



Consensus Tracking for Multiagent Systems Under Bounded Unknown External Disturbances Using Sliding- PID Control

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Abstract

This paper is devoted to the study of consensus tracking for multiagent systems under unknown but bounded external disturbances. A consensus tracking protocol which is a combination between the conventional PID controller and sliding mode controller named sliding-PID protocol is proposed. The protocol is applied to the consensus tracking of multiagent system under bounded external disturbances where results showed high effectiveness and robustness.

Keywords: consensus tracking; multiagent systems; PID controller; sliding mode control; robustness.

1. Introduction

In the most recent years, multiagent systems (MAS) seriously utilized as a part of front-line innovations to finish complex assignments past the abilities of solid frameworks. As of late, MAS have been effectively used in transportation, manufacturing, defense systems, seek and rescue/protect in threatening situations, mobile sensor systems, logistics, combat zone conditions, cooperated surveillance and large-scale assembly [1-5]. In reference [6], authors addressed the distributed consensus control of MAS via nonlinear coupling where they proposed an adaptive coupling protocol.

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The coupling strengths were adjusted and reduced through the adaptive coupling protocol but without reaching zero coupling strengths. In reference [7], authors investigated the leader-following consensus for multiagent systems by proposing three event-triggered protocol schemes; distributed, centralized and clustered. They showed that these event-triggered techniques can help reduce the frequency of information transmission and control update. In reference [8], authors studied the general multiagent networks for the leaderless and leader-follower consensus problems. They designed a distributed adaptive event-triggered protocols to achieve agents' consensus and to exclude the Zeno behavior. In reference [9], authors studied the output-feedback consensus control for a class of heterogeneous linear MAS in the presence of disturbance and uniform sampling processes. In reference [10] the authors designed a distributed adaptive fault-tolerant controller to solve the cooperative output regulation problem for linear MAS with actuator faults. In reference [11] authors developed a distributed specified-time consensus protocols to solve the consensus of MAS with general linear dynamics over directed graph. The communication between the agents is highly depended on the consensus protocol. The more connected map the efficient the communications between the agents. Other various hypothetical and technical issues are as yet unsolved, unraveled somewhat, or exceedingly dubious which also limits the abilities of MAS to effectively accomplish the allotted assignments. Self-automating, self-overseeing, and decision making are among these capabilities. Besides, the vast majority of existing cooperative control procedures and consensus systems have not been approved in simulation environments, which makes them challenging to approve. Industrial implementation of existing MAS protocols including those discussed earlier can show serious difficulties and challenges. The difficulty of implementation of such controllers is mainly due to the complex control architecture, computational cost, and mechanization. On the other hand, a large portion of the controllers used in the industry around the globe is PID controllers even though modern controllers have been effectively applied in various areas. PID controllers are straightforward in structure, simple to execute, and with acceptable execution performance. However, with all that superb capabilities, conventional PID controllers cannot give satisfactory performance and meet the control requirements when the controlled systems exhibit inherent nonlinear dynamics or experienced external disturbances. In other words, these controllers become less effective in robustness, reliability, and credibility [12]. Various late publications about PID controllers are creating deferent strategies to overcome the defects of the conventional PID [13-17]. In this paper a PID-sliding mode based MAS protocol is developed and results are compare to those provided by the protocol shown in [18]. The proposed approach guarantees high level of system performance, robustness against external disturbances, and non-chattering effect experienced by sliding mode controllers. The paper is organized as follows. In section 2, an overview of graph theory is presented. Section 3, briefly explains the problem statement. Section 4, presents and explains the main result and section 5 presents a simulation example for validation and comparison. Finally, conclusion is presented section 6.

2. Preliminaries

2.1. Graph theory

The correspondence topology among agents is presented by a graph $G = \{V, \varepsilon, A\}$, with the arrangement of nodes $V = f(v_1, v_2, \dots, v_n)$, the arrangement of edges $\varepsilon \subseteq V \times V$; and a weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$. Here, every node v_i in V relates to an agent i , and each edge $(v_i, v_j) \in \varepsilon$ in a weighted directed graph compares to a data interface from agent j to agent i , which implies that agent i can get data from agent j .

