derivatives. We have drawn a graphical comparison between the solutions of these three types of fractional models of jeffery fluid. Graphs of different parameters have been also plotted using MathCad software. Furthermore, comparison among ordinary and fractionalized velocity fields are made to observe the impact of fractional parameter γ. It is clear from graph that velocity obtained with ordinary derivative is higher than that obtained with fractional derivatives. It is also found that velocity obtained with Atangana-Baleanu (AB) fractional derivative is smaller than that obtained with Caputo and Caputo-Fabrizio fractional derivatives. Therefore, Atangana-Baleanu fractional derivative is the best choice to obtain controlled velocity.

Keywords: Jeffrey's fluid; Free convection; Chemical reaction; Heat generation; Newtonian heating Atangana-Baleanu; Caputo; and Caputo-Fabrizio fractional derivatives.

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1. Introduction

The study of Magnetohydrodynamics (MHD) convective flow in the existence of porosity has significant importance in the area of geophysics, solar physics, electrically power generation, and applied engineering. Sandeep [1] examined the impact of thermal diffusion of Magnetohydrodynamics (MHD) flow with non-Newtonian fluid over a sheet. Khan and his colleagues [2] studied the thermo effect of convection flow on MHD fluid in the presence of porous media through an oscillating plate and analyzed the physical interpretation of flow parameters. An inclined magnetic field and thermal radiation on unsteady magnetohydrodynamic convection flow of a fluid through a perpendicular plate immersed in a porous media is analyzed N. Sandeep and V. Sagunamma [3]. Kempannagari and his colleagues [4] is studied the MHD ferrofluid flow through a surface with chemical reaction, heat transfer, thermal radiation, Biot number, and fractional heating.

Ramandevi and his colleagues [5] are analyzed the viscous effects on two different non-Newtonian fluid of MHD flow with variable heat source through a sheet. A boundary layer flow of MHD fluid with electrical conductivity over a cone is observed by Kumar and his colleagues [6]. Reddy and his colleagues [7] examined the influence of cross diffusion on non-Newtonian fluids flow over a sheet. The effect of stagnation point flow with non-linear thermal radiation of a Casson fluid are discussed by Ananth and his colleagues [8]. Walters-B and Casson nano-fluids on MHD flow with cross diffusion and non-linear thermal radiation over a non-uniform thickness of sheet is analyzed by Lakshmi and his colleagues [9]. The impact of mass transfer and chemical reaction on Casson fluid over a plate immersed in a porous media are observed by Shehzad and his colleagues [10].

An exact solution for the impact of chemical reaction of order first and mass diffusion on Casson fluid MHD flow through an oscillating perpendicular plate in the presence of porosity is discussed by Kateria and his colleagues [11].

The effects of mass and heat transfer on MHD fluid over a plate with Newtonian heating is investigated by Das and his colleagues [12]. Reference [13] analyzed the effects of heat source/sink, mass diffusion, and Newtonian heating on free convection flow of magnetohydrodynamics fluid. An analytical solution for the effect of Newtonian heating on MHD fluid with heat source/sink and mass diffusion imbedded in a porous media is reported by Hussanan and his colleagues [14]. The effects of thermal radiation, chemical reaction, heat sink/source, and Newtonian heating on MHD fluid is studied by Hussanan and his colleagues [15]. The influence of Newtonian heating on free convection flow with mass diffusion is investigated by Vieru and his colleagues [16]. The impact of shear stress, heat source/sink, chemical reaction, mass diffusion, and magnetic field on unsteady boundary layer flow is studied by Fatecau and his colleagues [17]. However, The effect of relative magnetic field on fluid with heat absorption/generation and chemical reaction is discussed by [18].

An analytical solution of Duffer effects on magnetohydrodynamics flow through a vertical plate is obtained by Kumaresana and his colleagues [19]. The effects of shear stress on natural convection flow of a viscous fluid is investigated by Rubab and his colleagues [20]. The influence of thermal radiation, chemical reaction, and heat source on nanofluid is discussed by [21]. Reference [22] obtained the solution of MHD fluid immersed in a porous media through a moving plate.

Shah and his colleagues [23] investigated the solution for free convection flow of MHD flow with non-uniform temperature and first order chemical reaction. The effects of heat generation/absorption and chemical reaction on Jeffrey fluid are discussed by Mohanty and his colleagues [24]. Das and his colleagues [25] obtained the exact solution of Jeffrey fluid over a surface. An exact solution of Jeffrey fluid with Caputo-Fabrizio frictional derivative are obtained by Saqib and his colleagues [26]. Butt and his colleagues [27] obtained the analytical solution of Jeffrey fluid with mass and heat transfer effect over an infinite perpendicular plate. The viscous effects on Jeffrey fluid over a sheet with heat transfer are analyzed by [28]. Moreover, a new technique of Atangana-Baleanu fractional derivatives are presented by [29]. A new definition of Caputo-Fabrizio fractional derivatives are analyzed [30-31].

The purpose of present work is to analyze the effects of Newtonian heating, chemical reaction of order first, relative magnetic field, thermal radiation, and heat sink/source on Jeffrey fluid in the presence of porous media over a plate with Caputo, Caputo-Fabrizio, and Atangana-Baleanu frictional derivatives. Also a comparison is drawn between fractionalized and ordinary Jeffrey fluids. Relative magnetic field are used for the design of power plants, nuclear gas turbines, rolling of steels, missiles, and combustion etc.

2. Mathematical Formulation of the Problem

Consider a MHD flow of Jeffrey fluid over an infinite vertical plate. In addition, rate of heat sink/source, thermal radiation, and chemical reaction of order first were also considered. Plate lies in xy plane and z axis is normal to plate. A magnetic field of strength B_0, is applied perpendicular to plate. Initially plate and fluid both are in equilibrium at the temperature T_∞ and concentration C_∞, respectively. After time $t' = 0^+$, the plate starts to move in xz plane with velocity $U_0 f(t')'$. The magnetic Reynolds number is smaller, therefore flow is laminar. The influence of Joule heating parameter in energy equation is also neglected. Under these Boussinesq's approximation, we obtain the following governing boundary layer equation

$$\frac{\partial u(z,t')}{\partial t'} = (\frac{\nu}{1+\lambda_1})(1+\lambda_2\frac{\partial}{\partial t'})\frac{\partial^2 u(z,t')}{\partial z^2} - \frac{\sigma\beta_0^2}{\rho}u(z,t') - \frac{\nu\phi}{K_1}u(z,t') +$$

$$\rho g'\beta_T(T-T'_\infty) + \rho g'\beta_c(C-C'_\infty), \tag{1}$$

Eq. (1) is valid when the magnetic lines of force are set comparative to the fluid while if magnetic field are set comparative to the plate, Eq. (1) becomes by [32]

$$\frac{\partial u(z,t')}{\partial t'} = (\frac{\nu}{1+\lambda_1})(1+\lambda_2\frac{\partial}{\partial t'})\frac{\partial^2 u(z,t')}{\partial z^2} - \frac{\sigma\beta_0^2}{\rho}u(z,t') - \frac{\nu\phi}{K_1}u(z,t') + \frac{\sigma\beta_0^2}{\rho}U\epsilon f(t') +$$

$$\rho g'\beta_T(T-T'_\infty) + \rho g'\beta_c(C-C'_\infty), \tag{2}$$

$$\rho Cp\frac{\partial T}{\partial t'} = K_2\frac{\partial^2 T}{\partial z^2} + Q_0(T-T'_\infty)\,, \tag{3}$$

$$\frac{\partial C}{\partial t'} = D_1 \frac{\partial^2 C}{\partial z^2} - K_3(C - C'_\infty)\,, \tag{4}$$

With initial and boundary conditions

$$u(z,0) = 0,\ T(z,0) = T_\infty,\ C(z,0) = C_\infty,\quad z > 0, \tag{5}$$

$$u(0,t') = U_1 f(t'),\ \frac{\partial T(0,t')}{\partial z} = -h_s T(0,t'),\ C(0,t') = C_w,\quad t' > 0, \tag{6}$$

$$u(z,t') \to 0,\ T(z,t') \to 0,\ C(z,t') \to 0,\ z \to \infty,\ t' > 0. \tag{7}$$

where $\rho, K_1, g', \nu, B_0, \lambda_2, D_1, K_2, K_3$ and σ represents the density of fluid, porosity parameter, acceleration due to gravity, parameter of kinematic viscosity, magnetic field, parameter of Jeffrey fluid, mass diffusion, thermal conduction parameter, chemical reaction and parameter of electrical conductivity. Introducing the following non-dimensional variables and parameters

$$u^* = \frac{u}{U_1},\quad z^* = \frac{U_0^2 z}{\nu},\quad t^* = \frac{U_0^2 t'}{\nu'},\quad T^* = \frac{T - T'_\infty}{T'_\infty},\quad Pr = \frac{\mu Cp}{k_2},$$

$$Gr = \frac{\nu g' \beta_T T'_\infty}{U_1^3},\quad Gm = \frac{\nu g' \beta_C}{U_1^3}(C_w - C'_\infty),\quad M = \frac{\beta_0^2 \nu \sigma}{\rho U_1^2},\quad R = \frac{K_3 \nu}{U_1^2},$$

$$\frac{1}{K} = \frac{\nu^2 \phi}{K_1 U_1^2},\quad Q = \frac{Q_0 \nu}{U_1^2 K_2},\quad Sc = \frac{\nu}{D_1},\quad \lambda = \frac{h_s \nu}{U_1},\quad f(t^*) = \frac{\nu}{U_1^2} t'. \tag{8}$$

into Eqs.(1)-(6), and leaving the notation of stars,we obtain the following boundary value problem

$$\frac{\partial u(z,t)}{\partial t} = \left(\frac{1}{1+\lambda_1}\right)\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u(z,t)}{\partial z^2} - Mu(z,t) - \frac{1}{k}u(z,t) + M\epsilon f(t) + GrT(z,t) + GmC(z,t), \tag{9}$$

$$Pr\frac{\partial T(z,t)}{\partial t} = \frac{\partial^2 T(z,t)}{\partial z^2} + PrQT(z,t) \tag{10}$$

$$\frac{\partial C(z,t)}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C(z,t)}{\partial z^2} - RC(z,t), \tag{11}$$

with associated initial and boundary conditions

$$u(z,0) = 0, T(z,0) = 0, C(z,0) = 0,\quad z > 0, \tag{12}$$

$$u(0,t) = f(t), \frac{\partial T(0,t)}{\partial z} = -\lambda(1+T), C(0,t) = 1,\ t > 0, \tag{13}$$

$$u(z,t) \to 0,\ T(z,t) \to 0,\ C(z,t) \to 0,\ z \to \infty,\ t > 0. \tag{14}$$

3. Generalization of Local Model

The local model defined in equations (9-14) have been generalized by replacing ordinary time derivative by Atangana-Baleanu, Caputo and Caputo-Fabrizio, fractional derivative of order γ.

