Modelling Airport Efficiency With Distributions Of The Inefficient Error Term: An Application Of Time Series Data For Aircraft Departure Delay

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Abstract

The study employs determinants of the aircraft departure delay to estimate airport efficiency. Two main parameters were applied to fit the stochastic frontier model using transcendental logarithmic function where both frontier and inefficiency models were generated. The estimated airport efficiencies over a period of 1827 days applying the half-normal and exponential distributions for the inefficiency error terms were (0.7498; δ=0.1417, n=1827) and (0.8181; δ=0.1224, n=1827) respectively. The correlation coefficient for the efficiency estimates (ρ=0.9791, n=1827, p<0.05) between the half-normal and exponential distributions showed no significant statistical difference.

Further analysis showed that airport inefficiency was significantly associated with higher number of persons on board, lower visibility level, lower air pressure tendency, higher wind speed and a higher proportion of arrival aircraft delays. The study offers a contribution towards assessing the dynamics for the distribution of inefficient error term to estimate airport efficiency by employing both meteorological and aviation parameters. The study recommends that although either half-normal or exponential distributions could be used; the exponential distribution for the error term was found more suitable when estimating the efficiency score for the airport.

Keywords: departure delay; stochastic frontier; airport efficiency; half-normal; exponential.

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1) Introduction

The main objective of the study was to measure, analyse and estimate airport efficiency. Entebbe International Airport was used as a case study. Stochastic frontier modelling approach was used to compute the efficiency of aircraft departures at an airport in a consistent way while also shedding light on the factors associated with aircraft departure delay [1]. The study sought to determine the most appropriate distribution [2] to be used when estimating airport efficiency between half-normal, exponential and truncated normal distributions. Investigations into the relationship between flight schedule punctuality and aircraft turnaround efficiency at airports were done [3] who recommended proper use of buffer time in maintaining schedule punctuality performance. Parametric and nonparametric approaches to measure productive efficiency for eight American and eight European Airlines were applied [4]. Other deterministic approaches have used data envelopment analysis that presupposes deterministic approach to determine efficiency ratios for European airports [5]. However, none of these studies have comprehensively considered aircraft departure delay as a determinant of airport efficiency and the dynamics for the distribution of inefficiency error term.

In this study, stochastic frontier modelling approach was employed so as to take care of the stochastic nature of the parameters in determining airport efficiency. Specifically, efficiency of the airport was tagged to assessing output dynamics of departure delay. The output at departure was the daily proportion of delay for aircraft departure. The availability of such information wassuch that it informs the design of policies that could improve the overall air traffic flow management and the efficiency of aircraft operations at the airport.

2) Methodological framework

Airport operational data used was collected from a flat database file of manifest aircraft operational and meteorological records comprising of 1827 daily records. The stochastic frontier model proposed by [6, 7] and extended by [8, 9] was employed to study the dynamics of aircraft departure delay towards assessing airport efficiency. Consider departing aircraft on a certain day denoted by i whose operational output is determined by the following production function:

$$lny_i = \beta lnx_i + \varepsilon_i \qquad (1)$$

Where

$$\varepsilon_i = vu_i - mu_i$$

$$i = 1,2,\dots,1827$$

y_i = airport departure delay for on the ith day

x_i = vector of explanatory variables

β = vector of unknown scalar parameters to be estimated

vu_i = idiosyncratic error term which is assumed to be independently and identically distributed as $N(0, \sigma_{vu}^2)$. The term captures random variation in output due to factors beyond control of the aircraft such as weather, measurement errors in dependent variables and the omitted explanatory variables.

mu_i = non-negative random variable, accounting for the existence of technical inefficiency and it is identically distributed as half-normal $mu \sim |N(0, \sigma^2)|)$

The inefficiency effect of mu_i was assumed to consist of both observed and unobserved systematic effects, which vary across aircrafts on different days. The subtraction of the nonnegative random variable mu_i, from the random error vu_i, implied that the logarithm of the proportion of departure delay was smaller than it would otherwise be if technical inefficiency did not exist. A variety of distributions such as exponential, truncated-normal and gamma may be used to characterize the technical inefficiency term (mu_i). While models that involve two-distributional parameters, for example gamma and truncated normal could accommodate a wider range of possible distributional

shape, their application appeared to come as a potential cost of increased difficulty in identifying parameters [10]. Different simulations exercises by [11] indicated that half-normal, the most straightforward model is more appropriate. However, our analysis on the factors affecting airport's efficiency was based on both half-normal and exponential models so as to establish the differences in the estimates.