The adjacency matrix A of a weighted directed graph is defined to such an extent that $a_{ii} = 0$ for any $v_i \in V$; $a_{ij} > 0$ if $(v_j, v_i) \in \varepsilon$, and $a_{ij} = 0$ otherwise. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n \times n}$ of graph G is defined by $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ij} = \sum_{j=1, j \neq i}^n a_{ij}$, $i, j \in \{1, \dots, n\}$. For an undirected graph, L is symmetric semi-definite positive, while for directed graph L may be non-symmetric. I_n denotes the $n \times n$ identity matrix, $0_{m \times n}$ denotes the $m \times n$ zero matrix and $\mathbf{1}_n = [1, 1, \dots, 1]^T \in \mathbb{R}^n$. The eigenvalues $\lambda_{\min(A)}$ and $\lambda_{\max(A)}$ are the smallest and the biggest eigenvalues of the matrix A , respectively.

Definition 1 [19]: A directed path from node v_i to v_j is a sequence of edges $(v_i, v_{j_1}), (v_{j_1}, v_{j_2}), \dots, (v_{j_l}, v_j)$ in a directed graph G with distinct nodes v_{j_k} , $k = 1, \dots, l$.

Definition 2 [19]: The directed graph G is said to have a directed spanning tree if there is a root node that can reach all the other nodes following a directed path in graph G .

Lemma 2.1 [20]: An undirected graph G is connected if and only if its Laplacian matrix L satisfies the condition that an eigenvalue zero with multiplicity 1 has a corresponding eigenvector $\mathbf{1}$ and all other eigenvalue have positive real parts.

Lemma 2.2 [21]: For matrices M, N, V , and W with appropriate dimensions, the Kronecker product \otimes has the following properties

- (1) $(M + N) \otimes V = M \otimes V + N \otimes V$;
- (2) $(M \otimes N)(V \otimes W) = MV \otimes NW$;
- (3) $(M \otimes N)^T = M^T \otimes N^T$;
- (4) $(\xi M) \otimes N = M \otimes (\xi N)$, where ξ is a constant

3. Problem statement

For the purpose of improving tracking effectiveness and robustness, this paper aims to extend the sliding-PID controller in [12] to a certain class of MAS subject to external disturbances. For such purpose, consider the case of MAS with virtual leader labelled '0' and N followers labelled ' $i = 1, 2, \dots, N$ ' with dynamics modeled as follows

$$\begin{aligned} \dot{\xi}_i(t) &= f_i(\xi_i(t)) + g_i(\xi_i(t))[u_i(t) + d_i(t)] \\ y_i(t) &= h_i(\xi_i(t)), \end{aligned} \quad (i = 1, 2, \dots, N) \quad (1)$$

where $\xi_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, $d_i(t) \in \mathbb{R}^p$, and $y_i(t) \in \mathbb{R}^q$ denote the state, control input, the external disturbance, and the output of an agent ' i ', respectively; $f_i \in \mathbb{R}^{n \times n}$, $g_i \in \mathbb{R}^{n \times n}$ and $h_i \in \mathbb{R}^{q \times n}$ are smooth vector-valued functions.

The virtual leader dynamics are given as follows

$$\dot{\xi}_0(t) = f(\xi_0(t)) \tag{2}$$

where $\xi_0(t) \in \mathbb{R}^n$ is the leader's state and $f(\xi_0(t)) \in \mathbb{R}^m$ is the vector function which depicts its inherent dynamics. The sliding-PID controller presented in [12] is a controller that joins the conventional PID controller with the sliding mode controller as appeared in Fig. 1. In order to ensure robustness without chattering effect, the sliding variable is considered as a high-order function of Lie derivatives up to the relative degree r of the system dynamics

$$\sigma_r(e, \dot{e}, \dots, e^{(r-1)}) = \sum_{k=r-1}^0 \lambda_k L_f^k e(x, t) \tag{3}$$

