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## **Micropolar Fluid Through a Porous Medium Bounded by a Porous Plate**

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### **Abstract**

An analysis of the steady two-dimensional boundary layer flow of a micropolar fluid through a porous medium bounded by a porous plate by using a generalized Darcy's law is considered. Numerical solution for velocity field has been derived and the effect parameter of the problem is discussed.

**Keywords:** polar fluid; porous medium; steady flow; porous plate; boundary layer.

### **1. Introduction**

The micropolar fluid theory is the one of the most important non-Newtonian fluid models described by [1]. This theory shows microrotation effects as well as micro inertia and has many applications such as polymer fluids, liquid crystals, animal bloods, unusual lubricants, colloidal and suspension solutions, colloidal fluids, liquid crystals, polymeric suspension, and flow through a porous medium. The extensive reviews of the micropolar fluid theory and its applications can be found in [2]. The comprehensive literature on micropolar fluids, thermomicropolar fluids and their applications in engineering and technology were presented by [3, 4]. Srinivasacharya and his colleagues [5] analysed the unsteady stokes flow of micropolar fluid between two parallel porous plates. Reference [6] investigated fully developed magnetohydrodynamic flow of a micropolar and viscous fluid in a vertical porous space. Several authors have studied the characteristic of the boundary layer flow micropolar fluid under different boundary conditions by using Darcy's law [7-15].

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The purpose of this paper is to investigate the steady two-dimensional boundary layer flow of a micropolar fluid through a porous medium bounded by a porous plate by using a generalized Darcy's law [16, 17].

## 2. Mathematical Analysis

The boundary layer steady flow of an incompressible micro-polar fluid through a porous medium bounded by a semi-infinite horizontal porous plate is studied. The x-axis is taken along the plate and the y-axis normal to it. We assume that all the physical properties of the fluid are constant. The microscopic inertia term involving can be neglected for steady two-dimensional boundary layer flow in a micropolar fluid without introducing any appreciable error in the solution. Under these assumptions the governing two-dimensional boundary layer equations for micropolar fluids through the porous medium by using a generalized Darcy's law are [7-17]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + K_1 \frac{\partial \sigma}{\partial y} + \frac{\nu \varphi}{K} (U - u) + C \varphi^2 (U^2 - u^2) \quad (2)$$

$$G_1 \frac{\partial^2 \sigma}{\partial y^2} - 2\sigma - \frac{\partial u}{\partial y} = 0 \quad (3)$$

where  $u, v$  are the intrinsic average components of the velocity along  $x, y$  coordinates respectively,  $\varphi$  the porosity,  $\rho$  the fluid density,  $\nu = \frac{(\mu+S)}{\rho}$  is the apparent kinematic viscosity of the fluid,  $\mu$  the coefficient of dynamic viscosity of the fluid,  $S$  a constant characteristic of the fluid,  $\sigma$  the micro-rotation component,  $C$  Forchheimer's inertia coefficient,  $K_1 = \frac{S}{\rho}$ , ( $K_1 > 0$ ) the coupling constant, ( $G_1 > 0$  the microrotation constant,  $K > 0$  the permeability of the porous medium and  $U$  the free stream velocity.

The boundary conditions are:

$$u = 0 \quad , \quad v = -\frac{c}{\sqrt{x}} \quad , \quad c > 0 \quad , \quad \sigma = 0 \quad \text{at } y = 0 \quad (4)$$

$$u \rightarrow U \quad , \quad \sigma \rightarrow 0 \quad , \quad \text{as } y \rightarrow \infty$$

where  $c$  a constant.

