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Study of Peristaltic Flow of Blood in Artery

Dr. Smita Dey[a*], Anamol Kumar Lal[b]

[a]*University department of Mathematics, Morabadi Campus, Basic Science Building,Ranchi University, Ranchi – 834008, Jharkhand, India*

[b]*Department of Mathematics, Marwari College, Lake Road, Ranchi – 834001, Jharkhand, India*

[a]*Email :smitadey2000@yahoo.com*

[b]*Email :myid.anmol@gmail.com*

Abstract

In the functioning of ureters, intestines, esophagus and in medical instruments such as the heart-lung machine, fluid is transported by a unique process, arteries of contractions of the wall propagates along the length of the tube causing fluid to be transported in the direction of the wave. This is called peristaltic pumping. The problem of peristaltic transport in two dimensional channels and in axisymmetric tubes has received considerable attention. In the paper, we have studied the effect of asymmetric Peristalsis on blood flow through an artery with stenosis. We know that blood can be taken as a Newtonian fluid as well as a Non-Newtonian fluid. Here, we have considered the blood as a Newtonian and Non Newtonian fluid. Here, the basic equation is taken as Navier-stokes equation and we have transformed the stationary co-ordinates to moving Co-ordinates. The frictional force and the expression for pressure rise have been obtained.

Keywords:Peristalsis; Newtonian fluid; Non-Newtonian fluid

Nomenclature

C	=	Wave Propagation velocity.
h	=	Mean radius of the channel.
λ	=	Wavelength.
$\bar{q}$	=	Dimensionless volume flow.
p	=	Pressure.
b_1	=	Amplitude of the Peristaltic wave on upper wall.
b_2	=	Amplitude of the Peristaltic wave on lower wall.
$\varepsilon_1, \varepsilon_2$	=	Dimensionless Amplitude.

[*]Dr. Smita Dey. Tel.: +91-9835138838
E-mail Address : smitadey2000@yahoo.com

1.Introduction

Pumping of fluids through flexible tubes by means of the peristaltic wave motion of the tube wall has been the subject of engineering and scientific research for over four decades. Engineers and physiologists term the phenomenon of such flow as peristalsis. It is a form of induced by a progressive wave of area contraction or expansion along the walls of a distensible duct containing a liquid or mixture. Besides its various engineering applications (e.g., heart-lung machines, finger and roller pumps, etc.), it is known to be significant mechanism responsible for fluid transport in many biological organs including in swallowing food through esophagus, urine transport from kidney to bladder through the ureter, movement of chyme in gastrointestinal tract, transport of spermatozoa in the ductusefferentes of the male reproductive tracts, and, in cervical canal, in movement of ovum in the female fallopian tubes, transport of lymph in lymphatic vessels, and in the vasomotion of small blood vessels such as arterioles, venules and capillaries. Recently Tripathi & Srinivas [1,2] studied asymmetric Peristalsis for heat transfer.

Latham [3] was probably the first to study the mechanism of peristaltic pumping in his M.S. Thesis. Shapiro et al. [4] and Jaffrin and Shapiro [5] explained the basic principles and brought out clearly the significance of the various parameters governing the flow. The literature on the topic is quite extensive by now and a review of much of the literature up to the year 1983, arranged according to the geometry, the fluid, the Reynolds number, the wave number, the amplitude ratio and the wave shape was presented in an excellent article by Srivastava and Srivastava [6].

The significant contributions to the subject between the years 1984 and 1994 were well referenced by Srivastava and Saxena [7]. The important studies of the recent years include the investigations of Srivastava and Srivastava [6], Mekheimer and his colleagues [8], El-Shehawey and Husseny[11], El-Hakeem and El-Misery [10], Muthuet and his colleagues [9], Mishra and Rao [12], Medhavi[13],Medhavi and Singh (12) and a few others.

In both the mechanical and physiological situations, most of the studies conducted in the literature considered the transported fluid to be a Newtonian fluid. It is well accepted that a large number of fluids of practical importance behave like a non-Newtonian fluid also. The present article deals with the flow of a Newtonian and Non- Newtonian fluid.

