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An Estimation of the Entropy for a Fréchet Distribution Based on Generalized Hybrid Censored Samples

Moshera A M Ahmad*

El Gazeera High Institute for Computer & Management Information System

Email: moshera_ahmad@pg.cu.edu.eg

Abstract

In this paper, under generalized type-I hybrid censored samples; we derive the estimators for the entropy function of the Fréchet distribution. We also compare the introduced estimators in the sense of the relative mean squared error (RMSE) for various censored samples.

Keywords: Entropy; Fréchet distribution; Generalized Type-I and Type-II Hybrid Censoring; Types of censor.

1. Introduction

The genesis of the word "entropy" is in the physical sciences. One way in which the term may be used derives from information theory. The theory posits that we find out more from some messages than other messages, and there is a way of expressing the difference in the "information content" of different messages (see the example in [1]). Entropists are interested in how the receipt of a piece of information reduces uncertainty. Shannon in [2] introduced an entropy measure into the information theory. If $\boldsymbol{X} = (X_1, X_2, \dots, X_n)$ is a continuous random vector with joint probability density function f, then the entropy of $\boldsymbol{X}$ is defined as

$$H(X) = H(f) = -\int_{-\infty}^{\infty} f(\boldsymbol{x}) log f(\boldsymbol{x}),$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$ is the observed value of $\boldsymbol{X}$. This expression is useful in that, it provides a measure of ignorance or uncertainty about which of several possible outcomes will occur.

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* Corresponding author.

Many authors worked on the estimation of entropy for different life distributions. For example, [3] investigated the decomposition of entropy in both hybrids censoring schemes and applied to exponential, Weibull and Pareto distributions, and [4] derived the maximum likelihood estimators for the entropy of the Rayleigh distribution based on doubly-generalized type II hybrid censored samples. Also, [5] introduced an extend Fréchet distribution and derived the corresponding Shannon entropy, and [6] derived the estimators for the entropy function of the Lomax distribution under generalized type-I hybrid censored samples.

Consider the Fréchet distribution with cumulative distribution function (cdf):

$$F(x;\alpha,\lambda) = e^{-\left(\frac{\lambda}{x}\right)^{\alpha}} , x > 0, \alpha > 0, \lambda > 0, \tag{1}$$

and probability density function (pdf):

$$f(x;\alpha,\lambda) = \alpha\lambda^{\alpha}x^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x}\right)^{\alpha}} , x > 0, \alpha > 0, \lambda > 0. \tag{2}$$

For the pdf (2), the entropy simplifies to:

$$H(f) = \gamma\left(1 + \frac{1}{\lambda}\right) + \log\left(\frac{\alpha}{\lambda}\right) + 1, \tag{3}$$