$$D_t^{\gamma} u(z,t) = (\frac{1}{1+\lambda_1})(1+\lambda_2 D_t^{\gamma} u(z,t))\frac{\partial^2 u(z,t)}{\partial z^2} - Mu(z,t) - K^{-1}u(z,t) + M\epsilon f(t) +$$

$$GrT(z,t) + GmC(z,t), \tag{15}$$

$$PrD_t^{\gamma} T(z,t) = \frac{\partial^2 T(z,t)}{\partial z^2} + QPrT(z,t)\ , \tag{16}$$

$$D_t^{\gamma} C(z,t) = \frac{1}{Sc}\frac{\partial^2 C(z,t)}{\partial z^2} - RC(z,t)\ , \tag{17}$$

$$u(z,0) = 0, T(z,0) = 0, C(z,0) = 0, \quad z > 0, \tag{18}$$

$$u(0,t) = f(t), \frac{\partial T(0,t)}{\partial z} = -\lambda(1+T), C(0,t) = 1,\ t > 0, \tag{19}$$

$$u(z,t) \to 0,\ \ T(z,t) \to 0,\ \ C(z,t) \to 0,\ \ z \to \infty,\ \ t > 0. \tag{20}$$

where $D_t^{\gamma} u(z,t)$ represent the Caputo fractional derivative of $u(z,t)$ in the following form

$$D_t^{\gamma} u(z,t) = \{\frac{1}{\Gamma(1-\gamma)}\int_0^t \frac{1}{(t-\tau)^{\gamma}}\frac{\partial u(z,\tau)}{\partial \tau} d\tau,\ 0 \le \gamma < \ 1;\ \frac{\partial u(z,t)}{\partial t},\ \gamma = 1. \tag{21}$$

$D_t^{\gamma} u(z,t)$ represent the Caputo-Fabrizio fractional derivative of $u(z,t)$ in the following form

defined as

$$D_t^{\gamma} u(z,t) = \frac{1}{(1-\gamma)}\int_0^t e^{\frac{-\gamma(t-p)}{1-\gamma}}\frac{\partial u(z,p)}{\partial p} dp, 0 \le \gamma \le \ 1; \tag{22}$$

$D_t^{\gamma} u(z,t)$ represent the Atangana-Baleanu fractional derivative of $u(z,t)$ in the following form

defined as

$$D_t^{\gamma} u(z,t) = \frac{M(\gamma)}{(1-\gamma)}\int_0^t E_{\gamma}(-\gamma\frac{(t-p)^{\gamma}}{1-\gamma})\frac{\partial u(z,p)}{\partial p} dp, \tag{23}$$

4. Solution of Problem

Now we solved the model by applying Laplace transform technique. we can solve Eq. (15) for velocity profile, Eq. (16) for temperature profile and Eq. (17) for concentration profile respectively.

4.1. Calculation of Concentration With Caputo

Eq. (17) can be solved with the help of Laplace transform,we obtain

$$Scq^{\gamma}\underline{C}(z,q) = \frac{\partial^2 \underline{C}(z,q)}{\partial z^2} - ScR\underline{C}(z,q), \tag{24}$$

Boundary conditions satisfies Eq. (24) are

$$\underline{C}(0,q) = \frac{1}{q}, \quad \underline{C}(z,q) \to 0, \;\; z \to \infty. \tag{25}$$

The solution of partial differential Eq. (24), by using conditions of Eq. (25), is

$$\underline{C}(z,q) = \frac{1}{q} e^{-z\sqrt{sc(q^{\gamma}+R)}}. \tag{26}$$

Appropriate form of Eq. (26) is

$$\underline{C}(z,q) = [\frac{q^{\gamma}+R}{q}] \frac{e^{-z\sqrt{sc}\sqrt{(q^{\gamma}+R)}}}{q^{\gamma}+R} \tag{27}$$

By applying inverse Laplace transform on Eq. (27), we obtained the general solution in the following form

$$C(z,t) = \int_0^t \quad [\frac{(t-s)^{-\gamma}}{\Gamma(1-\gamma)} + R] F_1(z,s) ds. \tag{28}$$

where

$$F_1(z,s) = \int_0^{\infty} \quad e^{-Ru} erfc(\frac{z\sqrt{Sc}}{2\sqrt{u}}) s^{-1}(0,-\alpha,-us^{-\alpha}) du. \tag{29}$$

4.2. Sherwood Number

The expression for rate of mass transfer is calculated as

$$Sh = -\frac{\partial C}{\partial z}|_{z=0} = -L^{-1}\{\frac{\partial \underline{C}}{\partial z}|_{z=0}\} = \sqrt{Sc}\int_0^t \quad (\frac{(t-s)^{-\gamma}}{\Gamma(1-\gamma)} + R) s^{\frac{\gamma}{2}-1} E^{\frac{1}{2}}_{\gamma,\frac{\gamma}{2}}(Qs^{\gamma}) ds. \tag{30}$$

4.3. Calculation of Concentration With Caputo-Fabrizio

By taking Laplace transform on Eq. (17), we obtain

$$\frac{Scq}{(1-\gamma)q+\gamma}\underline{C}(z,q) = \frac{\partial^2 \underline{C}(z,q)}{\partial z^2} - ScR\underline{C}(z,q), \tag{31}$$

The solution of partial differential Eq. (31), by using conditions of Eqs. (19) and (20), is

$$\underline{C}(z,q) = \frac{1}{q} e^{-z\sqrt{\frac{(q+b_3)}{(q+b_1)}b_2}}. \tag{32}$$

The inverse Laplace transform of Eq. (32) is

$$C(z,t) = \phi_1(z,t), \tag{33}$$

where

$$\phi_1(z,t) = e^{-z\sqrt{b_2}} - \frac{z\sqrt{b_2}\sqrt{b_3-b_1}}{2\sqrt{\pi}} \int_0^\infty \int_0^t \frac{1}{\sqrt{t}} e^{\left(-b_1 t - \frac{z^2 b_2}{4u} - u\right)} \times I_1\left(2\sqrt{(b_3-b_1)ut}\right) dt du. \tag{34}$$

4.4. Sherwood Number

The rate of mass transfer is obtained from Eq. (32) by using partial fraction method in the following form

$$Sh = \sqrt{\frac{b_2 b_3}{b1}} \tag{35}$$

4.5. Calculation Of Concentration With Atangana-Baleanu

By taking Laplace transform on Eq. (17), we obtain

$$\frac{Scq^\gamma}{(1-\gamma)q^\gamma+\gamma}\underline{C}(z,q) = \frac{\partial^2 \underline{C}(z,q)}{\partial z^2} - ScR\underline{C}(z,q), \tag{36}$$

The solution of partial differential Eq. (36), by using conditions of Eqs. (19) and (20), is

$$\underline{C}(z,q) = \frac{1}{q} e^{-z\sqrt{\frac{(q^\gamma+b_3)}{(q^\gamma+b_1)}b_2}}. \tag{37}$$

Equivalent form of Eq. (37) is

$$\underline{C}(z,q) = \frac{1}{q^{1-\gamma}} \frac{1}{q^\gamma} e^{-z\sqrt{b_2}\sqrt{\frac{(q^\gamma+b_3)}{(q^\gamma+b_1)}}}. \tag{38}$$

The inverse Laplace transform of Eq. (38) is written as

$$C(z,t) = \int_0^t \left[\frac{(t-s)^{-\gamma}}{\Gamma(1-\gamma)}\right] \varphi_1(z,s) ds. \tag{39}$$

where

$$\varphi_1(z,s) =$$

$$\int_0^{\infty} [e^{-z\sqrt{b_2}} - \frac{z\sqrt{b_2}\sqrt{b_3-b_1}}{2\sqrt{\pi}} \int_0^{\infty} \int_0^{s} \frac{1}{\sqrt{s}} e^{(-b_1 s - \frac{z^2 b_2}{4u} - u)} \times I_1(2\sqrt{(b_3-b_1)us}) ds du] \times$$

$$s^{-1}\psi(0,-\gamma,-xs^{-\gamma})dx. \quad (40$$

4.6. Sherwood Number

The non-dimensional form of mass transfer rate is given by

$$Sh = \sqrt{\frac{b_2 b_3}{b1}} \quad (41)$$

4.7. Calculation of Temperature With Caputo

By taking Laplace transform on Eq. (16), we find

$$Prq^{\gamma}\underline{T}(z,q) = \frac{\partial^2 \underline{T}(z,q)}{\partial z^2} + QPr\underline{T}(z,q), \quad (42)$$

Boundary conditions satisfies Eq. (42) are

$$\frac{\partial \underline{T}(0,q)}{\partial z} = -\lambda(1+\underline{T}(0,q)), \quad \underline{T}(z,q) \to 0, \ z \to \infty. \quad (43)$$

The solution of partial differential Eq. (42), by using condition of Eq. (43) is

$$\underline{T}(z,q) = \frac{b_4}{q[\sqrt{q^{\gamma}-Q}-b_4]} e^{-z\sqrt{Pr(q^{\gamma}-Q)}}. \quad (44)$$

The suitable form of Eq. (44) is

$$\underline{T}(z,q) = \frac{b_4[\sqrt{q^{\gamma}-Q}+b_4]}{[q^{\gamma}-b_5]} \frac{[q^{\gamma}-Q]}{q} \frac{e^{-z\sqrt{Pr}\sqrt{(q^{\gamma}-Q)}}}{q^{\gamma}-Q}. \quad (45)$$

or $$\underline{T}(z,q) = (\frac{b_6}{q} + \frac{q^{\gamma} b_6^{\grave{0}}}{q(q^{\gamma}-b_5)}) \frac{e^{-z\sqrt{Pr}\sqrt{(q^{\gamma}-Q)}}}{q^{\gamma}-Q}. \quad (46)$$

Taking inverse Laplace transform of Eq. (46), we have

$$T(z,t) = \int_0^t (b_6 + g_1(t-s))F_2(z,s)ds. \quad (47)$$

where

$$g_1(t) = b_6^{\grave{0}} E_{\gamma}(b_5 t^{\gamma}) \quad (48)$$

$$F_2(z,s) = \int_0^\infty e^{Qu} erfc(\frac{z\sqrt{Pr}}{2\sqrt{u}}) s^{-1}(0,-\alpha,-us^{-\alpha})du. \tag{49}$$

4.8. Nusselt Number

The Nusselt number can be calculated from Eq. (46) as

$$Nu = -\frac{\partial T}{\partial z}|_{z=0} = -L^{-1}\{\frac{\partial \underline{T}}{\partial z}|_{z=0}\} = \sqrt{Pr}\int_0^t (b_6 + b_6^0 E_\gamma(b_5(t-s)^\gamma))s^{\frac{\gamma}{2}-1}E_{\gamma,\frac{\gamma}{2}}^{\frac{1}{2}}(Qs^\gamma)ds. \tag{50}$$

4.9. Calculation of Temperature With Caputo-Fabrizio

By taking Laplace transform on Eq. (16), we obtain

$$\frac{Prq}{(1-\gamma)q+\gamma}\underline{T}(z,q) = \frac{\partial^2\underline{T}(z,q)}{\partial z^2} + PrQ\underline{T}(z,q), \tag{51}$$