1.1 Basicequations and formulation of the problem (for Newtonian Flow):

Let the Equation of the tube surface (Fig. 1) be given by:-

$$h(z,t) = a\left[1 + \in \cos\left\{\left(\frac{2\pi}{\lambda}\right)(z - ct)\right\}\right]$$

Where a = undisturbed radius and $\in$ = the amplitude ratio.

Blood is taken as a Newtonian fluid with the assumptions of infinite wave length and inertia-free flow, the Navier-stokes equation reduces to:

$$\frac{dp}{dz} = \mu \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

The boundary conditions are:

$$u = -c \text{ at } \quad y = h + b_1.Cos\left(\frac{2\pi}{\lambda}.z\right) \;;\quad u = -c \text{ at } \quad y = -h - b_2.Cos\left(\frac{2\pi}{\lambda}.z\right) \qquad (2)$$

Where c is the wave propagation velocity, h is the mean radius of the channel, b_1 and $b_2$are the amplitudes of the peristaltic wave on the upper and lower walls respectively. λ is the wavelength.

Let us transform the stationary Co-ordinates to moving Co-ordinates as

z = Z - c.t, y = Y And u(z,y) = U(Z - c.t, Y) – c

for velocity component in the laboratory frame U

Integrating equation (i). We get

$$\frac{u}{c}=-1+\frac{h^2}{2.\mu.c}.\frac{dp}{dz}\left[\left(\frac{y}{h}\right)^2-(\in_1-\in_2)Cos2\pi\bar{z}\frac{y}{h}-\left(1+\in_1 Cos2\pi z\right).\left(1+\in_2 Cos2\pi z\right)\right] \qquad (3)$$

Where $\in_1=\frac{b_1}{h},\ \in_2=\frac{b_2}{h},\ Z=\frac{z}{\lambda},\qquad \in_1<1, \in_2<1$

The rate of volume flow through each section 'q' is given by

$$q=\int_{y_1}^{y_2} udy \qquad q=-cH-\frac{H^3}{12\mu}\frac{dp}{dz} \qquad \text{where} \quad H=2h+(b_1+b_2).Cos\frac{2\pi}{\lambda}z$$

$\bar{q}$ = dimensionless volume flow = $\frac{q}{2hc} = -\frac{H}{2h}-\frac{H^3}{24\mu c}\frac{dp}{dz}$ *(4)*

The instantaneous volume flow rate

i.e. $$Q(z,t)=\int_{y_1}^{y_2}(u+c)dy = q+cH \qquad (5)$$

The time-mean volume flow at each cross section is

$$\bar{Q} = \frac{1}{T}\int_0^T Qdt = q+2hc+(b_1+b_2).\frac{C}{T}\int_0^T Cos\frac{2\pi}{\lambda}(z-ct).dt = q+2hc$$

The pressure change per wavelength is

$$\Delta_{P_\lambda}=-\int_0^\lambda \frac{dp}{dz}.dz \quad \Rightarrow \frac{h^2}{12\mu c\lambda}\Delta_{p_\lambda}=\frac{(\in_1+\in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}}.\left[-3+\left\{\frac{8}{(\in_1+\in_2)^2}+1\right\}.\theta\right] \qquad (6)$$

Where $\theta=\frac{Q}{2hc}$

The dimensionless pressure rise $\left(\Delta_{P_\lambda}\right)_{\theta=0}$ for

Zero time -mean flow and the dimensionless time mean flow θ_0 for the zero pressure rise are given by

$$\frac{h^2}{12\mu c\lambda}\left(\Delta_{p_\lambda}\right)_{\theta=0}=\frac{-3(\in_1+\in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}} \qquad (7)$$

and $$\theta_0 = \frac{3(\in_1 + \in_2)^2}{8 + (\in_1 + \in_2)^2} \tag{8}$$

Frictional force at the wall a cross one wave length

i.e. $F' = -\int_0^{\lambda} \left(\frac{H}{2}\right)^2 . \frac{dp}{dx}.dx = \frac{6\mu c\lambda q}{\sqrt{4-(\in_1+\in_2)^2}} + 3\mu c\lambda, \qquad F = \frac{F'}{\mu\lambda c} = \frac{6\bar{q}}{\sqrt{4-(\in_1+\in_2)^2}} + 3$

Dimensionless frictional force i.e.