where γ is the Euler-Mascheroni constant. Lifetime data often come incomplete, they come with a feature that creates special problems in the analysis of the data. This feature is known as censoring and, occurs when exact lifetimes are known only for a portion of the individuals under study; the remainder of the lifetimes are known only to exceed certain values. Censoring arises in various ways. type I and type II censoring scheme are the two most common censoring schemes. In type I censoring, the experiments are run over a fixed period of time in such away that an individual's lifetime will be known exactly only if it is less than some predetermined value. For example, in a life test experiment n items may be placed on test, but a decision is made to terminate the test after a certain time T has elapsed. Lifetimes will then be known exactly only for those items that fail by time. The main disadvantage of this type of censoring is that, with high probability, far fewer failures may occur. This will have a bad effect on the efficiency of inferential procedures based on type I censoring. In type II censoring, only the r smallest observations in a random sample of n items are observed $(1 \leq r \leq n)$. For example, in life testing a total of n items is placed on test, but instead of continuing until all n items have failed, the test is terminated at the time of the r^{th} failure. Estimation of the parameters from censored samples has been investigated by many authors such as [7,8], and [9]. The main disadvantage of this type of censoring is that, most likely, it could take a long time before observing r failures. The mixing type I and type II censoring scheme is known as hybrid censoring scheme (HCS). If in a life test experiment n items are placed on test, but a decision made to terminate the test when a pre-fixed number, $r < n$, has failed, or when a pre-fixed time,T, has been reached, this is called type I hybrid censoring scheme (type-I HCS), and we can express that symbolically as $T_* = \min\{X_{r:n}, T\}$. However, if we terminate the experiment at the random time $T^* = \max\{X_{r:n}, T\}$, this called type II hybrid censoring scheme. It means that if the r failures occur before time T, then the experiment would continue up to time T , which may end up giving perhaps more than r failures in the data. On the other hand, if the r^{th} failure does not occur before time T, then the experiment would continue until the time when the r^{th} failure occurs, in which case we would observe exactly r failures in the data. As in the case of type-I censoring, the main disadvantage of type-I HCS is that, with high probability, fewer failures may occurring by the pre-fixed time T. This leads to bad results in the estimation of model parameters. Extensive work has been done on hybrid censoring scheme, see [10,11,12,13,14,15], and [16]. Although type-II HCS guaranteeing at least r failures to be observed by the end of the experiment, the main disadvantage is that it might take a long time to observe the desired r failures [for more details see, [17]]. To overcome the shortcoming of these schemes, [18] introduced two extensions, and called them generalized type-I and generalized type-II hybrid Censoring. The Fréchet distribution, also known as inverse Weibull distribution, is applied to extreme events such as natural calamities, wind speeds, sea currents, and annually maximum one-day rainfalls and river discharges. Many authors have studied different aspects of inferential procedures for the Fréchet distribution. Calabria and Pulcini in [19] deals with the problem of predicting, on the base of censored sampling, the ordered lifetimes in a future sample when samples are assumed to follow the inverse Weibull distribution. Kazmi and Azizpour in [20] presented the statistical inferences of the inverse Weibull distribution under Type-I hybrid censoring. Ateya in [21] studied point and interval estimation of the scale and shape parameters of the inverse Weibull distribution based on balakrishnan's unified hybrid censored scheme. Ramos and his colleagues in [22] discussed the problem of estimating the parameters of the Fréchet distribution from both frequentist and Bayesian points of view. Kumar and Kumar in [23] dealt with the parameter estimation and reliability characteristics of the inverse Weibull distribution based on the random censoring model. In this paper, under generalized type-I hybrid censored samples; we derive the estimators for the entropy function of the Fréchet distribution. We also compare the introduced estimators in terms of the relative mean squared error (RMSE) for various censored samples. The rest of this paper is organized as follows; Section 2, introduces the generalized type-I hybrid censoring scheme. Section 3, describes the computation of the entropy function using maximum likelihood. In Section 4, descriptions of different estimators of the entropy of the Frechet distribution are compared through simulation study. Finally, Section 5, concludes.

2. Generalized Type-I Hybrid Censoring

Consider a life-testing experiment with n identical units placed on a life-test at time 0. Assume that $X_1, X_2, \dots, X_n$ denote the corresponding lifetimes from a distribution with cdf $F(x)$ and pdf $f(x)$. A generalized Type I hybrid censoring scheme is described as follows. Fix integers $r_1, r_2 \in \{1, 2, \dots, n\}$ such that $r_1 < r_2 < n$, and time $T \in (0, \infty)$. If the ${r_1}^{\text{th}}$ failure occurs before time T, terminate the experiment at $min\{X_{r_2:n}, T\}$. If the ${r_1}^{\text{th}}$ failure occurs after time T, terminate the experiment at $X_{r_1:n}$. In other words;

- If the ${r_1}^{\text{th}}$ failure occurs after time T, terminate the experiment at $X_{r_1:n}$,

- If the ${r_1}^{\text{th}}$ failure occurs before time T, terminate the experiment at T,

- If the ${r_2}^{\text{th}}$ failure occurs before time T, terminate the experiment at $X_{r_2:n}$.