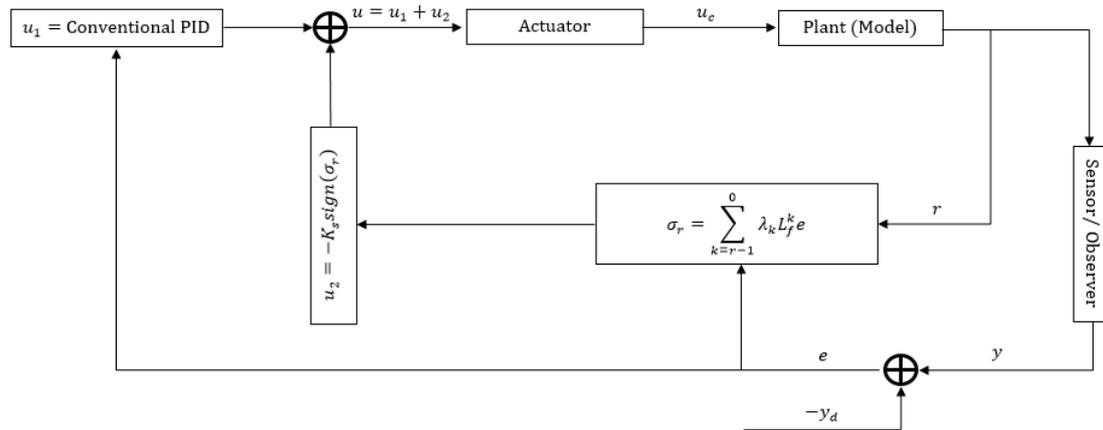


Figure 1: Block diagram of the sliding-PID controller

Lemma [12]: Suppose that the dynamic of the controlled system satisfy the assumptions 1 to 5 as described in [12]. For any initial state $x_0 \in X$ (operating space), the following controller assures that the closed-loop tracking error $e(x, t)$ as well as the sliding variable σ_r converge to zero with a proper choice of the controller gains

$$u_r = u_1 + u_2 = \left[K_p e(x, t) + K_d \dot{e}(x, t) + K_i \int_{t_0}^t e(x, \tau) d\tau \right] + \left[-K_s \text{sign} \left(\sigma_r(e, \dot{e}, \dots, e^{(r-1)}) \right) \right] \tag{4}$$

The proposed MAS protocols is an extension of the controller (4) to a team of more than one agent such that the agents' protocols are computed as follows

$$u_{r,i} = u_{1,i} + u_{2,i} = \left[K_p e_i(x, t) + K_d \dot{e}_i(x, t) + K_i \int_{t_0}^t e_i(x, \tau) d\tau \right] + \left[-K_s \text{sign} \left(\sigma_{r,i}(e_i, \dot{e}_i, \dots, e_i^{(r-1)}) \right) \right] \tag{5}$$

where

$$e_i(x, t) = \sum_{j=0}^n (\xi_i(t) - \xi_j(t)) \tag{6}$$

$$\sigma_{r,i}(e_i, \dot{e}_i, \dots, e_i^{(r-1)}) = \sum_{k=r-1}^0 \lambda_k L_f^k e_i(x, t) = \sum_{j=0}^n \sum_{k=r-1}^0 \lambda_k L_f^k a_{ij} (\xi_i(t) - \xi_j(t)) \tag{7}$$

4. Main result

The consensus tracking issue of MAS is to design, for any each 'i' and for any starting position states, a control input $u_i(t), i = \{1, \dots, n\}$ such that

$$\lim_{t \rightarrow \infty} \|\xi_i(t) - \xi_0(t)\|_2 = 0 \tag{8}$$

To achieve this asymptotic behavior, consider the class of MAS where the virtual leader has the equivalent inherent dynamics with all agents, and these inherent dynamics fulfill a Lipchitz-type condition given by the following definition.

Definition 1: $\forall \xi, \zeta \in \mathbb{R}^m; \forall t \geq 0$, there exists a nonnegative constant l such that

$$\|f(t, \xi) - f(t, \zeta)\|_2 \leq l \|\xi - \zeta\|_2 \tag{9}$$

To satisfy condition (3), the control input (5) is applied to individual agents as follows

$$u_i(t) = - \left[K_p \sum_{j=0}^n a_{ij} (\xi_i(t) - \xi_j(t)) + K_i \sum_{j=0}^n \int_{t_0}^t a_{ij} (\xi_i(\tau) - \xi_j(\tau)) d\tau + K_d \sum_{j=0}^n \sum_{k=r-1}^1 \lambda_k L_f^k a_{ij} (\xi_i(t) - \xi_j(t)) \right] - K_s \operatorname{sgn} \left(\sum_{j=0}^n \sum_{k=r-1}^0 \lambda_k L_f^k a_{ij} (\xi_i(t) - \xi_j(t)) \right) \tag{10}$$

where K_p, K_i and K_d are the PID gains, K_s is the sliding mode gain, and λ_k is design parameter