In this study, generalized likelihood ratio tests were used to help confirm the functional form and specification of the estimated models. The correct critical values of the test statistic came from a χ^2 distribution at the 0.05 level of significance and a mixed χ^2 distribution [12]. The study employed the following transcendental logarithmic stochastic production function.

$$\ln(mY_i) = \beta_0 + \beta_1 \ln(mNOPS) + \beta_2 \ln(mVIS) + \beta_{11} \ln(mNOPS)^2 + \beta_{22} \ln(mVIS)^2 + \beta_{12} \ln(mVIS) \ln(mNOPS) + vu_i + mu_i \tag{2}$$

Where i indicates day of departure, ln the known natural logarithm and $\beta_{ij} = 0$ for all $i \leq j = 1,2$ thus the Cobb-Douglas production function. Symmetry has also been imposed by $\beta_{ij} = \beta_{ji}$ and inputs are number of operations and visibility. The variable mYrepresents an interaction between the month of operation and the proportion of aircrafts that delay to depart on a given day; mNOPSrepresents an interaction between the number of operations and month of operation at the airport per day; mVISrepresents an interaction between the average value of visibility and month of operation per day; β_s are unknown parameters to be estimated, vu_i are random stochastic disturbance terms and mu_i stands for the technical inefficiency term.

The study applied the following model to estimate determinants of airport technical inefficiency. The model was specified as:

$$mu_i = \delta_0 + \delta_1 POBout + \delta_2 QNH + \delta_3 AHPDRATE + \delta_4 WINDSPED \tag{3}$$

Where POBout represents total number of passengers on board of a departing aircraft, QNH is pressure in queen's nautical miles; AHPDRATE is the proportion of aircraft that delay to arrive and WINDSPED represents the daily average wind speed. All parameters are daily aggregates [1].

Equation (3) could only be estimated if the technical inefficiency effects, mu_i are stochastic and had particular distribution properties [13]. Therefore, the following null hypotheses were of interest: no airport technical inefficiency, hence, $\gamma = \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$ or airport technical inefficiency effects are non-stochastic; and the aircraft departure delay specific factors do not influence the airport technical inefficiencies$\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$. Under $\gamma = 0$, the stochastic frontier model reduces to a traditional average response function that is without inefficiency. Various tests of null hypotheses for parameters in the frontier functions as well as in the inefficiency model could be performed using generalized likelihood-ratio test statistic defined by;

$$\lambda = -2[\ln\{L(H_0)\} - \ln\{L(H_1)\}] \tag{4}$$

Where $L(H_0)$ and $L(H_1)$ represented the value of the likelihood function under the null H_0 and the alternative H_1 hypotheses, respectively. Thus, if the null hypothesis was true, the test statistic would have approximately a chi-square or a mixed chi-square distribution with the degree of freedom equal to the difference between parameters involved in the null and alternative hypotheses.

3) Results and discussions

The stochastic frontier and inefficiency models with the inefficiency model following half-normal and exponential distributions are presented in Table 1. In both models, the coefficients for most parameters had the anticipated effects on airport efficiency. In the frontier model, the coefficients of daily number of aircraft operations, squared number of aircraft operations are significant and have the anticipated positive signs which implied that any increase in the number of aircraft operations resulted into an increase in airport inefficiency. The coefficients for airport daily average visibility, though significant was negative implying that an increase in the airport's daily average visibility would result into a reduction in the airport's inefficiency.