We can note that this type of HCS is allowing the experiment to continue beyond time T if very few failures had been observed up to that time point, since the experimenter would like to observe r_2 failures, but is willing to settle for a bare minimum of r_1 failures. We will observe one of the following forms of observations, under such a generalized type I HCS:

$$Case\ I: \{x_{1:n} < x_{2:n} < \cdots < T < \cdots < x_{r_1:n} < \ldots < x_{r_2:n}\} \quad if\ x_{r_1:n} > T,$$

$$Case\ II: \{x_{1:n} < x_{2:n} < \cdots < x_{r_1:n} < \ldots < T < \cdots < x_{r_2:n}\} \quad if\ x_{r_1:n} < T < x_{r_2:n},$$

$$Case\ III: \{x_{1:n} < x_{2:n} < \cdots < x_{r_1:n} < \ldots < x_{r_2:n} < \cdots < T\} \quad if\ T > x_{r_2:n}.$$

A schematic representation of the generalized type-I hybrid censoring scheme is presented in Figure 1.

Given a generalized type-I hybrid censored sample, the likelihood functions for three different cases are as follows:

Case I

$$\frac{n!}{(n-r_1)!}\prod_{i=1}^{r_1} f(x_{\mathrm{i}:n})\,[S(x_{\mathrm{r}:n})]^{n-r_1};\ d = 0,1,2,\ldots,(r_1-1),$$

Case II

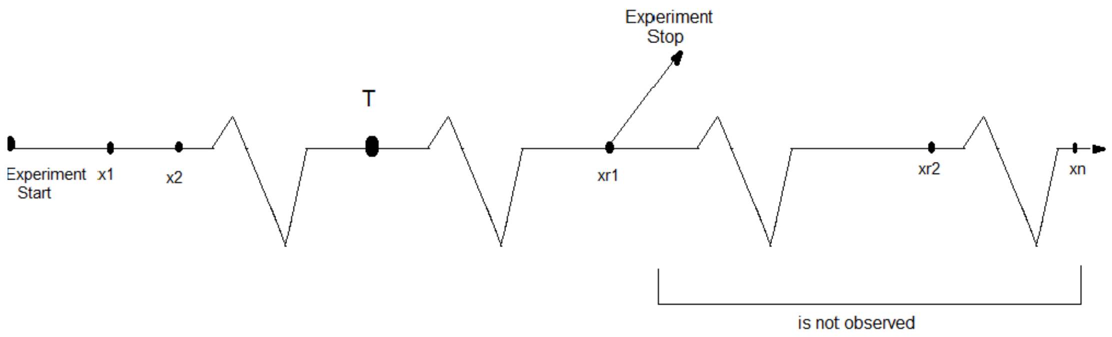
$$\frac{n!}{(n-d)!}\prod_{i=1}^{d} f(x_{\mathrm{i}:n})\,[S(\mathrm{T})]^{n-d}; d = r_1, or\ r_1+1, or, \ldots, or\ (r_2-1),$$

Case III

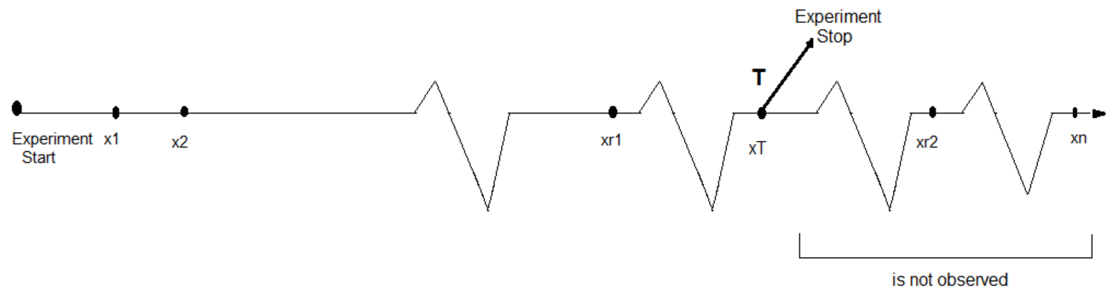
$$\frac{n!}{(n-r_2)!}\prod_{i=1}^{r_2} f(x_{\mathrm{i}:n})\,\left[S(x_{r_2:n})\right]^{n-r_2}; d = r_2\ ,$$

where d is a number of observed failures up to time T.