The solution of partial differential Eq. (51), by using conditions of EqS. (19) and (20), is

$$\underline{T}(z,q) = \frac{\lambda\sqrt{q+b_1}}{q(\sqrt{(q-b_8)b_7}-\lambda\sqrt{q+b_1})} e^{-z\sqrt{\frac{(q-b_8)}{(q+b_1)}b_7}}. \tag{52}$$

Eq. (52) can be written in suitable form as

$$\underline{T}(z,q) = [\frac{b_{11}}{q} + \frac{b_{12}}{q-b_{10}}]e^{-z\sqrt{\frac{(q-b_8)}{(q+b_1)}b_7}} \tag{53}$$

The inverse Laplace transform of Eq. (53) is

$$T(z,t) = \int_0^t (b_{11}\phi_2(y,s) + b_{12}\phi_3(z,s))ds, \tag{54}$$

where

$$\phi_2(z,s) = e^{-z\sqrt{b_7}} - \frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^\infty \int_0^s \frac{1}{\sqrt{s}} e^{(-b_1 s - \frac{z^2 b_7}{4u} - u)} \times I_1(2\sqrt{(-b_8-b_1)us})dsdu, \tag{55}$$

$$\phi_3(z,s) = e^{b_{10}t}e^{-z\sqrt{b_7}} - \frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^\infty \int_0^s \frac{e^{b_{10}s}}{\sqrt{s}} e^{(-b_{10}s - b_1 s - \frac{z^2 b_7}{4u} - u)} \times I_1(2\sqrt{(-b_8-b_1)us})dsdu. \tag{56}$$

4.10. Nusselt Number

With the help of partial fraction, dimensionless form of (Nu) is given by

$$Nu = b_{11}D_1 + b_{12}D_2 e^{b_{10}t}. \tag{57}$$

4.11. Calculation of Temperature With Atangana-Baleanu

By taking Laplace transform on Eq. (16), we obtain

$$\frac{Prq^{\gamma}}{(1-\gamma)q^{\gamma}+\gamma}\underline{T}(z,q)=\frac{\partial^2\underline{T}(z,q)}{\partial z^2}+PrQ\underline{T}(z,q), \tag{58}$$

The solution of partial differential Eq. (58), by using conditions of Eqs. (19) and (20), is

$$\underline{T}(z,q)=\frac{\lambda\sqrt{q^{\gamma}+b_1}}{q(\sqrt{(q^{\gamma}-b_8)b_7}-\lambda\sqrt{q^{\gamma}+b_1})}e^{-z\sqrt{\frac{(q^{\gamma}-b_8)}{(q^{\gamma}+b_1)}b_7}}. \tag{59}$$

Equivalently form of Eq. (59) is

$$\underline{T}(z,q)=\frac{b_{11}}{q^{1-\gamma}}\frac{e^{-z\sqrt{b_7}\sqrt{\frac{(q^{\gamma}-b_8)}{(q^{\gamma}+b_1)}}}}{q^{\gamma}}+\frac{b_{12}}{q^{1-\gamma}}\frac{e^{-z\sqrt{b_7}\sqrt{\frac{(q^{\gamma}-b_8)}{(q^{\gamma}+b_1)}}}}{q^{\gamma}-b_{10}}, \tag{60}$$

Inverse Laplace transform of Eq. (60) is

$$T(z,t)=\int_0^t \quad (b_{11}\frac{(t-s)^{-\gamma}}{\Gamma(\gamma-1)}\varphi_2(z,s)+b_{12}\frac{(t-s)^{-\gamma}}{\Gamma(\gamma-1)}\varphi_3(z,s))ds, \tag{61}$$

where

$$\varphi_2(z,s)=\int_0^{\infty} \quad [e^{-z\sqrt{b_7}}-\frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^{\infty} \quad \int_0^{s} \quad \frac{1}{\sqrt{s}}e^{(-b_1s-\frac{z^2b_7}{4u}-u)}\times$$

$$I_1(2\sqrt{(-b_8-b_1)us})dsdu]s^{-1}\psi(0,-\gamma,-xs^{-\gamma})dx. \tag{62}$$

$$\varphi_3(z,s)=\int_0^{\infty} \quad [e^{b_{10}s-z\sqrt{b_7}}-\frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^{\infty} \quad \int_0^{s} \quad \frac{e^{b_{10}s}}{\sqrt{s}}e^{(-b_{10}s-b_1s-\frac{z^2b_7}{4u}-u)}\times$$

$$I_1(2\sqrt{(-b_8-b_1)us})dsdu]s^{-1}\psi(0,-\gamma,-xs^{-\gamma})dx. \tag{63}$$

4.12. Nusselt Number

By using partial fraction method, we find the (Nu) of Eq. (60) in the following form

$$Nu=b_{11}D_1+b_{12}D_2E_{\gamma}(b_{10}t^{\gamma}). \tag{64}$$

4.13. Calculation of Velocity With Caputo

By taking Laplace transform on Eq.(15) and using initial and boundary condition, we obtaine

$$q^{\gamma}\underline{u}(z,q) = (\frac{1}{1+\lambda_1})(1+\lambda_2 q^{\gamma})\frac{\partial^2 \underline{u}(z,q)}{\partial z^2} - M\underline{u}(z,q) - \frac{1}{k}\underline{u}(z,q) + M\epsilon f(q) + Gr\underline{T}(z,q) + Gm\underline{C}(z,q), \quad (65)$$

$$\underline{u}(0,q) = \frac{1}{(q-a)}, \quad \underline{u}(z,q) \to 0, \;\; z \to \infty. \quad (66)$$

where $\underline{u}(y,q)$ is the Laplace transform of the function $u(y,t)$ and q is the transform variable. The solution of partial differential Eq. (65), by using condition of Eq. (66), is

$$\underline{u}(z,q) = (1 - \frac{M\epsilon}{q^{\gamma}+H})\frac{1}{(q-a)} e^{-z\sqrt{\frac{q^{\gamma}+H}{(b_{13}+b_{14}q^{\gamma})}}} + \frac{M\epsilon}{(q^{\gamma}+H)(q-a)} +$$

$$\frac{Grb_4[\sqrt{q^{\gamma}-Q}+b_4]}{q(q^{\gamma}-b_5)[(b_{13}+b_{14}q^{\gamma})(Pr(q^{\gamma}-Q))-(q^{\gamma}+H)]}[e^{-z\sqrt{\frac{q^{\gamma}+H}{(b_{13}+b_{14}q^{\gamma})}}} - e^{-z\sqrt{Pr(q^{\gamma}-Q)}}] +$$

$$\frac{Gm}{q[(b_{13}+b_{14}q^{\gamma})(Sc(q^{\gamma}+R))-(q^{\gamma}+H)]}[e^{-z\sqrt{\frac{q^{\gamma}+H}{(b_{13}+b_{14}q^{\gamma})}}} - e^{-z\sqrt{Sc(q^{\gamma}+R)}}]. \quad (67)$$

The appropriate form of Eq. (67) is

$$\underline{u}(z,q) = \frac{q^{\gamma}}{(q-a)}\frac{e^{-\frac{z}{\sqrt{b_{14}}}\sqrt{\frac{q^{\gamma}+H}{(q^{\gamma}+c_1)}}}}{q^{\gamma}} - \frac{M\epsilon}{(q-a)}\frac{e^{-\frac{z}{\sqrt{b_{14}}}\sqrt{\frac{q^{\gamma}+H}{(q^{\gamma}+c_1)}}}}{(q^{\gamma}+H)} + \frac{M\epsilon}{(q^{\gamma}+H)(q-a)} +$$

$$[\frac{b_{15}q^{\gamma}}{q(q^{\gamma}-m_1)} + \frac{b_{16}q^{\gamma}}{q(q^{\gamma}-m_2)} + \frac{b_{17}q^{\gamma}}{q(q^{\gamma}-b_5)} + \frac{b_{21}q^{\gamma}}{q(q^{\gamma}-m_3)} + \frac{b_{22}}{q(q^{\gamma}-m_4)}]\frac{e^{-\frac{z}{\sqrt{b_{14}}}\sqrt{\frac{q^{\gamma}+H}{(q^{\gamma}+c_1)}}}}{q^{\gamma}} -$$

$$[\frac{b_{18}}{q(q^{\gamma}-m_1)} + \frac{b_{19}}{q(q^{\gamma}-m_2)} + \frac{b_{20}}{q(q^{\gamma}-b_5)}]\frac{e^{-z\sqrt{Pr(q^{\gamma}-Q)}}}{(q^{\gamma}-Q)} -$$

$$[\frac{b_{23}}{q(q^{\gamma}-m_3)} + \frac{b_{24}}{q(q^{\gamma}-m_4)}]\frac{e^{-z\sqrt{Sc(q^{\gamma}+R)}}}{(q^{\gamma}+R)}. \quad (68)$$

where

$$(m_1, m_2) = \frac{-(b_{13}Pr - b_{14}PrQ - 1) \pm \sqrt{(b_{13}Pr - b_{14}PrQ - 1)^2 - 4b_{14}Pr(-H - b_{13}PrQ)}}{(2b_{14}Pr)}, \quad (69)$$

$$(m_3, m_4) = \frac{-(b_{13}Sc + b_{14}PrR - 1) \pm \sqrt{(b_{13}Sc + b_{14}ScR - 1)^2 - 4b_{14}Sc(-H + b_{13}ScR)}}{(2b_{14}Sc)}. \quad (70)$$

Taking inverse Laplace transform of Eq. (68), we find

$$u(z,t) = \int_0^t \quad [g_2(t-s)\varphi_4(z,s) - e^{a(t-s)}M\epsilon\varphi_5(z,s) + e^{a(t-s)}M\epsilon g_3 s + [b_{15}g_4(t-s) +$$

$$b_{16}g_5(t-s) + b_{17}g_6(t-s) + b_{21}g_7(t-s) + b_{22}g_8(t-s)]\varphi_4(z,s) - [b_{18}g_4(t-s) +$$

$$b_{19}g_5(t-s)b_{20}g_6(t-s)]F_2(z,s)-[b_{23}g_7(t-s)+b_{24}g_8(t-s)]F_1(z,s)]ds \quad (71)$$

where

$$\varphi_4(z,s)=\int_0^\infty [e^{-\frac{z}{\sqrt{b_{14}}}}-\frac{z\sqrt{H-c_1}}{2\sqrt{b_{14}}\sqrt{\pi}}\int_0^\infty \int_0^s \frac{1}{\sqrt{s}}e^{(-c_1s-\frac{z^2}{4ub_{14}}-u)}\times I_1(2\sqrt{(H-c_1)us})dsdu]\times s^{-1}\psi(0,-\gamma,-xs^{-\gamma})dx. \quad (72)$$

$$\varphi_5(z,s)=\int_0^\infty [e^{-Hs-\frac{z}{\sqrt{b_{14}}}}-\frac{z\sqrt{H-c_1}}{2\sqrt{b_{14}}\sqrt{\pi}}\int_0^\infty \int_0^s \frac{e^{-Hs}}{\sqrt{s}}e^{(Hs-c_1s-\frac{z^2}{4ub_{14}}-u)}\times I_1(2\sqrt{(H-c_1)us})dsdu]\times s^{-1}\psi(0,-\gamma,-xs^{-\gamma})dx. \quad (73)$$

$$g_2t=-t^{\gamma}E_{1,1-\gamma}(at),\ \ g_3s=s^{\gamma-1}E_{\gamma,\gamma}(-Hs^{\gamma}), \quad (74)$$

$$g_4t=E_{\gamma}(m_1t),\ \ g_5t=E_{\gamma}(m_2t),\ \ g_6t=E_{\gamma}(b_5t),\ \ g_7t=E_{\gamma}(m_3t),\ \ g_8t=E_{\gamma}(m_4t). \quad (75)$$