Theorem 4.1: Suppose that the fixed undirected graph G is connected and at least one $a_{i0} > 0$, and let $M = L + \operatorname{diag}(a_{10}, \dots, a_{n0})$, where L is the Laplacian matrix of G . MAS achieve the consensus metric (8) under the individual protocols (10) if the control gains are selected to fulfill the following condition

$$\begin{cases} (K_p + K_i) \geq \frac{(l + d_{max})}{\lambda_{max}(M)} \\ K_d \lambda_{min}(M) \sum_{k=r-1}^1 \lambda_k \geq K_s \end{cases}$$

Proof:

Let $\tilde{\xi}_i(t) = \xi_i(t) - \xi_0(t), i = \{1, \dots, n\}$. Using (10) in (1), one can get

$$\begin{aligned} \dot{\tilde{\xi}}_i(t) = & f(\tilde{\xi}_i(t)) - \left[K_p \sum_{j=0}^n a_{ij} (\tilde{\xi}_i(t) - \tilde{\xi}_j(t)) + K_i \sum_{j=0}^n \int_{t_0}^t a_{ij} (\tilde{\xi}_i(\tau) - \tilde{\xi}_j(\tau)) d\tau \right. \\ & \left. + K_d \sum_{j=0}^n \sum_{k=r-1}^1 \lambda_k L_f^k a_{ij} (\tilde{\xi}_i(t) - \tilde{\xi}_j(t)) \right] \\ & - K_s \operatorname{sgn} \left(\sum_{j=0}^n \sum_{k=r-1}^0 \lambda_k L_f^k a_{ij} (\tilde{\xi}_i(t) - \tilde{\xi}_j(t)) \right) + d(\tilde{\xi}_i(t)) \end{aligned} \tag{11}$$

Suppose that

$$\begin{cases} \tilde{\xi}(t) = [\tilde{\xi}_1^T(t), \tilde{\xi}_2^T(t), \dots, \tilde{\xi}_n^T(t)]^T \\ F(\tilde{\xi}(t)) = \left[\left(f(\xi_1(t)) - f(\xi_0(t)) \right)^T, \dots, \left(f(\xi_n(t)) - f(\xi_0(t)) \right)^T \right]^T \\ D(\tilde{\xi}(t)) = \left[\left(d(\xi_1(t)) \right)^T, \dots, \left(d(\xi_n(t)) \right)^T \right]^T \end{cases} \quad (12)$$

where $\tilde{\xi}(t), F, D \in \mathbb{R}^n$. The dynamic model (11) can be written as follows

$$\begin{aligned} \dot{\tilde{\xi}}(t) = F(\tilde{\xi}(t)) - \left[K_p(M \otimes I_m)\tilde{\xi}(t) - K_i(M \otimes I_m) \int_{t_0}^t \tilde{\xi}(t) d\tau - K_d(M \otimes I_m) \sum_{k=r-1}^1 \lambda_k L_f^k \tilde{\xi}(t) - \right. \\ \left. K_s \operatorname{sgn} \left((M \otimes I_m) \sum_{k=r-1}^0 \lambda_k L_f^k \tilde{\xi}(t) \right) \right] + D(\tilde{\xi}(t)) \end{aligned} \quad (13)$$

According to the graph theory; M is symmetric positive definite if the fixed undirected graph G is connected and at least one a_{i0} is positive. Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2} \tilde{\xi}^T(t) (M \otimes I_m) \tilde{\xi}(t) \quad (14)$$

Tracking the time derivative of $V(t)$ along the trajectory (13), it gives

$$\begin{aligned} \dot{V}(t) = \tilde{\xi}^T(t) (M \otimes I_m) \dot{\tilde{\xi}}(t) \\ = \tilde{\xi}^T(t) (M \otimes I_m) \left[F(\tilde{\xi}(t)) - K_p(M \otimes I_m)\tilde{\xi}(t) - K_i(M \otimes I_m) \int_{t_0}^t \tilde{\xi}(t) d\tau \right. \\ \left. - K_d(M \otimes I_m)\tilde{\xi}(t) - K_s \operatorname{sgn} \left((M \otimes I_m) \sum_{k=r-1}^0 \lambda_k L_f^k \tilde{\xi}(t) \right) + D(\tilde{\xi}(t)) \right] \end{aligned} \quad (15)$$