Table 1: Parameter estimation for the stochastic frontier and inefficiency models when the error term follows half-normal and exponential distributions

Dependent: Departure delay	Half-Normal distribution		Exponential distribution	
a) Stochastic Frontier Model	Estimation	S.E	Estimation	S.E
Constant	24.6108***	1.7166	24.8949***	1.7628
Daily number of aircraft operations	1.0438***	0.1393	0.9787***	0.1402
Airport daily average visibility	-2.3335***	0.2768	-2.4459*	0.2838
Square of number of aircraft operations	0.0179**	0.0076	0.0208**	0.0077
Square of airport visibility	0.1625***	0.0116	0.1673***	0.0119
Interaction between aircraft operations and visibility	-1.3320***	0.0799	-1.2951***	0.0794
b) Inefficiency Model				
Constant	-2.9758**	1.0972	-4.7732**	1.6906
Persons on board of departing aircrafts	-0.0005***	0.0001	-0.0006***	0.0002
Queen's nautical height	0.0021*	0.0010	0.0029*	0.0016
Arrival delay	-0.0106***	0.0020	-0.0166***	0.0031
Wind speed	0.0302	0.0164	0.0458	0.0251

Note: *; **; *** indicate 0.1; 0.05 and 0.01 levels of significance respectively.

3.1   Determinants of technical inefficiency

In Table 1 coefficients of the explanatory variables of the technical inefficiency model are presented. As expected, the coefficient of daily number of persons on board is negative, which means that the busier the airport, the more likely is an aircraft departure delay. Thus, more aircrafts shall delay to depart as a result of airport congestion. Hence, the likelihood that an aircraft delays to depart was found to be influenced by accumulated possibilities of passengers' delays as demonstrated by its effect on increased number of persons on board of a departing aircraft (β = -0.0005, $p<0.01$). Similarly, the parameter arrival delay has a negative sign meaning that increased proportions of aircraft arrival delay directly affects the tendency for departure delays (β = -0.0106, $p<0.01$) on the airport inefficiency level.

3.2   Summary of efficiency at the airport

Findings show that the airport mean efficiency over the period of study for the exponentially distributed inefficient error term was only six percent greater than that when the error term was estimated to follow half-normal distribution. The third possible distribution for the error term, truncated normal distribution did not offer a feasible solution for this study, hence not presented. However, from the computations, the coefficients in Table 1 and based on standard deviations as presented in Table 2, the exponentially distributed inefficiency error term seemed to have provided a better efficiency estimate than the half-normal distribution under the same conditions.

Table 2: Efficiency comparison between half-normal and exponential distributions (N=1827)

Inefficiency term	Mean	Std. Dev.	Min	Max
Efficiency with half-normal distribution	0.7498	0.1417	0.2485	0.9680
Efficiency with exponential distribution	0.8181	0.1224	0.2624	0.9630

A non-parametric test [14] for trend of the daily airport efficiency scores generated from the two stochastic models in Table 1 over the five year study period was performed. Analysis of airport efficiency showed that the estimates when the inefficient term followed the exponential distribution were consistently greater than when the estimates were based on the half-normal distributions. Estimates from both models were not significantly different over the period and provided no possibility of intersection, Figure 1.