Introducing the dimensionless variable,

$$n = \sqrt{\frac{U}{2\nu x}} y$$

and the expressions for  $u, v$  and  $\sigma$  as

$$u = Uf'(n) \quad , \quad v = \sqrt{\frac{\nu U}{2x}} [-f(n) + nf'(n)] \quad , \quad \sigma = \left(\frac{U}{2\nu x}\right)^{\frac{1}{2}} Ug(n)$$

into equations (1), (2) and (3), we get

[Equation (1) is identically satisfied]

$$f''' + ff'' + Lg' + \frac{1}{M}(1 - f') + N(1 - f'^2) = 0 \quad (5)$$

$$Gg'' - 2(2g + f'') = 0 \quad (6)$$

where

$$L = \frac{K_1}{\nu} \quad (\text{coupling constant parameter})$$

$$M = \frac{KU}{2\phi\nu x} \quad (\text{permeability parameter})$$

$$N = 2C\phi^2 x \quad (\text{inertia coefficient parameter})$$

$$G = \frac{G_1 U}{\nu x} \quad (\text{micro-rotation parameter})$$

$$m = c \sqrt{\frac{2}{\nu U}} \quad (\text{absorption parameter})$$

The boundary conditions for  $f$  and  $g$  can be expressed as

$$f(0) = m \quad , \quad f'(0) = 0 \quad , \quad g(0) = 0 \quad (7)$$

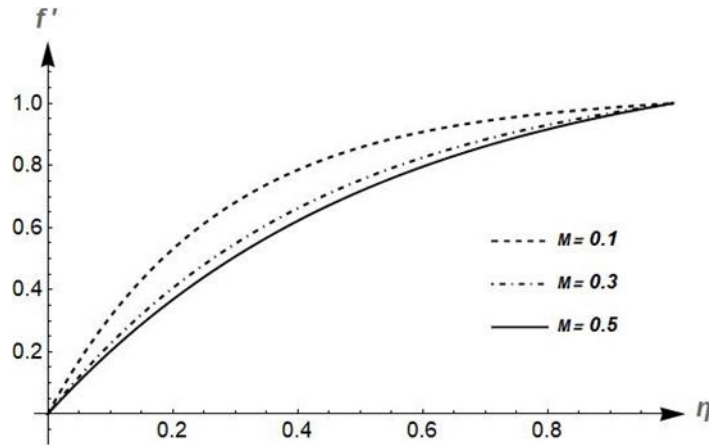
$$f'(\infty) = 1 \quad , \quad g(\infty) = 0$$

Equations (5) and (6) governed by the boundary conditions (7) are solved numerically using shooting technique.

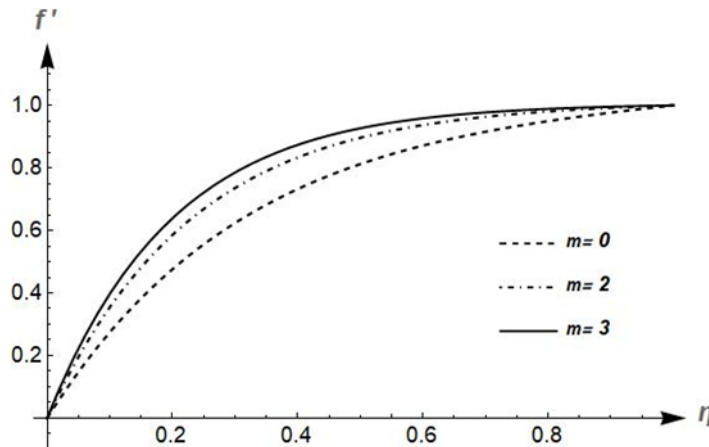
### 3. Conclusions

In Figure 1 we have plotted the dimensionless velocity  $f'$  profiles showing the effect of the permeability parameter, when  $L = 0.1$ ,  $N = 0.01$ ,  $G = 3$  and  $m = 1$ . From this figure we observe that when the permeability parameter  $M$  increases the velocity decreases.

In Figure 2 we have plotted the dimensionless velocity  $f'$  profiles showing the effect of the absorption parameter  $m$ , when  $L = 0.1$ ,  $N = 0.01$ ,  $G = 3$  and  $M = 0.1$ . From this figure we observe that when the absorption parameter  $m$  increases the velocity increases.



**Figure 1:** Velocity profiles  $f'$  for different values the permeability parameter  $M$ .



**Figure 2:** Velocity profiles  $f'$  for different values of the absorption parameter  $m$ .

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