Developing the expression (15), it gives

$$\begin{aligned} \dot{V}(t) = \tilde{\xi}^T(t) (M \otimes I_m) F(\tilde{\xi}(t)) - K_p \tilde{\xi}^T(t) (M \otimes I_m)^2 \tilde{\xi}(t) - K_i \tilde{\xi}^T(t) (M \otimes I_m)^2 \int_{t_0}^t \tilde{\xi}(t) d\tau \\ - K_d \tilde{\xi}^T(t) (M \otimes I_m)^2 \sum_{k=r-1}^1 \lambda_k L_f^k \tilde{\xi}(t) - K_s \tilde{\xi}^T(t) (M \otimes I_m) \operatorname{sgn} \left((M \otimes I_m) \sum_{k=r-1}^0 \lambda_k L_f^k \tilde{\xi}(t) \right) \\ + \tilde{\xi}^T(t) (M \otimes I_m) D(\tilde{\xi}(t)) \end{aligned} \quad (16)$$

Using norms properties, the function $\dot{V}(t)$ is bounded as follows

$$\begin{aligned} \dot{V}(t) \leq \lambda_{\max}(M) \|\tilde{\xi}(t)\|_2^2 - K_p \lambda_{\min}^2(M) \|\tilde{\xi}(t)\|_2^2 - K_i \lambda_{\min}^2(M) \left\| \int_{t_0}^t \tilde{\xi}(t) d\tau \right\|_2^2 \\ - K_d \lambda_{\min}^2(M) \left\| \sum_{k=r-1}^1 \lambda_k L_f^k \tilde{\xi}(t) \right\|_2^2 - K_s \lambda_{\min}(M) \|\tilde{\xi}^T(t)\|_2 + d_{\max} \lambda_{\max}(M) \|\tilde{\xi}(t)\|_2^2 \\ \leq \lambda_{\max}(M) \|\tilde{\xi}(t)\|_2^2 - K_p \lambda_{\min}^2(M) \|\tilde{\xi}(t)\|_2^2 - K_i \lambda_{\min}^2(M) \int_{t_0}^t \|\tilde{\xi}(t)\|_2^2 d\tau \\ - K_d \lambda_{\min}^2(M) \sum_{k=r-1}^1 \lambda_k \left\| L_f^k \tilde{\xi}(t) \right\|_2^2 - K_s \lambda_{\min}(M) \|\tilde{\xi}^T(t)\|_2 + d_{\max} \lambda_{\max}(M) \|\tilde{\xi}(t)\|_2^2 \end{aligned} \quad (17)$$

Since $\tilde{\xi}(t)$ is a measure of the taking error, we suppose that

$$\begin{cases} \sum_{k=r-1}^1 \lambda_k \|L_f^k \tilde{\xi}(t)\|_2^2 \leq \|\tilde{\xi}(t)\|_2^2 \sum_{k=r-1}^1 \lambda_k \leq \|\tilde{\xi}(t)\|_1^2 \sum_{k=r-1}^1 \lambda_k \\ \int_{t_0}^t \|\tilde{\xi}(t)\|_2^2 d\tau \leq \|\tilde{\xi}(t)\|_2^2 \end{cases} \quad (18)$$

Using conditions (18), the function (17) is bounded as follows

$$\begin{aligned} \dot{V}(t) \leq & -[(K_p + K_i)\lambda_{min}^2(M) - (l + d_{max})\lambda_{max}(M)]\|\tilde{\xi}(t)\|_2^2 \\ & - (K_d\lambda_{min}^2(M) \sum_{k=r-1}^1 \lambda_k + K_s\lambda_{min}(M))\|\tilde{\xi}(t)\|_1^2 \end{aligned} \quad (19)$$

It results that the Lyapunov function V continuously decays if the gains of the protocols (5) fulfill the following conditions:

$$\begin{cases} [(K_p + K_i)\lambda_{min}^2(M) - (l + d_{max})\lambda_{max}(M)] \geq 0 \\ (K_d\lambda_{min}^2(M) \sum_{k=r-1}^1 \lambda_k + K_s\lambda_{min}(M)) \geq 0 \end{cases} \quad (20)$$

If $\lambda_k > 0$ $k = 1, r - 1$ are selected such as $\sum_{k=r-1}^1 \lambda_k > 0$ it results,

$$\begin{cases} (K_p + K_i) \geq \frac{(l+d_{max})}{\lambda_{max}(M)} \\ K_d\lambda_{min}(M) \sum_{k=r-1}^1 \lambda_k \geq K_s \end{cases} \quad (21)$$

End of proof.