Airport departure efficiency trend

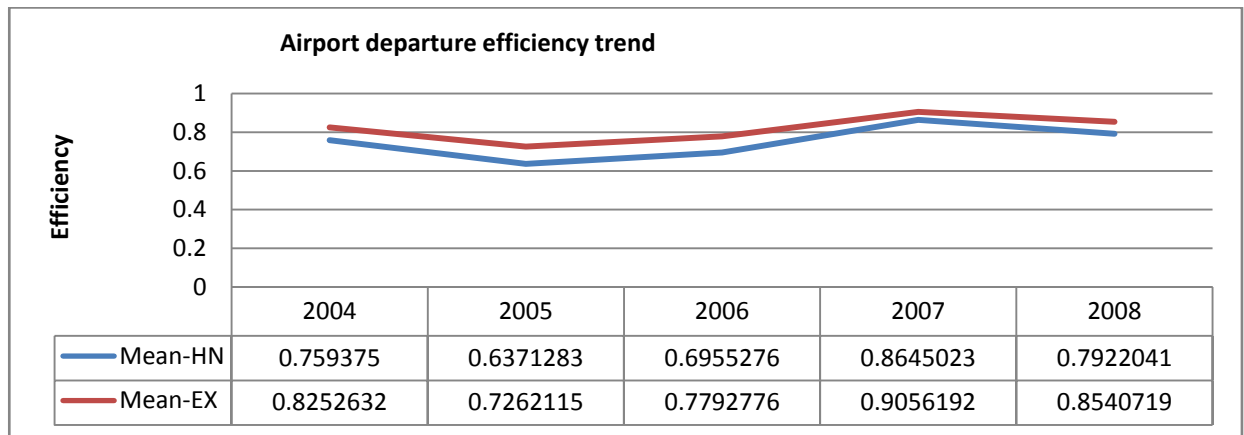|	2004	2005	2006	2007	2008
Mean-HN	0.759375	0.6371283	0.6955276	0.8645023	0.7922041
Mean-EX	0.8252632	0.7262115	0.7792776	0.9056192	0.8540719

Figure 1: Annualized airport departure efficiency for half-normal and exponential distributions for the period from 2004 to 2008.

Rank analysis disaggregated by year was performed to test for trend in efficiency scores. Figure 2 shows that the two annualized model estimates for half-normal and exponential revealed no considerable deviation. However, the airport efficiencies for the year 2005 had the least sum of ranks while the highest score was realized in the year 2007.
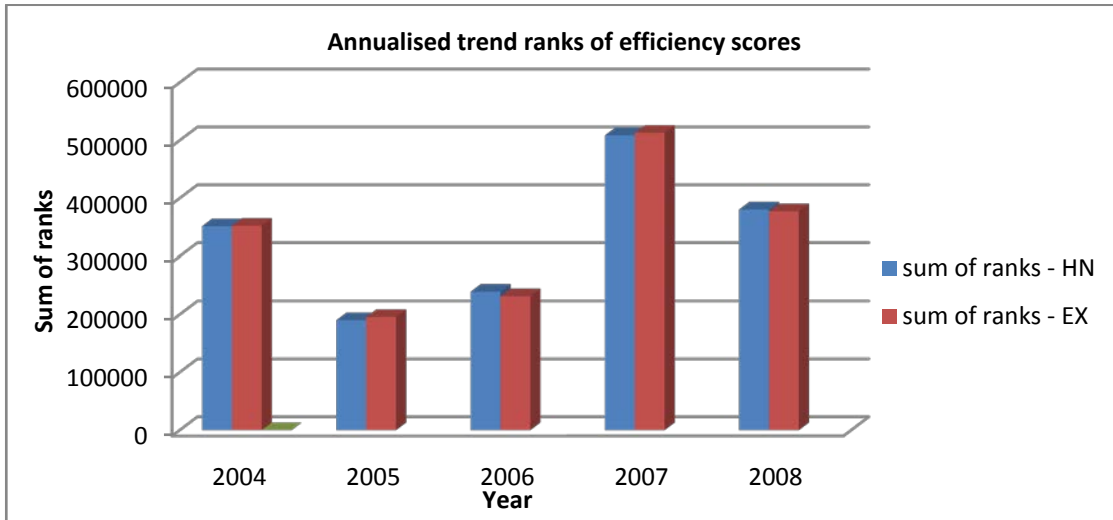
Figure 2: Annualized airport departure efficiency trend sum of ranks for half-normal (HN) and exponential (EX) distributions for the period from 2004 to 2008.

In Table 3, descriptive statistics were computed for each of variables interacted with month of the year for both the stochastic and inefficiency models.