CaseI



CaseII



CaseIII

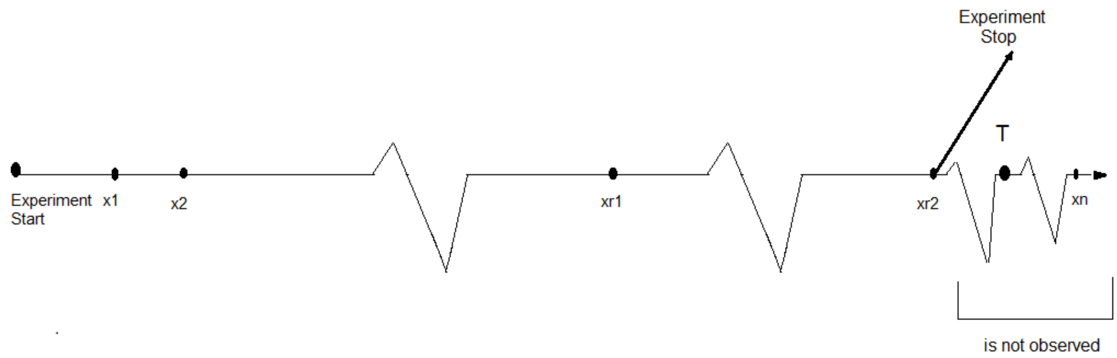


Figure 1: Schematic representation of the generalized hybrid censoring scheme Type-I

3. Maximum Likelihood Estimation

Now let us assume that the lifetimes of the experimental units are i.i.d. Fréchet random variables with pdf (2) and cdf (1). If d denotes the number of failures that occur by time point T, then based on the three forms of the generalized type I HCS sample, the likelihood functions of α and λ are given by:

Case I

$$L_I(\alpha,\lambda)=\frac{n!}{(n-r_1)!}\left(\prod_{i=1}^{r_1}\alpha\lambda^{\alpha}x_i^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha}}\right)^{n-r_1},$$

Case II

$$L_{II}(\alpha,\lambda)=\frac{n!}{(n-d)!}\left(\prod_{i=1}^{d}\alpha\lambda^{\alpha}x_i^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{T}\right)^{\alpha}}\right)^{n-d},$$

Case III

$$L_{III}(\alpha,\lambda) = \frac{n!}{(n-r_2)!}\left(\prod_{i=1}^{r_2} \alpha\lambda^{\alpha} x_i^{-(\alpha+1)} e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha}}\right)^{n-r_2}.$$

Additionally, the corresponding log likelihood functions are:

Case I

$$l_I(\alpha,\lambda) \equiv k_1 + r_1(log\alpha + \alpha log\lambda) - (\alpha+1)\sum_{i=1}^{r_1} logx_i - \sum_{i=1}^{r_1}\left(\frac{\lambda}{x_i}\right)^{\alpha} + (n-r_1)\log\left(1-e^{-\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha}}\right),$$

Case II

$$l_{II}(\alpha,\lambda) \equiv k_2 + d(log\alpha + \alpha log\lambda) - (\alpha+1)\sum_{i=1}^{d} logx_i - \sum_{i=1}^{d}\left(\frac{\lambda}{x_i}\right)^{\alpha} + (n-d)\log\left(1-e^{-\left(\frac{\lambda}{T}\right)^{\alpha}}\right),$$

Case III

$$l_{III}(\alpha,\lambda) \equiv k_3 + r_2(log\alpha + \alpha log\lambda) - (\alpha+1)\sum_{i=1}^{r_2} logx_i - \sum_{i=1}^{r_2}\left(\frac{\lambda}{x_i}\right)^{\alpha} + (n-r_2)\log\left(1-e^{-\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha}}\right),$$

where k_1, k_2, and k_3 are constants that don't depend on the parameters.