Resistance to flow $$R = \frac{P_0 - P_\lambda}{\theta} = \frac{12\mu c\lambda}{h^2\theta} \frac{(\in_1+\in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}} . \left[-3 + \left\{\frac{8}{(\in_1+\in_2)^2} + 1\right\}\theta\right] \tag{9}$$

When there is no. Peristaltic wave, then $$R_0 = \frac{3\mu c\lambda}{h^2} \tag{10}$$

Dimensionless Resistance to flow is

$$\bar{R} = \frac{R}{R_0} = \frac{-4(\in_1+\in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}} . \left[\frac{3}{\theta} - \left\{\frac{8}{(\in_1+\in_2)^2} + 1\right\}\right] \tag{11}$$

1.2 For Non-Newtonian Flow

The geometry of the wall surface {figure (vi) } is described as

$$H(x', t') = a(x') + b.Sin\frac{2\pi m}{\lambda}(x' - ct') \tag{12}$$

With $$a(x') = a_0 + Kx' \tag{13}$$

Where, $a(x')$ is the radius of the tube at any axial distance from inlet, a_0 is the radius of the inlet, K is a constant whose magnitude depends on the length of the tube, exit and inlet dimension, and b is the amplitude of the wave. Assuming that Wave number α and Re are very small; appropriate Navier-Stokes equations describing the flow are

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r.\left(\frac{\partial u}{\partial r}\right)^n\right] \tag{14}$$

and $$\frac{\partial p}{\partial r} = 0 \tag{15}$$

When a no slip condition is imposed as a boundary condition for the U velocity the particle is assumed to move only in the radial direction along the boundary.

$$\frac{\partial u}{\partial r} = 0 \quad \text{at} \quad r = 0 \tag{16}$$

$$u = 0 \quad \text{at} \quad r = h = 1 + \frac{\lambda kx}{a_0} + \phi Sin2\pi m(x-t) \tag{17}$$

Where $h = \frac{H}{a_0}; \phi = \frac{b}{a_0} < 1$

Solving equationsafter applying the boundary condition,

The expression for velocity profileis

$$u(x,r,t) = \frac{n}{n+1}\left(-\frac{1}{2}\frac{dp}{dx}\right)^{\frac{1}{n}}\left[h^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right] \tag{18}$$

The instantaneous volume flow rate $Q(x,t)$ is obtained as

$$Q(x,t) = \int_0^h 2\pi r u dr = \frac{\pi n}{3n+1}\left(\frac{-1}{2}\frac{dp}{dx}\right)^{\frac{1}{n}} h^{\frac{3n+1}{n}} \tag{19}$$

$$\Rightarrow \frac{dp}{dx} = -2\frac{Q^n(x,t)\left(\frac{3n+1}{n\pi}\right)^n}{h^{3n+1}} \tag{20}$$

The Pressure rise $\Delta P_L(t)$ in the tube of length L in the non-dimensional form is given by

$$\Delta P_L(t) = \int_0^{L/\lambda} \frac{dp}{dx} dx$$

$$= -2\left(\frac{3n+1}{n\pi}\right)^n \int_0^{L/\lambda} \frac{Q^n(x,t)}{\left[1 + \frac{\lambda kx}{a_0} + \phi Sin2\pi m(x-t)\right]^{3n+1}} .dx$$

$$\Rightarrow \left(\frac{n}{3n+1}\right)^n \Delta P_L(t) = -2\int_0^{L/\lambda} \frac{\left[\frac{Q(x,t)}{\pi}\right]^n}{\left[1 + \frac{\lambda kx}{a_0} + \phi .Sin2\pi m(x-t)\right]^{3n+1}} dx \tag{21}$$