Table 3: Descriptive characteristics of parameters in the frontier and inefficiency models (N=1827)

Stochastic frontier parameters (ln)	Mean	Std. Dev.	Min	Max
Month interaction with departure delay	6.973734	1.312651	2.302585	8.631414
Month interaction with visibility	12.23239	0.9026097	8.940236	13.29423
Month interaction with number of aircraft operations	6.977467	1.09385	2.564949	8.886824
Number of aircraft operations self- interaction	49.8809	14.60144	6.578965	78.97564
Month interaction with visibility self- interaction	150.4457	20.98274	79.92783	176.7366
Month interaction with number of aircraft operations and visibility	16.0653	1.099453	11.54613	17.94568
Inefficiency parameters				
Persons on board of a departing aircraft	1257.384	399.7012	130	3277
Queen's nautical height-Pressure	1033.922	34.5126	975	1098
Rate of arrival delay	48.21182	17.5628	6	100
Wind speed	5.441708	2.181841	2	9

3.3   Analysis of annualized airport efficiency

The aircraft departure delay concept towards airport efficiency estimation also sought to establish the behaviour of annualized efficiency at the airport. Figure 3 shows the variations from normal of the airport's efficiencies when the inefficient term follows half-normal distribution. The density plots showed that except for the year 2007, the efficiencies for the other years portray negatively skewed distributions, indicating a slight tendency towards higher airport efficiencies.
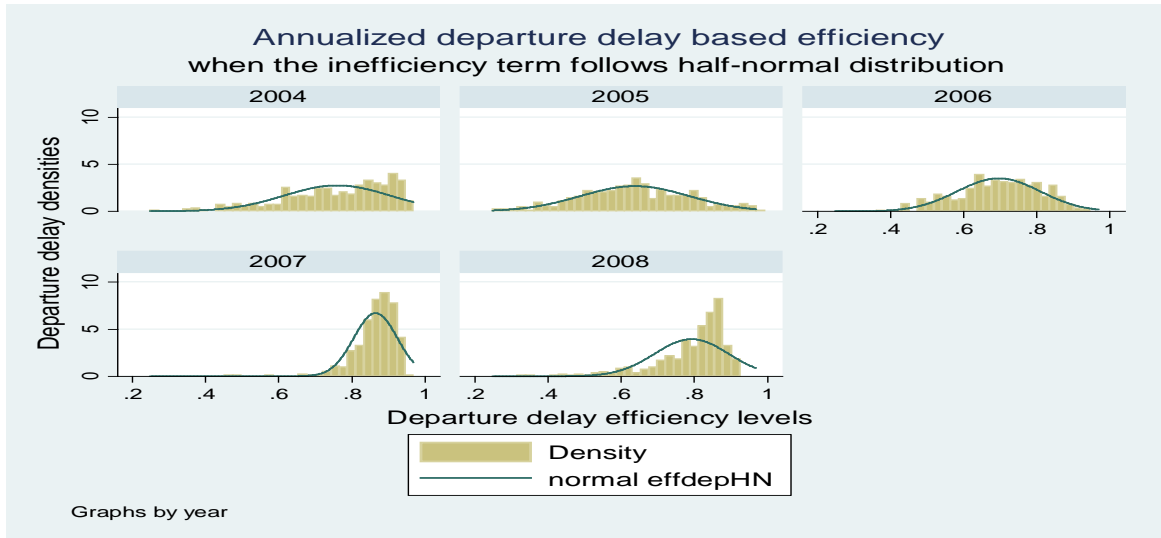
Figure 3: Annualized departure delay based efficiency of an airport for half-normally distributed inefficiency term