The corresponding likelihood equations are:

CaseI

$$\frac{d\ln l(\alpha,\lambda)}{d\alpha} \equiv r_1\left(\frac{1}{\alpha} + ln\lambda\right) - \sum_{i=1}^{r_1}\ln x_i - \sum_{i=1}^{r_1}\left(\frac{\lambda}{x_i}\right)^{\alpha} ln\left(\frac{\lambda}{x_i}\right) + (n-r_1)\frac{e^{-\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha}}\right)}\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha} ln\left(\frac{\lambda}{x_{r_1}}\right) = 0,$$

$$\frac{d\ln l(\alpha,\lambda)}{d\lambda} \equiv \frac{\alpha}{\lambda}\left(r_1 - \sum_{i=1}^{r_1}\left(\frac{\lambda}{x_i}\right)^{\alpha} - (n-r_1)\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha}\frac{e^{-\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{x_{r_1}}\right)^{\alpha}}\right)}\right) = 0,$$

CaseII

$$\frac{d\ln l(\alpha,\lambda)}{d\alpha} \equiv d\left(\frac{1}{\alpha}+ln\lambda\right)-\sum_{i=1}^{d}\ln x_i-\sum_{i=1}^{d}\left(\frac{\lambda}{x_i}\right)^{\alpha} ln\left(\frac{\lambda}{x_i}\right)+(n-d)\frac{e^{-\left(\frac{\lambda}{T}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{T}\right)^{\alpha}}\right)}\left(\frac{\lambda}{T}\right)^{\alpha} ln\left(\frac{\lambda}{T}\right)=0,$$

$$\frac{d\ln l(\alpha,\lambda)}{d\lambda} \equiv \frac{\alpha}{\lambda}\left(d-\sum_{i=1}^{d}\left(\frac{\lambda}{x_i}\right)^{\alpha}-(n-d)\left(\frac{\lambda}{T}\right)^{\alpha}\frac{e^{-\left(\frac{\lambda}{T}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{T}\right)^{\alpha}}\right)}\right)=0,$$

CaseIII

$$\frac{d\ln l(\alpha,\lambda)}{d\alpha} \equiv r_2\left(\frac{1}{\alpha}+ln\lambda\right)-\sum_{i=1}^{r_2}\ln x_i-\sum_{i=1}^{r_2}\left(\frac{\lambda}{x_i}\right)^{\alpha} ln\left(\frac{\lambda}{x_i}\right)+(n-r_2)\frac{e^{-\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha}}\right)}\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha} ln\left(\frac{\lambda}{x_{r_2}}\right)=0,$$

$$\frac{d\ln l(\alpha,\lambda)}{d\lambda} \equiv \frac{\alpha}{\lambda}\left(r_2-\sum_{i=1}^{r_2}\left(\frac{\lambda}{x_i}\right)^{\alpha}-(n-r_2)\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha}\frac{e^{-\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{x_{r_2}}\right)^{\alpha}}\right)}\right)=0.$$

These equations cannot be solved analytically and we solve them numerically to obtain the MLE of $\hat{\alpha}$ and $\hat{\lambda}$ of α and λ respectively.