The above equation gives the Pressure flow rate relationship. The frictional force $F_L(t)$ at the wall in the channel of length L in their Non dimensional forms are given by:

$$\Rightarrow\left(\frac{n\pi}{3n+1}\right)^{n} F_L(t) = +2\int_0^{L/\lambda} \frac{Q^n(x,t)}{\left[1+\frac{\lambda kx}{a_0}+\phi Sin2\pi m(x-t)\right]^{3n}}dx \tag{22}$$

2. Results and Discussion

2.1 For Newtonian Flow

In orderto see the effects of various parameters on friction, resistance to flow etc, the following values of the parameters are taken

Table 1

ε_1	0	0.1	0.2	0.3	0.4	0.5	0.6
ε_2	0	0.1	0.2	0.3	0.4	0.5	0.6
$\bar{q}$	0.5	0.55					
θ	0.5	0.1					

It has been observed that

1) On increasing the value of $\bar{q}$ from .5 to .55, there is an increase in the value of F with ε_1 [(fig (2)].
2) On increasing the value ofθ from .5 to 1, there is increase in the value of $\bar{R}$with ε_2 [fig (3)].
3) On decreasing the value of ε_1+ $\varepsilon_2$from 1 to 0.1, F also decreases with $\bar{q}$[fig (4)].
4) On increasing the value of ε_2 from .1 to .2, the value of $\theta_0$increases with $\varepsilon_1(\Delta P_\lambda = 0)$[fig (5)].

2.2 For Non-Newtonian Flow

In order to see the effect of various parameters on friction, pressure rise etc, the following values of the parameters are taken:

Table 2

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\bar{Q}$	0	0.2	0.4	0.6	0.8	1	1.2	1.4			
a_0	0.012 cm										
m	2										
L	λ = 20 cm										
K	$3a_0/L = 0.018$										

Equation (21) gives the formulas for pressure change and
Equation (22) gives the formulas for frictional force change and
Here; for numerical integration, Gaussian quadrature formula has been used. Here, it has been assumed that

$$Q^n(x,t) = (\overline{Q})^n + \phi Sin2\pi m(x-t)$$

where $\overline{Q}$ is the time average of the flow over one period of the wave. This form of $Q(x,t)$ has been assumed due to the fact that constant value of $Q(x,t)$ gives $\Delta P_L(t)$ always negative and hence, there will be no pumping action.
It has been observed that

1) On increasing the value of *n* from $\frac{1}{3}$ to $\frac{2}{3}$,there is a decrease in the value of ΔP_L with increasing value of time (as time increase). (fig 7)

2) On increasing the value of *n* from $\frac{1}{3}$ to $\frac{2}{3}$,there is an increase in the value of ΔP_L with increasing value of $\overline{Q}$ (as $\overline{Q}$ increase). (fig 8)

3) When value of amplitude ratio changes from $\phi = 0$ to $\phi = .7$ then at $n = \frac{1}{3}$; F_L increases with increasing value of $\overline{Q}$. (fig 9)

4) When $n = \frac{1}{3}$, then the frictional forces F_L changes much more when $\phi = 0$ compared to $\phi = .7$. (as time increase) (fig 10)

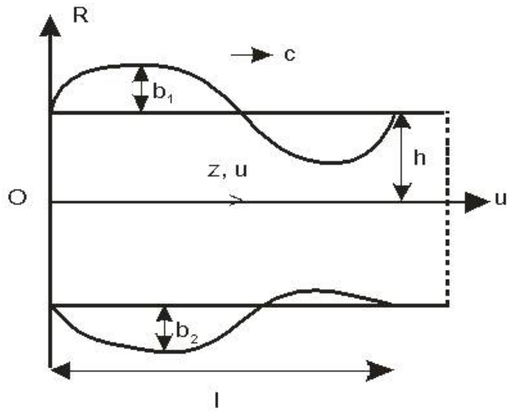


Fig. 1. Peristaltic Flow through a Channel (Newtonian)