Skin friction of Eq. (68) is defined as

$$\tau=-\frac{\partial u}{\partial z}|_{z=0}=-L^{-1}\{\frac{\partial \underline{u}}{\partial z}|_{z=0}\}=\frac{\sqrt{H}}{\sqrt{c_1b_{14}}}e^{at}-\int_0^t [\frac{M\epsilon}{\sqrt{b_{14}}}e^{a(s)}H(t-s)+D_3b_{15}(1-E_{\gamma}(m_1s^{\gamma}))+ b_{16}D_4(1-E_{\gamma}(m_2s^{\gamma}))+b_{17}D_5(1-E_{\gamma}(b_5s^{\gamma}))+b_{21}D_6(1-E_{\gamma}(m_3s^{\gamma}))+b_{22}D_7(1-E_{\gamma}(m_4s^{\gamma}))-\sqrt{Pr}(t-s)^{\frac{\gamma}{2}-1}E^{\frac{1}{2}}_{\gamma,\frac{\gamma}{2}}(Q(t-s)^{\gamma})[\frac{b_{18}}{m_1}(1-E_{\gamma}(m_1s^{\gamma}))+\frac{b_{19}}{m_2}(1-E_{\gamma}(m_2s^{\gamma}))+ \frac{b_{20}}{b_5}(1-E_{\gamma}(b_5s^{\gamma}))]-\sqrt{Sc}(t-s)^{\frac{\gamma}{2}-1}E^{\frac{1}{2}}_{\gamma,\frac{\gamma}{2}}(-R(t-s)^{\gamma})[\frac{b_{23}}{m_3}(1-E_{\gamma}(m_3s^{\gamma}))+\frac{b_{24}}{m_4}(1-E_{\gamma}(m_4s^{\gamma}))]]ds. \quad (76)$$

4.14. Calculation of Velocity With Caputo-Fabrizio

By taking Laplace transform on Eq. (15), we obtain

$$\frac{q}{(1-\gamma)q+\gamma}\underline{u}(z,q)=(b_{13}+\frac{qb_{14}}{(1-\gamma)q+\gamma})\frac{\partial^2\underline{u}(z,q)}{\partial z^2}-M\underline{u}(z,q)-K^{-1}\underline{u}(z,q)+ M\epsilon f(q)+Gr\underline{T}(z,q)+Gm\underline{C}(z,q)\ , \quad (77)$$

The solution of partial differential Eq. (77) is obtained in the following form by using initial and boundary conditions

$$\underline{u}(z,q) = (1 - \frac{M\epsilon(q+b_1)}{(q+b_{26})b_{25}})\frac{1}{(q-a)}e^{-z\sqrt{\frac{q+b_{26}}{(q+b_{28})}b_{29}}} + \frac{M\epsilon}{(q+b_{26})(q-a)b_{25}} +$$

$$\frac{Gr[(q+b_1)^2\lambda\sqrt{q+b_1}]}{q(\sqrt{(q-b_8)b_7}-\lambda\sqrt{q+b_1})[b_7b_{27}(q+b_{28})(q-b_8)-(q+b_{26})(q+b_1)b_{25}]}[e^{-z\sqrt{\frac{q+b_{26}}{(q+b_{28})}b_{29}}} -$$

$$e^{-z\sqrt{\frac{q-b_8}{(q+b_1)}b_7}}] + \frac{Gm(q+b_1)^2}{q[b_2b_{27}(q+b_3)(q+b_{28})-(q+b_{26})(q+b_1)b_{25}]}[e^{-z\sqrt{\frac{q+b_{26}}{(q+b_{28})}b_{29}}} - e^{-z\sqrt{\frac{q+b_3}{(q+b_1)}b_2}}]. \quad (78)$$

Eq. (78) can be written equivalently as

$$\underline{u}(z,q) = [1 - b_{30} - \frac{b_{31}}{q+b_{26}}]\frac{1}{(q-a)}e^{-z\sqrt{\frac{q+b_{26}}{(q+b_{28})}b_{29}}} + b_{32}(\frac{1}{(q-a)} - \frac{1}{q+b_{26}}) +$$

$$[\frac{b_{33}}{q} + \frac{b_{34}}{q-n_1} + \frac{b_{35}}{q-n_2} + \frac{b_{36}}{q-b_{10}}][e^{-z\sqrt{\frac{q+b_{26}}{(q+b_{28})}b_{29}}} - e^{-z\sqrt{\frac{q-b_8}{(q+b_1)}b_7}}] +$$

$$[\frac{b_{37}}{q} + \frac{b_{38}}{q-n_3} + \frac{b_{39}}{q-n_4}][e^{-z\sqrt{\frac{q+b_{26}}{(q+b_{28})}b_{29}}} - e^{-z\sqrt{\frac{q+b_3}{(q+b_1)}b_2}}]. \quad (79)$$

where

$$(n_1, n_2) = \frac{-(b_7b_{27}(b_{28}-b_8)-b_{25}(b_1+b_{26}))}{2(b_7b_{27}-b_{25})} \pm$$

$$\frac{\sqrt{(b_7b_{27}(b_{28}-b_8)-b_{25}(b_1+b_{26}))^2+4(b_7b_{27}-b_{25})(b_7b_{27}b_8b_{28}+b_1b_{25}b_{26})}}{2(b_7b_{27}-b_{25})}, \quad (80)$$

$$(n_3, n_4) = \frac{-(b_2b_{27}(b_{28}+b_3)-b_{25}(b_1+b_{26}))}{2(b_2b_{27}-b_{25})} \pm$$

$$\frac{\sqrt{(b_2b_{27}(b_{28}+b_3)-b_{25}(b_1+b_{26}))^2-4(b_2b_{27}-b_{25})(b_2b_{27}b_3b_{28}-b_1b_{25}b_{26})}}{2(b_2b_{27}-b_{25})}. \quad (81)$$

The solution of velocity field is obtained in the following form by using inverse Laplace transform on Eq. (79)

$$u(z,t) = \phi_4(z,s)(1-b_{30}) - b_{31}\int_0^t e^{-b_{26}(t-s)}\phi_4(z,s)ds + b_{32}(e^{at} - e^{-b_{26}t}) +$$

$$(b_{33}+b_{37})\phi_5(z,t) + b_{34}\phi_6(z,t) + b_{35}\phi_7(z,t) + b_{36}\phi_8(z,t) + b_{38}\phi_9(z,t) +$$
$$b_{39}\phi_{10}(z,t) - b_{33}\phi_2(z,t) - b_{34}\phi_{11}(z,t) - b_{35}\phi_{12}(z,t) - b_{36}\phi_3(z,t) - b_{37}\phi_1(z,t) -$$
$$b_{38}\phi_{13}(z,t) - b_{339}\phi_{14}(z,t), \quad (82)$$

$$\phi_4(z,t) =$$
$$e^{as}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \int_0^s \frac{e^{as}}{\sqrt{s}}e^{\left(-as-b_{28}s-\frac{z^2b_{29}}{4u}-u\right)} \times$$
$$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu. \quad (83$$

$$\phi_5(z,t) = e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{1}{\sqrt{t}} e^{(-b_{28}t-\frac{z^2 b_{29}}{4u}-u)} \times I_1(2\sqrt{(b_{26}-b_{28})ut})dtdu, \quad (84)$$

$$\phi_6(z,t) = e^{n_1 t}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_1 t}}{\sqrt{t}} e^{\left(-n_1 t-b_{28}t-\frac{z^2 b_{29}}{4u}-u\right)} \times I_1(2\sqrt{(b_{26}-b_{28})ut})dtdu. \quad (85)$$

$$\phi_7(z,t) = e^{n_2 t}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_2 t}}{\sqrt{t}} e^{\left(-n_2 t-b_{28}t-\frac{z^2 b_{29}}{4u}-u\right)} \times I_1(2\sqrt{(b_{26}-b_{28})ut})dtdu. \quad (86)$$

$$\phi_8(z,t) = e^{b_{10} t}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{b_{10} t}}{\sqrt{t}} e^{\left(-b_{10} t-b_{28}t-\frac{z^2 b_{29}}{4u}-u\right)} \times I_1(2\sqrt{(b_{26}-b_{28})ut})dtdu. \quad (87)$$

$$\phi_9(z,t) = e^{n_3 t}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_3 t}}{\sqrt{t}} e^{\left(-n_3 t-b_{28}t-\frac{z^2 b_{29}}{4u}-u\right)} \times I_1(2\sqrt{(b_{26}-b_{28})ut})dtdu. \quad (88)$$

$$\phi_{10}(z,t) = e^{n_4 t}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_4 t}}{\sqrt{t}} e^{\left(-n_4 t-b_{28}t-\frac{z^2 b_{29}}{4u}-u\right)} \times I_1(2\sqrt{(b_{26}-b_{28})ut})dtdu. \quad (89$$

$$\phi_{11}(z,t) = e^{n_1 t}e^{-z\sqrt{b_7}} - \frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_1 t}}{\sqrt{t}} e^{\left(-n_1 t-b_1 t-\frac{z^2 b_7}{4u}-u\right)} \times I_1(2\sqrt{(-b_8-b_1)ut})dtdu. \quad (90)$$

$$\phi_{12}(z,t) = e^{n_2 t}e^{-z\sqrt{b_7}} - \frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_2 t}}{\sqrt{t}} e^{\left(-n_2 t-b_1 t-\frac{z^2 b_7}{4u}-u\right)} \times I_1(2\sqrt{(-b_8-b_1)ut})dtdu. \quad (91)$$

$$\phi_{13}(z,t) = e^{n_3 t}e^{-z\sqrt{b_2}} - \frac{z\sqrt{b_2}\sqrt{b_3-b_1}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_3 t}}{\sqrt{t}} e^{\left(-n_3 t-b_1 t-\frac{z^2 b_2}{4u}-u\right)} \times I_1(2\sqrt{(b_3-b_1)ut})dtdu. \quad (92)$$

$$\phi_{14}(z,t) = e^{n_4 t}e^{-z\sqrt{b_2}} - \frac{z\sqrt{b_2}\sqrt{b_3-b_1}}{2\sqrt{\pi}}\int_0^\infty \int_0^t \frac{e^{n_4 t}}{\sqrt{t}} e^{\left(-n_4 t-b_1 t-\frac{z^2 b_2}{4u}-u\right)} \times I_1(2\sqrt{(b_3-b_1)ut})dtdu. \quad (93)$$