5. Simulation

In order to measure the effectiveness and robustness of the proposed sliding-PID consensus, the consensus of five agents used in [18] is selected to simulate the protocols (10). In this scenario, '0' denotes the leader, the dotted lines denote the links between the leader and the followers, and the solid lines denote the links among the followers as shown in Fig. 2. With the same external disturbances.

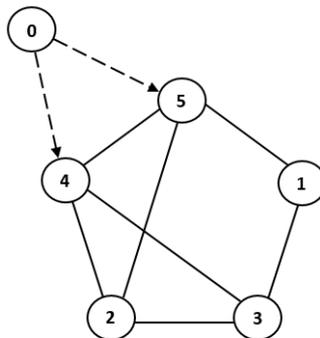
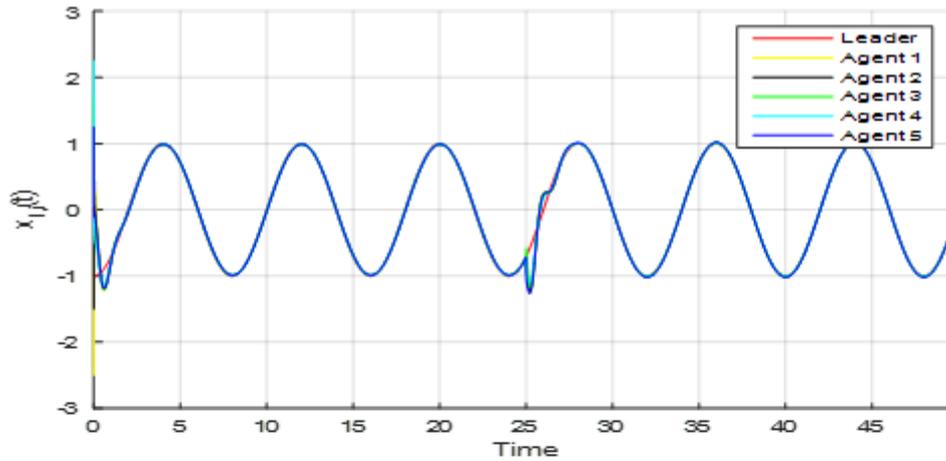
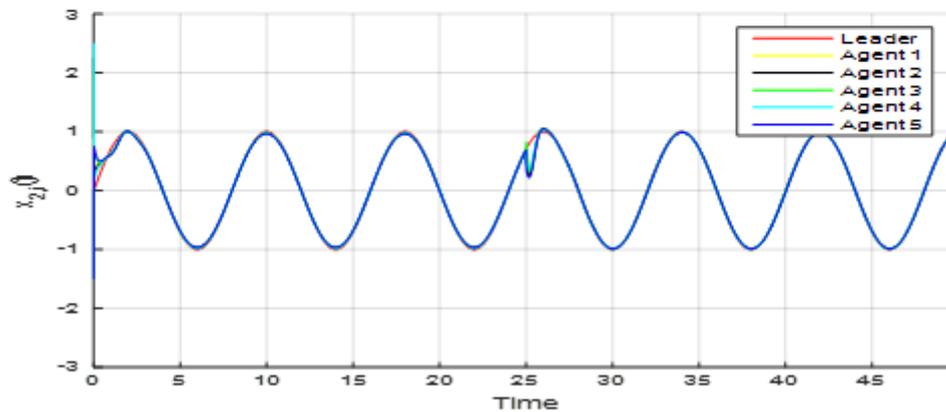


Figure 2: Communication topology

The state trajectories $(x_{1,i}(t), x_{2,i}(t))$ of the five agents using the sliding-PID protocol are graphed in Fig. 3. It can be seen that with the sliding-PID protocol (10), the consensus of the multiagent system in the presence of external disturbances is robustly achieved.