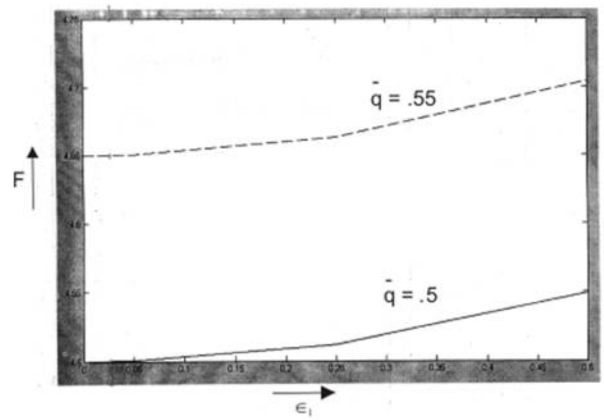


Fig. 2. Variation of F with ϵ_1 for different $\bar{q}$, $\epsilon_2 = 0$

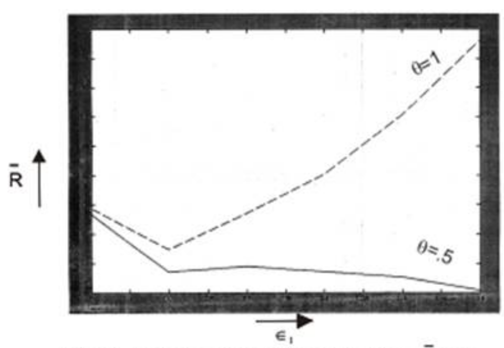


Fig. 3. Variation of Resistance to Flow $\bar{R}$ with ϵ_1 for different θ; $\epsilon_2 = 0$

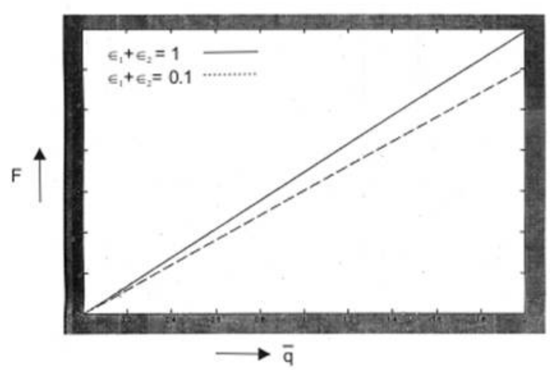


Fig. 4. Variation of F with $\bar{q}$ for different $\epsilon_1 + \epsilon_2$

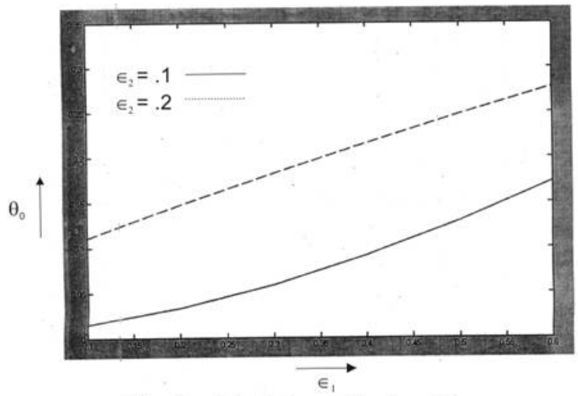
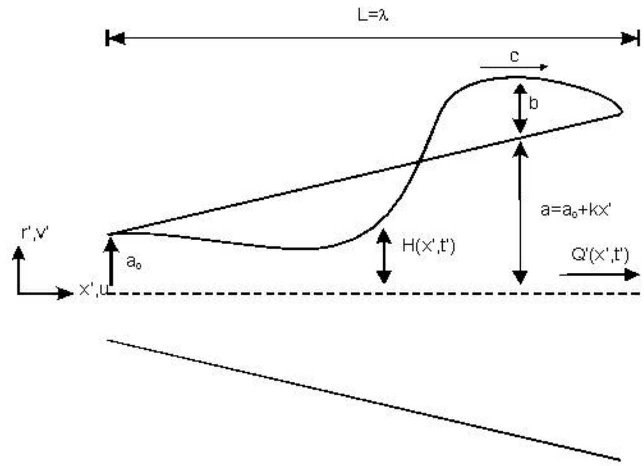