4.15. Skin friction

The dimensionless form of Skin friction is

$$\tau = (1-b_{30})D_8 e^{at} - b_{31}D_9 e^{at} + D_{10}(b_{33}+b_{37}) + b_{34}D_{11}e^{n_1 t} + b_{35}D_{12}e^{n_2 t} + b_{36}D_{13}e^{b_{10}t}$$

$$+b_{38}D_{14}e^{n_3 t} + b_{38}D_{15}e^{n_4 t} - b_{33}D_1 - b_{34}D_{16}e^{n_1 t} - b_{35}D_{17}e^{n_2 t} - b_{36}D_2 e^{b_{10}t} -$$

$$b_{37}\sqrt{\frac{b_3 b_2}{b_1}} - b_{38}D_{18}e^{n_3 t} - b_{35}D_{19}e^{n_4 t} \quad (94)$$

4.16. Calculation of Velocity With Atangana-Baleanu

By taking Laplace transform on Eq. (15), we obtain

$$\frac{q^\gamma}{(1-\gamma)q^\gamma+\gamma}\underline{u}(z,q) = \left(b_{13} + \frac{q^\gamma b_{14}}{(1-\gamma)q^\gamma+\gamma}\right)\frac{\partial^2 \underline{u}(z,q)}{\partial z^2} - M\underline{u}(z,q) - \frac{1}{k}\underline{u}(z,q) + M\epsilon f(q) + Gr\underline{T}(z,q) + Gm\underline{C}(z,q)\ , \quad (95)$$

The solution of partial differential Eq. (95), by using initial and boundary conditions, we have

$$\underline{u}(z,q) = \left(1 - \frac{M\epsilon(q^\gamma+b_1)}{(q^\gamma+b_{26})b_{25}}\right)\frac{1}{(q-a)}e^{-z\sqrt{\frac{q^\gamma+b_{26}}{(q^\gamma+b_{28})}b_{29}}} + \frac{M\epsilon(q^\gamma+b_1)}{(q^\gamma+b_{26})(q-a)b_{25}} +$$

$$\frac{Gr[(q^\gamma+b_1)^2\lambda\sqrt{q^\gamma+b_1}]}{q(\sqrt{(q^\gamma-b_8)b_7}-\lambda\sqrt{q^\gamma+b_1})[b_7 b_{27}(q^\gamma+b_{28})(q^\gamma-b_8)-(q^\gamma+b_{26})(q^\gamma+b_1)b_{25}]}\times$$

$$\left[e^{-z\sqrt{\frac{q^\gamma+b_{26}}{(q^\gamma+b_{28})}b_{29}}} - e^{-z\sqrt{\frac{q^\gamma-b_8}{(q+b_1)}b_7}}\right] + \frac{Gm(q^\gamma+b_1)^2}{q[b_2 b_{27}(q^\gamma+b_3)(q^\gamma+b_{28})-(q^\gamma+b_{26})(q^\gamma+b_1)b_{25}]}\times$$

$$\left[e^{-z\sqrt{\frac{q^\gamma+b_{26}}{(q^\gamma+b_{28})}b_{29}}} - e^{-z\sqrt{\frac{q^\gamma+b_3}{(q^\gamma+b_1)}b_2}}\right]. \quad (96)$$

Equivalently form of Eq. (96) is

$$\underline{u}(z,q) = (1-b_{30})\frac{q^\gamma}{(q-a)q^\gamma}e^{-z\sqrt{\frac{q^\gamma+b_{26}}{(q^\gamma+b_{28})}b_{29}}} - \frac{b_{31}}{q^\gamma+b_{26}}\frac{1}{(q-a)}e^{-z\sqrt{\frac{q^\gamma+b_{26}}{(q^\gamma+b_{28})}b_{29}}} + \frac{b_{30}}{q-a} +$$

$$\frac{b_{31}}{(q^\gamma+b_{26})(q-a)} + \left[\frac{b_{40}}{q^\gamma-n_1} + \frac{b_{41}}{q^\gamma-n_2} + \frac{b_{42}}{q^\gamma-b_{10}}\right]\left(\frac{q^\gamma+b_1}{q}\right)\left[e^{-z\sqrt{\frac{q^\gamma+b_{26}}{(q^\gamma+b_{28})}b_{29}}} -\right.$$

$$\left.e^{-z\sqrt{\frac{q^\gamma-b_8}{(q^\gamma+b_1)}b_7}}\right] + \left[\frac{b_{43}}{q^\gamma-n_3} + \frac{b_{44}}{q^\gamma-n_4}\right]\left(\frac{q^\gamma+b_1}{q}\right)\left[e^{-z\sqrt{\frac{q^\gamma+b_{26}}{(q^\gamma+b_{28})}b_{29}}} - e^{-z\sqrt{\frac{q^\gamma+b_3}{(q^\gamma+b_1)}b_2}}\right]. \quad (97)$$

By taking inverse Laplace transform on Eq. (97), we find

$$u(z,t) = \int_0^t \left[(1-b_{30})g_2(t-s)\varphi_6(z,s) - b_{31}e^{a(t-s)}\varphi_7(z,s)\right]ds + b_{30}e^{at} +$$

$\int_0^t \quad [b_{31}e^{a(t-s)}g_9(s) +$

$(\frac{(t-s)^{-\gamma}}{\Gamma(\gamma-1)} + b_1)(b_{40}\varphi_8(z,s) + b_{41}\varphi_9(z,s) + b_{42}\varphi_{10}(z,s) + b_{43}\varphi_{11}(z,s) +$

$b_{44}\varphi_{12}(z,s)) - (\frac{(t-s)^{-\gamma}}{\Gamma(\gamma-1)} + b_1)(b_{40}\varphi_{13}(z,s) + b_{41}\varphi_{14}(z,s) + b_{42}\varphi_3(z,s) +$

$b_{43}\varphi_{15}(z,s) + b_{44}\varphi_{16}(z,s))]ds$ (98)

$\varphi_6(z,s) = \int_0^\infty \quad [e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{1}{\sqrt{s}}e^{(-b_{28}s-\frac{z^2b_{29}}{4u}-u)} \times$

$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du$ (99)

$\varphi_7(z,t) = \int_0^\infty \quad [e^{-b_{26}s}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{-b_{26}s}}{\sqrt{s}}e^{(b_{26}s-b_{28}s-\frac{z^2b_{29}}{4u}-u)} \times$

$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du$ (100)

$\varphi_8(z,s) = \int_0^\infty \quad [e^{n_1s}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_1s}}{\sqrt{s}}e^{\left(-n_1s-b_{28}s-\frac{z^2b_{29}}{4u}-u\right)} \times$
$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du$ (101)

$\varphi_9(z,s) = \int_0^\infty \quad [e^{n_2s}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_2s}}{\sqrt{s}}e^{(-n_2s-b_{28}s-\frac{z^2b_{29}}{4u}-u)} \times$

$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du$ (102)

$\varphi_{10}(z,s) = \int_0^\infty \quad [e^{b_{10}s}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{b_{10}s}}{\sqrt{s}}e^{(-b_{10}s-b_{28}s-\frac{z^2b_{29}}{4u}-u)} \times$

$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du$ (103)

$\varphi_{11}(z,s) = \int_0^\infty \quad [e^{n_3s}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_3s}}{\sqrt{s}}e^{(-n_3s-b_{28}s-\frac{z^2b_{29}}{4u}-u)} \times$

$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})$ (104)

$\varphi_{12}(z,s) = \int_0^\infty \quad [e^{n_4s}e^{-z\sqrt{b_{29}}} - \frac{z\sqrt{b_{29}}\sqrt{b_{26}-b_{28}}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_4s}}{\sqrt{s}}e^{(-n_4s-b_{28}s-\frac{z^2b_{29}}{4u}-u)} \times$

$I_1(2\sqrt{(b_{26}-b_{28})us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du$ (105)

$\varphi_{13}(z,s) = \int_0^\infty \quad [e^{n_1s}e^{-z\sqrt{b_7}} - \frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_1s}}{\sqrt{s}}e^{(-n_1s-b_1s-\frac{z^2b_7}{4u}-u)} \times$

$$I_1(2\sqrt{(-b_8-b_1)us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du, \tag{106}$$

$$\varphi_{14}(z,s)=\int_0^\infty \quad [e^{n_2 s}e^{-z\sqrt{b_7}}-\frac{z\sqrt{b_7}\sqrt{-b_8-b_1}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_2 s}}{\sqrt{s}}e^{(-n_2 s-b_1 s-\frac{z^2 b_7}{4u}-u)}\times$$

$$I_1(2\sqrt{(-b_8-b_1)us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du, \tag{107}$$

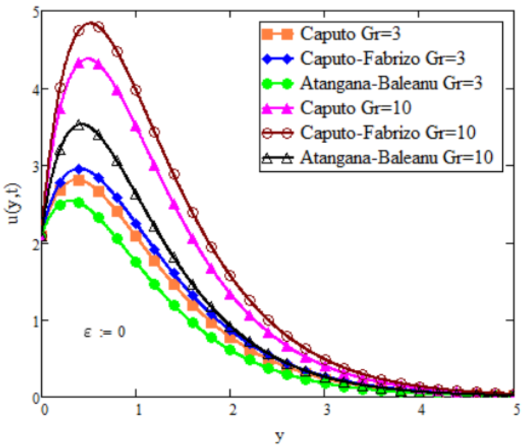
$$\varphi_{15}(z,s)=\int_0^\infty \quad [e^{n_3 s}e^{-z\sqrt{b_2}}-\frac{z\sqrt{b_2}\sqrt{b_3-b_1}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_3 s}}{\sqrt{s}}e^{(-n_3 s-b_1 s-\frac{z^2 b_2}{4u}-u)}\times$$

$$I_1(2\sqrt{(b_3-b_1)us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du, \tag{108}$$

$$\varphi_{16}(z,s)=\int_0^\infty \quad [e^{n_4 s}e^{-z\sqrt{b_2}}-\frac{z\sqrt{b_2}\sqrt{b_3-b_1}}{2\sqrt{\pi}}\int_0^\infty \quad \int_0^s \quad \frac{e^{n_4 s}}{\sqrt{s}}e^{(-n_4 s-b_1 s-\frac{z^2 b_2}{4u}-u)}\times$$

$$I_1(2\sqrt{(b_3-b_1)us})dsdu]s^{-1}(0,-\gamma,-us^{-\gamma})du, \tag{109}$$

$$g_9(s)=s^{\gamma-1}E_{\gamma,\gamma}(-b_{26}s^{\gamma}) \tag{110}$$



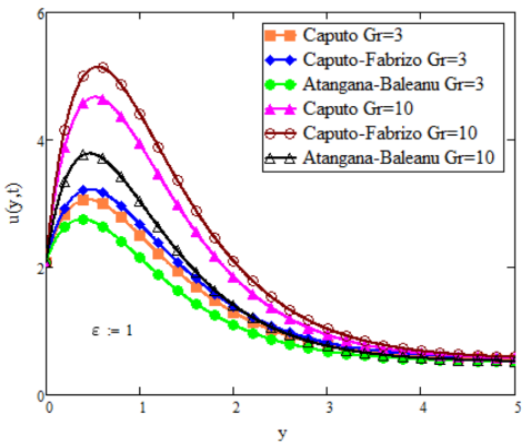


Figure 1: Velocity profile against y due to Gr, where the values of other parameters are Gm=6, M=0.5, t=0.5, K=1.4, γ=0.5, Sc=1.5, Pr=6.5, Q=0.25, R=0.9, $\lambda_1 = 0.4$, $\lambda_2 = 0.2$.