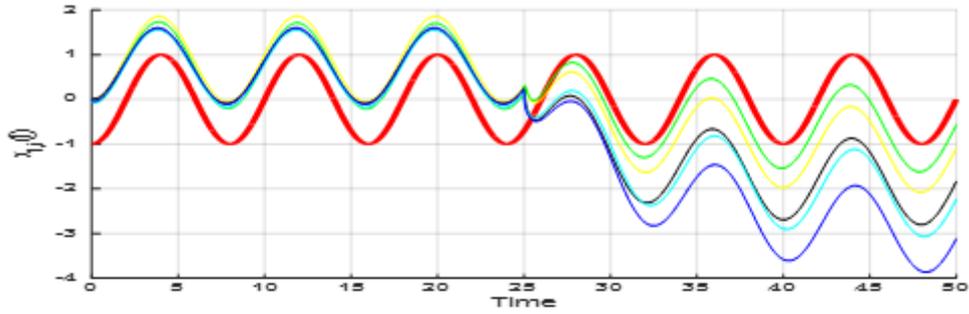
(a) Agent states $x_{1,i}(t)$



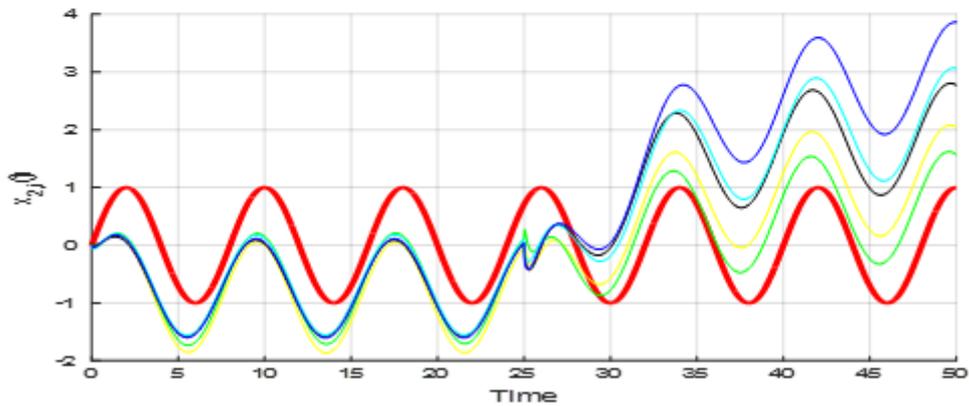
(b) Agent states $x_{2,i}(t)$

Figure 3: State trajectory of the multiagent system with presence of unknown bounded external disturbances

To show the advantage of the protocols (10) against other ones found in literature, the same scenario is run using the protocols in [18]. Fig. 4 shows the consensus tracking with the protocols [18]. It is shown that the agents didn't complete a consensus from start to a time before the 25th second. At the 25th second disturbances appear and the agents lost connections and diverged. Unlike what happened to the MAS using sliding-PID control in Fig.3, the agents performed a robust consensus from start and continued even after disturbance occurred.



(a) Agent states $x_{1,i}(t)$



(b) Agent states $x_{2,i}(t)$

Figure 4: State trajectory of the multiagent system using the controller in [18]

6. Conclusion

In this paper, a new sliding-PID distributed consensus was designed to robustly achieve consensus tracing among a certain class of multi-agent systems. The proposed controllers achieved high-precision robust tracking in presence of unknown bounded external disturbances. Through a leader-follower consensus scenario, the sliding-PID consensus strategy showed more effectiveness as compared to the control approach used in [18]. Issues related to multi-agent systems such as time-varying connectivity, communication delay, and communication loss would be investigated using sliding-PID consensus in future work.

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7. Recommendations

The developed sliding-PID control has to enable systems to perform assignments with complex necessities and

to work in open, dynamic, and dubious conditions. The results expected hold enhanced autonomy, self-overseeing, and decision making capabilities of MAS. And give high state execution and robustness strategies and algorithms into distributed consensus and control process of MAS in the presence of bounded external disturbances. Authors would recommend for future work on managing the more difficult and challenging situations of MAS.

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