Fig. 5. Variation of θ_0 with $\in_1$



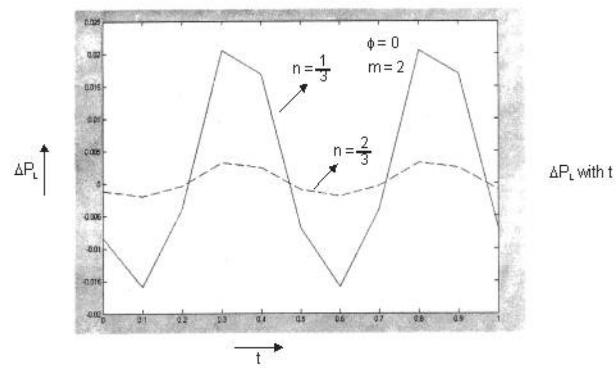


Fig. 7 .Variation of ' Δ_{P_L} ' with 't' for different '*n*'

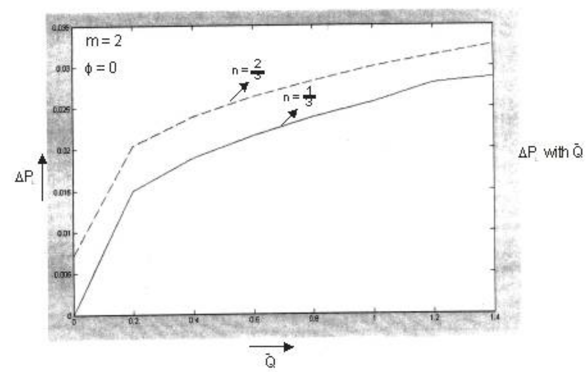


Fig. 8. Variation of ' Δ_{P_L} ' with ' $\bar{Q}$ ' for different '*n*'

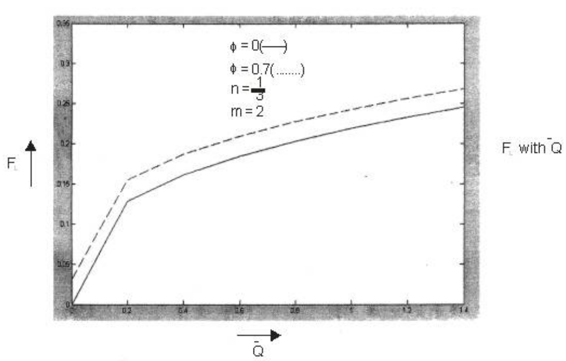


Fig. 9. Variation of ' F_L ' with ' $\bar{Q}$ ' for different ' $\bar{\phi}$ '

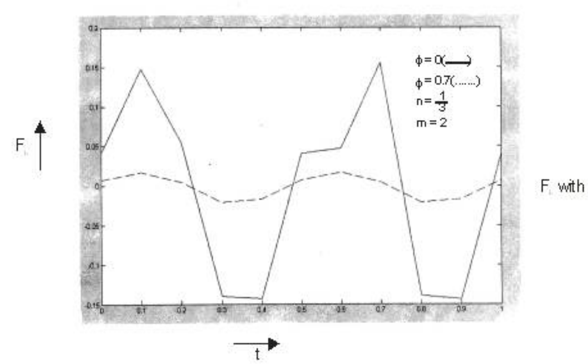


Fig. 10. Variation of ' F_L ' with '*t*' for different ' $\bar{\phi}$ '

3. Conclusions:

A Newtonian and Non Newtonian fluid model have been applied & study of the flow induced by means of sinusoidal peristaltic waves using long wavelength approximation has been taken into account.Comparison can be made for particular cases (Bingham, Power law, Newtonian fluids) both qualitatively and quantitatively, pressure drop increases with the flow rate and yield stress. Frictional force possesses the similar character with flow rate. It is strongly believed that the results of the analysis may be applied to discuss the peristaltic flow of blood.
The improvement can be done by introducing the elasticity parameter in the governing equation, it will be then more realistic work.

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