Similarly, Figure 4 shows the airport efficiency model estimate with exponential distribution indicated a greater inclination towards normality of the density plots for the year 2007. However, the airport efficiency density plot for the year 2005 is flatter than the one showed in Figure 3, indicating a greater level of kurtosis.
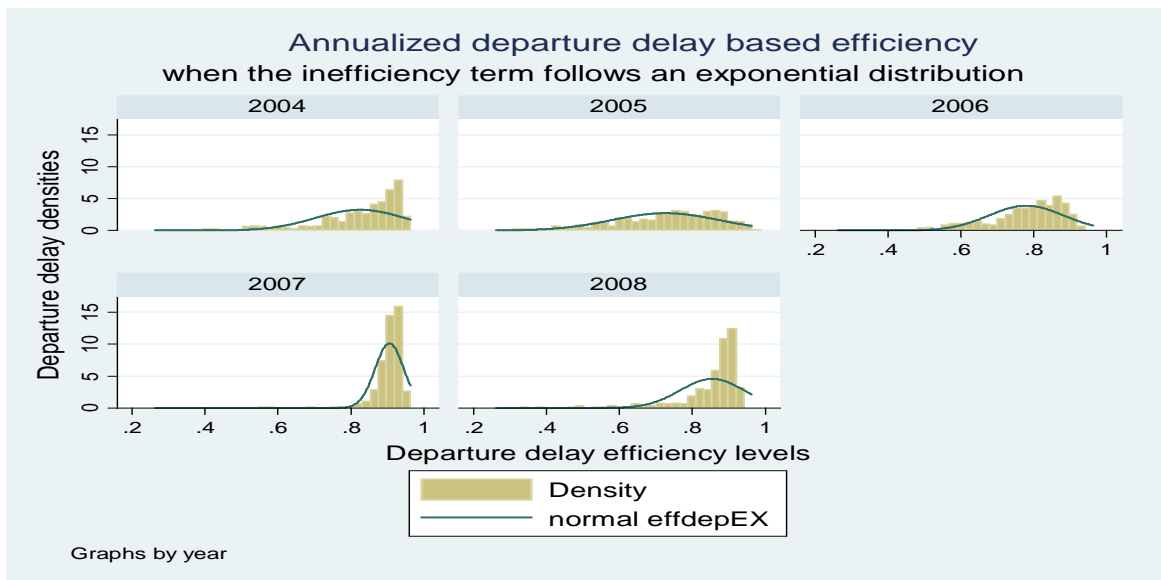

Figure 4: Annualized departure delay based efficiency of an airport for exponentially distributed inefficiency term

The estimated value of the γ-parameter, which is associated with the variance of the technical inefficiency effects in the stochastic frontier was estimated to be 0.90. This result suggested that technical inefficiency effects are significant components of total variability of airport efficiency score [9]. The null hypothesis, which specified that the explanatory variables in the technical inefficiency model were not stochastic, was thus rejected. Hence, aircraft departures at the airport used as a case study are not efficient, implying that inefficiency effects were present. Thus, it can be concluded that the explanatory variables in the technical efficiency model do contribute significantly to the explanation of the inefficiency effects for aircraft departure delays at the airport.

4. Conclusions

Estimation of efficiency of the airport using a stochastic frontier modelling approach was performed. Information was provided to guide comprehension of the efficiency performance of the airport by uniquely applying aircraft daily departure delay [15] against daily number of operations and daily average visibility. The results obtained showed that the average technical efficiency levels of the airport did not vary significantly when inefficiency term was modelled either as half-normal or exponential distributions. Comparing the average efficiency levels of the airport based on aircraft departure performance, we found a high level of heterogeneity. There are days when the airport efficiency went up to 96 percent from efficiencies of about 25 percent for both half-normal and exponential distributions. In the extreme ends when faced with a choice to select between the distribution for the error term, exponential distribution was recommended over half-normal and truncated normal because it presented a smaller variance.

The inefficiency model enabled identification of determinants of aircraft departure delay. The findings indicated that the number of persons on board of a departing aircraft, queen's nautical height and proportion of arrival delays at the airport were significant parameters for explaining airport's efficiency. This suggests that airport efficiency-enhancing policies need to focus on both aviation and meteorological factors. In addition, the results show that the high number of daily aircraft operations and lower levels of visibility are significantly associated with low aircraft departure efficiency. These findings suggest that in order to promote efficiency of air traffic flow at the airport, preventive measures need to be taken so as to mitigate aircraft departure delays by monitoring both meteorological and aviation parameters.

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