Once we obtain the MLE of α , say $\hat{\alpha}$, and MLE of λ say $\hat{\lambda}$, the MLEs of entropy are obtained as:

$$\widehat{H}(f)=\gamma\left(1+\frac{1}{\hat{\lambda}}\right)+\log\left(\frac{\hat{\alpha}}{\hat{\lambda}}\right)+1$$

4. Simulation Study

In this section, a simulation study is conducted to compare the performance of different estimators. We consider different α , λ, r_1, r_2 , and T. Using Fréchet distribution, a generalized hybrid censored data can be generated as follows; if $x_{r_1:n} > T$ then we have a case I and the corresponding generalized hybrid censor sample be comes ($x_{1:n} < x_{2:n} < \cdots < T < \cdots < x_{r_1:n} < \ldots < x_{r_2:n}$). If $x_{r_1:n} < T < x_{r_2:n}$ then we have a case II. Continue the experiment up to time T and find d, a number of observed failures up to time T. Note that d would take one of the values r_1, r_1+1 ,…, or (r_2-1) and the corresponding generalized hybrid censor sample would be ($x_{1:n} < x_{2:n} < \cdots < x_{r_1:n} < \ldots < T < \cdots < x_{r_2:n}$). If $T > x_{r_2:n}$ then we have a case III, where we stop the experiment at $x_{r_2,n}$, and the corresponding generalized hybrid censor sample would be ($x_{1:n} < x_{2:n} < \cdots < x_{r_1:n} < \ldots < x_{r_2:n} < \cdots < T$). In each case the process is replicated 10,000 times. The associated MLEs are computed. The MLE estimates of the entropy are derived. Finally, different schemes are taken into consideration to compute the relative mean square error (RMSE) of all estimates, and these values are tabulated in Tables (1), (2), and (3). We note the following from Tables (1) to (3):

• In Table (1) RMSEs values of all estimates of entropy are presented for sample size n=200, and No. of failures $r_1 = 80$ and $r_2 = 120$, and various choices of α , λ , and T. In general, we observed that:

- The RMSE of ML estimates of $\widehat{H}(x)$ at $\alpha = 2$, $\lambda = 9$ and 10 has the smallest value compared to the RMSE of ML estimates for the corresponding other sets of parameters.

- For a fixed α, the RMSE values decrease generally as the scale parameter λ increases.

- For a fixed n, α, λ, r_1 and r_2, the RMSE values of $\widehat{H}(x)$ decrease as the stopping time T increases.

• In Table (2), for a fixed $n, \alpha, \lambda,$ and r_1, the RMSE values of $\widehat{H}(x)$ decrease generally as the No. of failures r_2 increases.

• In general, we observe that the RMSE values of $\widehat{H}(x)$ decrease as the sample size n increases and Table (3) showed that.

Table 1: Entropy estimates and relative MSEs for $\hat{\alpha}$, $\hat{\lambda}$, and $\widehat{H}$ for selected values of α, λ, and T

n	r_1	r_2	α	λ	T	$\widehat{H}$	RMSE $\widehat{H}$	RMSE $\hat{\alpha}$	RMSE $\hat{\lambda}$
200	80	120	1.5	7	5	6.068	0.732	0.825	0.635
					7	6.024	0.719	0.824	0.641
					10	7.338	1.095	0.822	0.371
					15	5.502	0.579	0.823	0.777
					18	5.481	0.565	0.824	0.789
					20	5.453	0.557	0.823	0.793
				8	5	6.059	0.666	0.819	0.642
					7	6.049	0.663	0.819	0.646
					10	5.811	0.598	0.803	0.638
					15	5.315	0.462	0.820	0.835
					18	5.179	0.424	0.819	0.852
					20	5.121	0.408	0.817	0.856
				9	5	6.046	0.610	0.813	0.653
					7	3.753	6.086	0.813	0.639
					10	5.952	0.585	0.802	0.627
					15	4.941	0.316	0.815	0.888
					18	4.961	0.321	0.813	0.883
					20	4.878	0.299	0.812	0.890
			2	7	5	4.985	0.792	0.814	0.524
					7	7.102	1.277	0.867	0.082
					10	4.493	0.440	0.790	0.720
					15	4.417	0.416	0.786	0.729
					18	4.356	0.396	0.781	0.730
					20	4.408	0.413	0.787	0.735
				8	5	6.523	1.005	0.862	0.402
					7	5.997	0.844	0.862	0.647
					10	5.716	0.757	0.858	0.710
					15	4.430	0.362	0.787	0.764