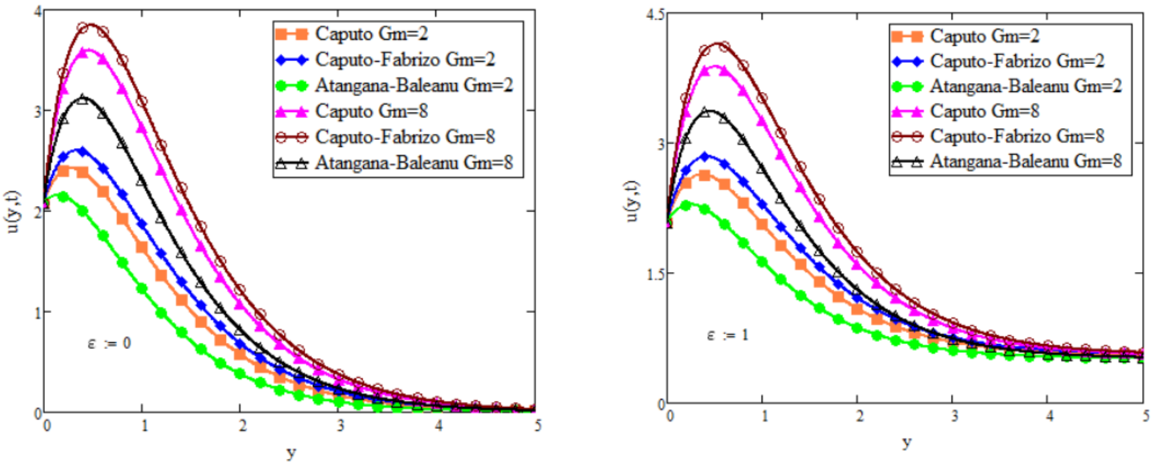


Figure 2: Velocity profile against y due to Gm, where the values of other parameters are Gr=5, M=0.5, t=0.5, K=1.4, γ=0.5, Sc=1.5, Pr=6.5, Q=0.25, R=0.9, $\lambda_1 = 0.4$, $\lambda_2 = 0.2$.

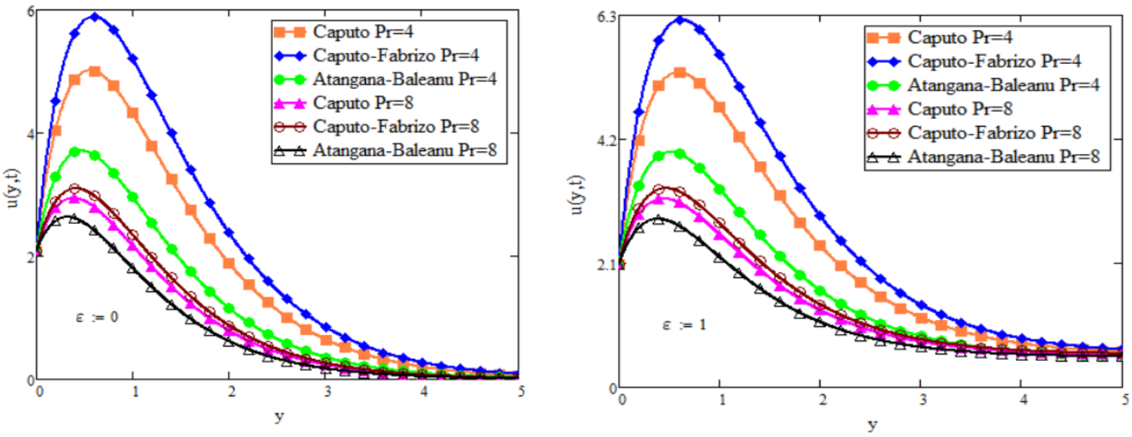


Figure 3: Velocity profile against y due to Pr, where the values of other parameters are Gr=5, M=0.5, t=0.5, K=1.4, γ=0.5, Sc=1.5, Gm=6, Q=0.25, R=0.9, $\lambda_1 = 0.4$, $\lambda_2 = 0.2$.

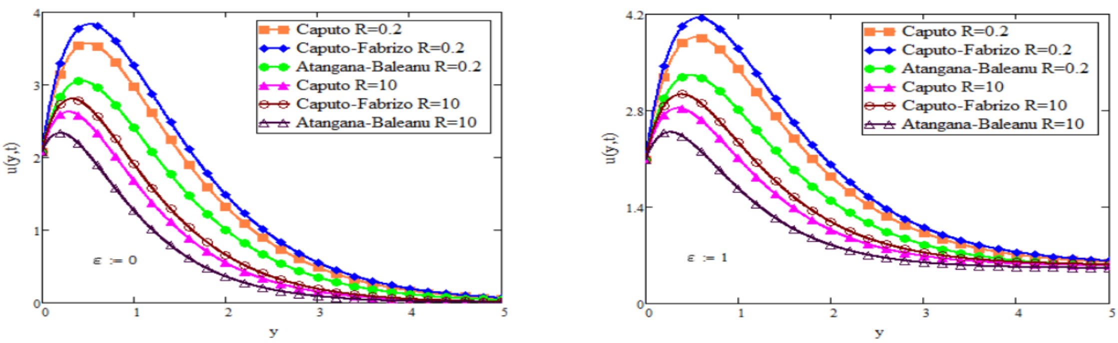


Figure 4: Velocity profile against y due to R, where the values of other parameters are Gr=5, M=0.5, t=0.5, K=1.4, γ=0.5, Sc=1.5, Gm=6, Q=0.25, Pr=6.5, $\lambda_1 = 0.4$, $\lambda_2 = 0.2$.

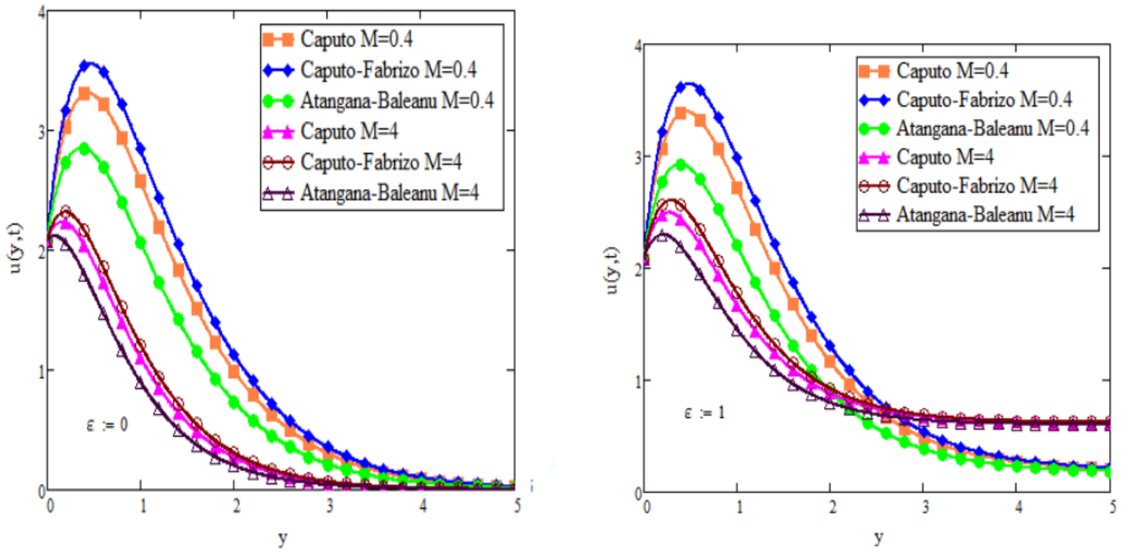


Figure 5: Velocity profile against y due to M, where the values of other parameters are Gr=5, R=0.9, t=0.5, K=1.4, γ=0.5, Sc=1.5, Gm=6, Q=0.25, Pr=6.5, $\lambda_1 = 0.4$, $\lambda_2 = 0.2$.

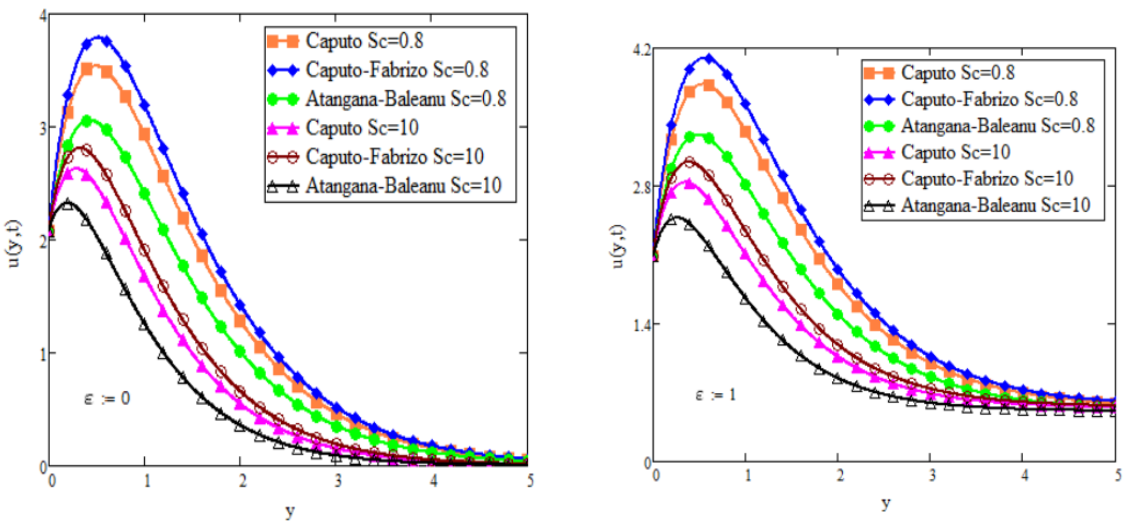


Figure 6: Velocity profile against y due to Sc, where the values of other parameters are Gr=5, R=0.9, t=0.5, K=1.4, γ=0.5, M=0.5, Gm=6, Q=0.25, Pr=6.5, $\lambda_1 = 0.4$, $\lambda_2 = 0.2$.