			18	4.507	0.385	0.791	0.754
			20	4.522	0.390	0.791	0.751
		9	5	5.992	0.778	0.856	0.650
			7	6.011	0.783	0.856	0.643
			10	6.398	0.898	0.852	0.421
			15	4.614	0.369	0.797	0.773
			18	4.579	0.358	0.797	0.783
			20	4.570	0.356	0.796	0.782
		10	5	5.992	0.778	0.856	0.650
			7	6.011	0.783	0.856	0.643
			10	6.398	0.898	0.852	0.421
			15	4.614	0.369	0.797	0.773
			18	4.579	0.358	0.797	0.783
			20	4.570	0.356	0.796	0.782
	3	7	5	5.993	0.724	0.852	0.652
			7	6.008	0.728	0.852	0.647
			10	6.019	0.732	0.852	0.643
			15	4.694	0.350	0.808	0.808
			18	4.495	0.293	0.800	0.825
			20	4.568	0.314	0.802	0.818
		8	5	8.608	2.130	0.917	2.399
			7	8.633	2.139	0.917	2.436
			10	4.555	0.656	0.811	0.499
			15	3.653	0.328	0.766	0.694
			18	3.654	0.328	0.767	0.695
			20	3.633	0.321	0.765	0.697
		9	5	6.719	1.342	0.911	0.426
			7	6.784	1.365	0.913	0.424
			10	5.007	0.745	0.853	0.592
			15	3.761	0.311	0.767	0.698
			18	3.758	0.310	0.767	0.698
			20	3.794	0.307	0.764	0.695

Table 2: Entropy estimates and relative MSEs for $\hat{\alpha}$,$\hat{\lambda}$, and $\widehat{H}$ for selected values of r_2

n	r_1	α	λ	T	r_2	$\widehat{H}$	RMSE $\widehat{H}$	RMSE $\hat{\alpha}$	RMSE $\hat{\lambda}$
200	80	2	9	5	140	6.018	0.785	0.856	0.641
					160	6.000	0.780	0.856	0.647
					180	5.984	0.775	0.856	0.651
				15	140	4.063	0.205	0.550	0.369
					160	11.591	2.439	0.951	0.387
					180	12.156	2.607	0.955	0.396
				20	140	3.229	0.041	0.192	0.345
					160	3.401	0.009	0.191	0.220
					180	6.509	0.931	0.841	0.213

Table 3: Entropy estimates and relative MSEs for $\hat{\alpha}$,$\hat{\lambda}$, and $\widehat{H}$ for selected values of n, r_1, r_2, and T

α	λ	r_1	r_2	n	T	$\widehat{H}$	RMSE $\widehat{H}$	RMSE $\hat{\alpha}$	RMSE $\hat{\lambda}$
2	9	40	120	150	7	6.134	0.820	0.859	0.616
					18	6.315	0.874	0.829	0.208
				100	5	5.984	0.775	0.855	0.643
					7	5.978	0.774	0.855	0.644
					10	6.016	0.785	0.855	0.627
					15	7.639	1.266	0.898	0.439
		15	35	50	5	6.113	0.814	0.859	0.623
					7	6.128	0.818	0.859	0.621
					10	5.071	0.505	0.805	0.677

5. Conclusions

Entropy estimates were computed using the MLE of α and λ in the Fréchet distribution based on generalized type I hybrid censored samples and compared them in terms of their RMSE. Although in this article we focused on the entropy estimate of the Fréchet distribution under the generalized type I hybrid censored samples, the proposed estimation can be extended to other distributions. Estimation of the entropy from other distributions under generalized hybrid censoring is of potential interest in future research.

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