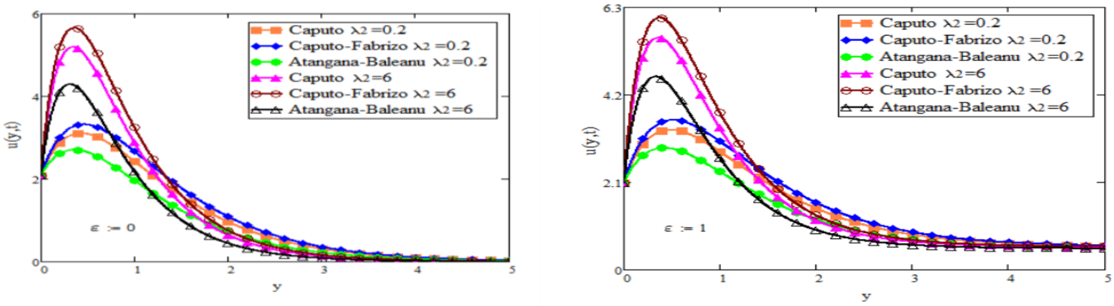


Figure 7: Velocity profile against y due to λ_2, where the values of other parameters are Gr=5, R=0.9, t=0.5, K=1.4, γ=0.5, M=0.5, Gm=6, Q=0.25, Pr=6.5, λ_1 = 0.4, Sc= 0.2.

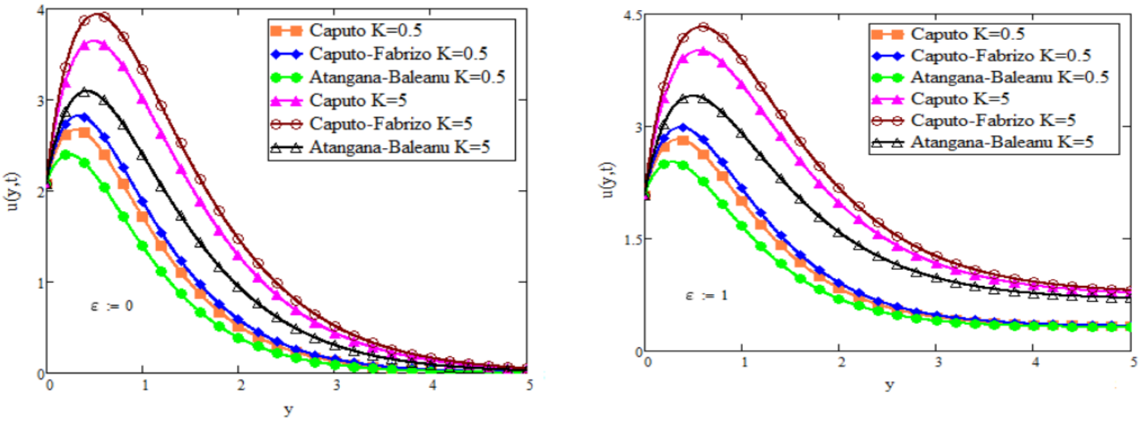


Figure 8: Velocity profile against y due to K, where the values of other parameters are Gr=5, R=0.9, t=0.5, λ_2 =0.2, γ=0.5, M=0.5, Gm=6, Q=0.25, Pr=6.5, λ_1 = 0.4, Sc= 0.2.

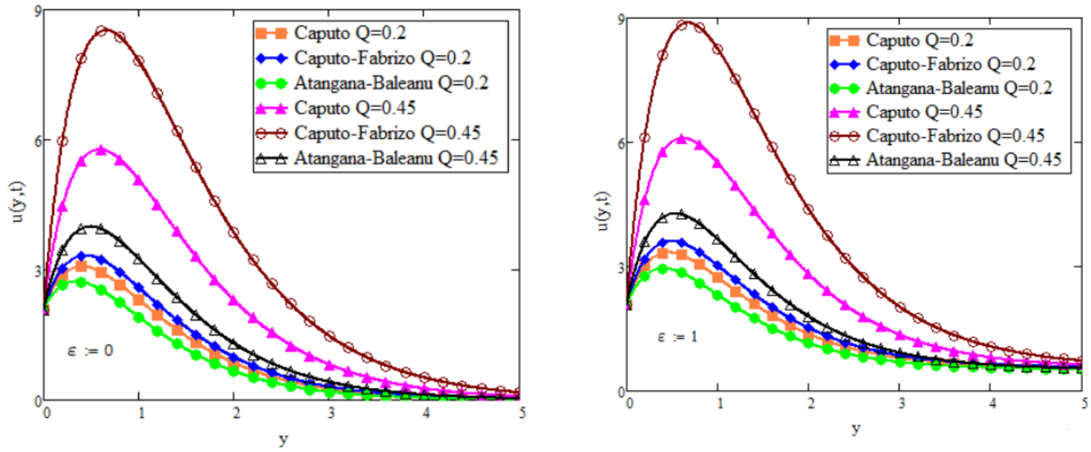


Figure 9: Velocity profile against y due to Q, where the values of other parameters are Gr=5, R=0.9, t=0.5, λ_2 =0.2, γ=0.5, M=0.5, Gm=6, Q=0.25, Pr=6.5, λ_1 = 0.4, Sc= 0.2.

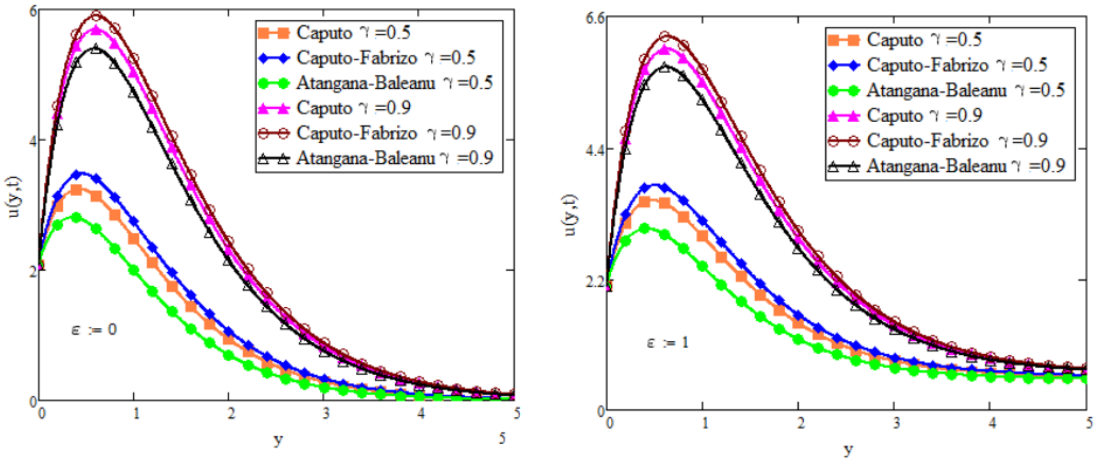


Figure 10: Velocity profile against y due to γ, where the values of other parameters are Gr=5, R=0.9, t=0.5, λ_2 =0.2, Q=0.25, M=0.5, Gm=6, Q=0.25, Pr=6.5, λ_1 = 0.4, Sc= 0.2.

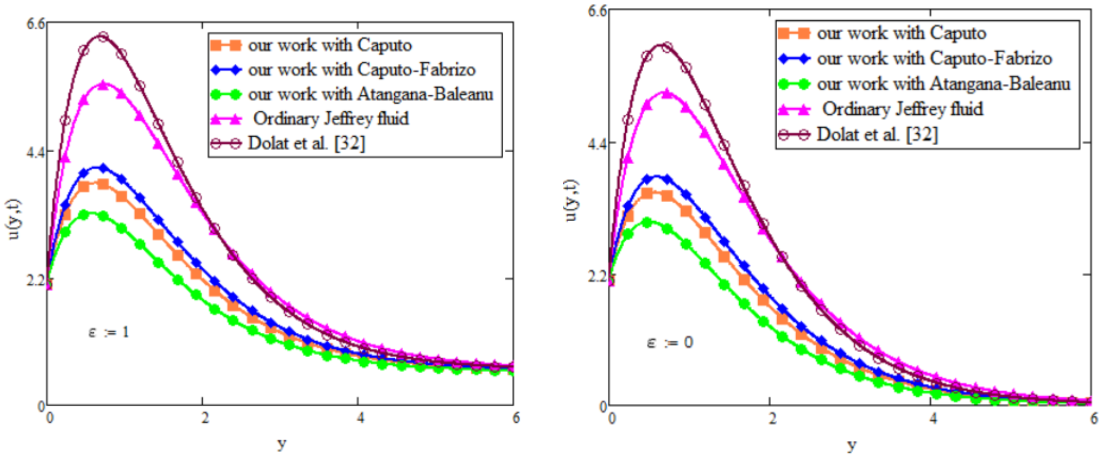


Figure 11: Comparison of velocity profile

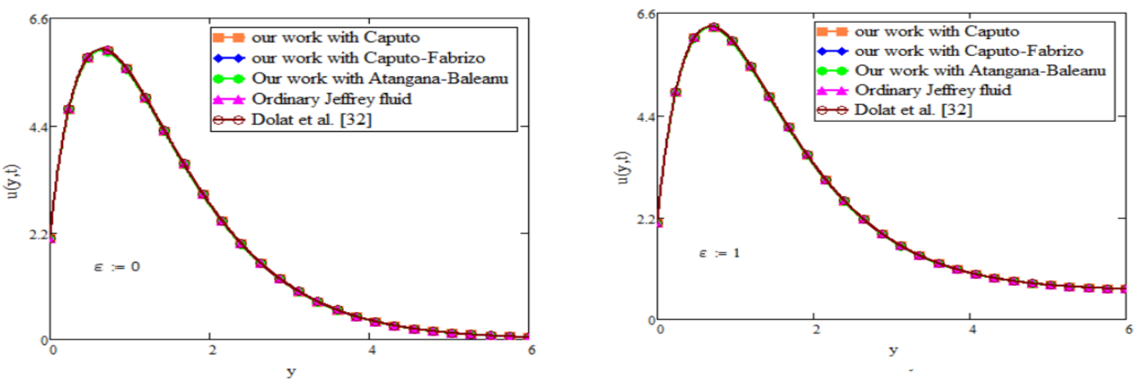


Figure 12: Comparison of velocity profile with $\gamma \longrightarrow 1$

5. Results and discussion

The solution for the influence of relative magnetic field, heat sink/source and thermal radiation on magnetohydrodynamic flow of Jeffrey's fluid through an exponentially accelerated plate in the existence of porosity are developed by using Laplace transform technique. The influence of numerous parameters used in the governing equations of velocity fields have analyzed in Figures.

Figure. 1 represent the influence of Gr on fluid velocity u(z,t). The fluid motion rises up with the maximizing values of Gr. physically, Gr signify the effect of thermal buoyancy force to viscous force. Therefore maximizing the values of Gr exceed the temperature gradient due to which velocity field rises. The impact of Gm on fluid velocity u(z,t) is depicted in Figure. 2. It is highlighted that velocity profile increases with increasing values of Gm. Physics behind this is that higher the values of Gm increase the concentration gradients which make the buoyancy force significant and hence rise in velocity field is observed. The impact of Pr on fluid velocity $u(z,t)$ is depicted in Figure.3. The velocity profile shows decreasing behavior with maximizing values of Pr due to thermal boundary layer. An increasing value of R decreases the fluid velocity as shown in Figure.4. Fluid velocity falls down for greater values of M as depicted in Figure.5. Physically, increasing values of M, exceeds the drag force which decreases the fluid motion. The effect of relative magnetic field due to plate and the fluid respectively is also shown graphically. The velocity profiles for magnetic field due to plate are higher as compared to the magnetic field due to fluid.

The influence of Sc on fluid motion is depicted in Figure. 6. It is observed that maximize the value of Sc slow down the fluid motion due to decay of molecular diffusion. Figure. 7 indicate the impact of λ_2 on velocity field u(z,t). Figure. shows that the motion of fluid rises by rising the value of λ_2. The influence of porosity K on fluid is shown in Figure.8. It is clear from Figure. that fluid motion rises with maximizing the value of K. The influence of heat generation/absorption on u(z,t) is depicted in Figure.9. It is analyzed that motion of fluid increases with an increasing values of fractional parameter γ as depicted in Figure. 10. Figure.11 represent the comparison of velocity distribution of present work with ordinary Jeffrey's fluid and Dolat and his colleagues [32]. It is clear from Figure. that Atangana-Baleanu fractional derivative is best choice for controlled velocity field. The comparison of velocity profile of present work is made with Dolat and his colleagues [32]. If we take fractional parameter $\gamma \to 1$, $\lambda_1 = \lambda_2 = 0$ of present work and Casson parameter $1/\beta \to 0$ of Dolat and his colleagues [32], then the both results are identical which shows the validation of our results.

6. Conclusion

An analytical solution for free convection MHD Jefrey's fluid flow through a vertical plate in the existence of first order chemical reaction and thermal radiation subject to newtonian heating condition is analyzed with the help of Caputo, Caputo-Fabrizio, and Atangana-Baleanu fractional derivatives. Laplace transform solutions for dimensionless energy, momentum, and concentration equations have been obtained. The expressions for Skin friction, Sherwood number and Nustle number are also studied. The graphs for various parameters used in the governing equation are plotted and discussed. From this work, we arrive at the following conclusions

• Velocity of fluid is higher with Caputo-Fabrizio derivative as compared to Caputo Fractional derivative, whereas Velocity of fluid is lower with Atangana-Baleanu derivative as compared to Caputo derivative,

• Velocity of fluid is higher, if magnetic field is fixed relative to plate,

• Velocity of fluid is lower, if magnetic field is fixed relative to fluid,

• Fluid velocity increases with increasing values of Gm, Gr, λ_2, Q, and fractional parameter γ,

• Fluid velocity decreases with increasing values of Pr, Sc, and chemical reaction parameter R.

7. Appendix

$$b_1 = \frac{\gamma}{(1-\gamma)},\quad b_2 = \frac{Sc+ScR(1-\gamma)}{(1-\gamma)},\quad b_3 = \frac{\gamma RSc}{Sc+ScR(1-\gamma)},\quad b_4 = \frac{\lambda}{\sqrt{Pr}},\quad b_5 = Q + b_4^2,$$

$$b_6 = \frac{b_4[(-Q)^{\frac{3}{2}}-b_4Q]}{-b_5}, b_6^0 = \frac{b_4[(b_5-Q)^{\frac{3}{2}}+b_4(b_5-Q)]}{b_5}, b_7 = \frac{Pr-PrQ(1-\gamma)}{(1-\gamma)}, b_8 = \frac{\gamma QPr}{Pr-PrQ(1-\gamma)},$$

$$b_9 = b_7 - \lambda^2, b_{10} = \frac{b_7b_8+\lambda^2b_1}{b_9}, b_{11} = \frac{\lambda^2b_1+\lambda\sqrt{-b_1b_7b_8}}{-b_{10}}, b_{12} = \frac{\lambda^2(b_{10}+b_1)+\lambda\sqrt{(b_1+b_{10})(b_{10}-b_8)b_7}}{b_{10}},$$

$$b_{13} = \frac{1}{1+\lambda_1},\quad b_{14} = \lambda_2 b_{13},\quad b_{15} = \frac{Grb_4(\sqrt{m_1-Q}+b_4)}{(m_1-m_2)(m_1-b_5)},\quad b_{16} = \frac{Grb_4(\sqrt{m_2-Q}+b_4)}{(m_2-m_1)(m_2-b_5)},$$

$$b_{17} = \frac{Grb_4(\sqrt{b_5-Q}+b_4)}{(b_5-m_1)(b_5-m_2)}, b_{18} = \frac{Grb_4(m_1-Q)(\sqrt{m_1-Q}+b_4)}{(m_1-m_2)(m_1-b_5)}, b_{19} = \frac{Grb_4(m_2-Q)(\sqrt{m_2-Q}+b_4)}{(m_2-m_1)(m_2-b_5)},$$

$$b_{17} = \frac{Grb_4(b_5-Q)(\sqrt{b_5-Q}+b_4)}{(b_5-m_1)(b_5-m_2)},\quad b_{21} = \frac{Gm}{m_3-m_4},\quad b_{22} = \frac{Gm}{m_4-m_3},\quad b_{23} = \frac{Gm(m_3+R)}{m_3-m_4},$$

$$b_{24} = \frac{Gm(m_4+R)}{m_4-m_3},\quad c_1 = \frac{b_{13}}{b_{14}},\quad b_{25} = \frac{1+H(1-\gamma)}{(1-\gamma)},\quad b_{26} = \frac{H\gamma}{1+H(1-\gamma)},\quad b_{27} = \frac{b_{14}+b_{13}(1-\gamma)}{(1-\gamma)},$$

$$b_{28} = \frac{b_{13}\gamma}{b_{14}+b_{13}(1-\gamma)},\quad b_{29} = \frac{b_{25}}{b_{27}},\quad b_{30} = \frac{M\epsilon}{b_{25}},\quad b_{31} = b_{30}(b_1 - b_{26}),\quad b_{32} = \frac{b_{30}}{a+b_{26}},$$

$$b_{33} = \frac{Grb_1^2[\lambda^2b_1+\lambda\sqrt{-b_1b_7b_8}]}{-b_9b_{10}n_1n_2},\quad b_{34} = \frac{Gr(n_1+b_1)^2[\lambda^2(n_1+b_1)+\lambda\sqrt{(n_1+b_1)(n_1-b_8)b_7}]}{b_9(n_1-b_{10})n_1(n_1-n_2)},$$

$$b_{35} = \frac{Gr(n_2+b_1)^2[\lambda^2(n_2+b_1)+\lambda\sqrt{(n_2+b_1)(n_2-b_8)b_7}]}{b_9(n_2-b_{10})n_2(n_2-n_1)},\quad b_{37} = \frac{Gmb_1^2}{n_3n_4},\quad b_{38} = \frac{Gm(n_3+b_1)^2}{n_3(n_3-n_4)},$$

$$b_{36} = \frac{Gr(b_{10}+b_1)^2[\lambda^2(b_{10}+b_1)+\lambda\sqrt{(b_{10}+b_1)(b_{10}-b_8)b_7}]}{b_9(b_{10}-n_1)b_{10}(b_{10}-n_2)},\quad b_{39} = \frac{Gm(n_4+b_1)^2}{n_3(n_4-n_3)},$$

$$b_{40} = \frac{Gr(n_1+b_1)^2[\lambda^2(n_1+b_1)+\lambda\sqrt{(n_1+b_1)(n_1-b_8)b_7}]}{b_9(n_1-b_{10})(n_1-n_2)},\quad b_{43} = \frac{Gm(n_3+b_1)}{n_3-n_4}, c_1 = \frac{b_{13}}{b_{14}}$$

$b_{41} = \frac{Gr(n_2+b_1)^2[\lambda^2(n_2+b_1)+\lambda\sqrt{(n_2+b_1)(n_2-b_8)b_7}]}{b_9(n_2-b_{10})(n_2-n_1)}, b_{44} = \frac{Gm(n_4+b_1)}{n_4-n_3},$

$b_{42} = \frac{Gr(b_{10}+b_1)^2[\lambda^2(b_{10}+b_1)+\lambda\sqrt{(b_{10}+b_1)(b_{10}-b_8)b_7}]}{b_9(b_{10}-n_1)(b_{10}-n_2)}, D_1 = \sqrt{\frac{-b_7b_8}{b_1}}, D_2 = \sqrt{\frac{b_7(b_{10}-b_8)}{(b_1+b_{10})}},$

$D_3 = \frac{1}{m_1\sqrt{b_{14}}}\sqrt{\frac{(m_1+H)}{(m_1+c_1)}},\ D_4 = \frac{1}{m_2\sqrt{b_{14}}}\sqrt{\frac{(m_2+H)}{(m_2+c_1)}}, D_5 = \frac{1}{b_5\sqrt{b_{14}}}\sqrt{\frac{(b_5+H)}{(b_5+c_1)}},\ D_6 = \frac{1}{m_3\sqrt{b_{14}}}\sqrt{\frac{(m_3+H)}{(m_3+c_1)}}$

$, D_7 = \frac{1}{m_4\sqrt{b_{14}}}\sqrt{\frac{(m_4+H)}{(m_4+c_1)}},\ D_8 = \sqrt{\frac{(b_{26}+a)}{(b_{28}+a)}},\ D_9 = \sqrt{\frac{1}{(a+b_{26})(a+b_{28})}},$

$D_{10} = \frac{b_{26}b_{29}}{b_{28}},\ D_{11} = \sqrt{\frac{(n_1+b_{26})b_{29}}{(n_1+b_{28})}},\ D_{12} = \sqrt{\frac{(n_2+b_{26})b_{29}}{(n_2+b_{28})}},\ D_{13} = \sqrt{\frac{(b_{10}+b_{26})b_{29}}{(b_{10}+b_{28})}},$

$D_{14} = \sqrt{\frac{(n_3+b_{26})b_{29}}{(n_3+b_{28})}},\ D_{15} = \sqrt{\frac{(n_4+b_{26})b_{29}}{(n_4+b_{28})}},\ D_{16} = \sqrt{\frac{(n_1-b_8)b_7}{(n_1+b_1)}},\ D_{17} = \sqrt{\frac{(n_2-b_8)b_7}{(n_2+b_1)}},$

$D_{18} = \sqrt{\frac{(n_3+b_3)b_2}{(n_3+b_1)}},\ D_{19} = \sqrt{\frac{(n_4+b_3)b_{22}}{(n_4+b_1)}},$

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