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## **Photometric Analysis of Two Young Open Clusters: NGC 1545 and NGC 2099**

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### **Abstract**

We carried out a photometric study on two young open clusters NGC 1545 and NGC 2099 in the near-infrared region by making use of JHK<sub>s</sub> of the 2MASS catalog and the GAIA DR2 survey. In this study, we utilized a method that is able to distinguish cluster stars from background stars. Our calculated results indicate that for both cluster NGC 1545 and NGC 2099, the number of members are 3059 and 2932, respectively. We have computed the cluster center of the two clusters and found it to be estimated at  $\alpha = 04^{\text{h}}20^{\text{m}}59^{\text{s}}.87$ ,  $\delta = 50^{\circ}15'58''.46$ ,  $\alpha = 05^{\text{h}}52^{\text{m}}17^{\text{s}}.8$ ,  $\delta = 32^{\circ}33'5''.58$ , respectively. The limiting radius for both clusters is about  $3.5 \pm 1.1$  arcmin and  $29.2 \pm 1.3$  arcmin, respectively. The Color Magnitude Diagram, in GAIA DR2 Bands, for NGC 1545, enabled us to estimate the reddening at RP band to be  $E(\text{BP}-\text{RP}) = 0.09$  mag, and  $E(\text{BP}-\text{RP}) = 0.08$  mag, for G band. For NGC 2099, the reddening at RP band is estimated to be  $E(\text{BP}-\text{RP}) = -0.08$  mag, and for the G band, it's estimated to be  $E(\text{BP}-\text{RP}) = -0.08$  mag. While in the 2MASS Bands, the reddening at K<sub>s</sub> band is estimated to be  $E(\text{J}-\text{K}_s) = 0.00205$  mag, and at J band it's estimated to be  $E(\text{J}-\text{H}) = 0.001$  mag for NGC 1545. For NGC 2099, the reddening at K<sub>s</sub> band is estimated to be  $E(\text{J}-\text{K}_s) = -0.09$  mag, however, and  $E(\text{J}-\text{H}) = -0.09$  mag at J band. We obtained the distance modulus at RP band to be  $(m - M)_{\text{RP}} = 9.6$  mag and 10.6 mag, and  $(m - M)_{\text{G}} = 10.4$  mag and 10.6 mag, for the G band, whereas the distance modulus at K<sub>s</sub> band  $(m - M)_{\text{K}_s} = 9.5$  mag and 10.6 mag, and  $(m - M)_{\text{J}} = 9.5$  mag and 10.6 mag, for the J band., for NGC 1545 and NGC 2099, respectively.

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In contrast, the luminosity and mass functions of these two open clusters, NGC 1545 and NGC 2099, have been estimated to be  $1997.85842 M_{\odot}$  (for GAIA DR2 Bands) and  $1847.36496 M_{\odot}$  (for 2MASS Bands) and  $3289.40412 M_{\odot}$  (for GAIA DR2 Bands) and  $3317.5277 M_{\odot}$  (for 2MASS Bands), respectively, while the mass function slopes are  $-2.69397 \pm 0.24234$  (for GAIA DR2 Bands) and  $-2.69142 \pm 0.26602$  (for 2MASS Bands) and  $-3.37104 \pm 0.65729$  (for GAIA DR2 Bands) or  $-3.55212 \pm 0.28216$  (for 2MASS Bands) for NGC 1545 and NGC 2099, respectively. Finally, the dynamical state of these two clusters shows that NGC 1545 and NGC 2099 are clusters that can be considered as dynamically relaxed.

**Keywords:** Photometric; star clusters; young open clusters; GAIA DR2; 2MASS; NGC 1545; NGC 2099.

## **1. Introduction**

Studying open star clusters is considered as, practically, to be one of the fundamental ways of understanding star formation. Therefore, determining the physical parameters of an open cluster (including: distance, reddening, age, etc.) using observational data helps us to understand and reveal the galactic structure and evolution [1]. The fundamental parameters resulted from previous studies of the two open star clusters NGC 1545 and NGC 2099 are listed below in Table 1.

The images of these two clusters taken from the LEDAS Digitized Sky Survey (DSS) are shown in figure. In our study, we use the fundamental parameters taken from Cantat-Gaudin and his colleagues (2018) [2] and Kharchenko (2016) [3], with the observational data taken from the 2MASS catalog and GAIA DR2 survey, are used to determine the basic photometric and astrometric properties of the open star clusters NGC 1545 and NGC 2099. By combining the GAIA Data 2<sup>nd</sup> Release (GAIA DR2) and the two Micron All Sky Survey (2MASS) data, we obtain a complete worksheet data of right ascension ( $\alpha$ ) and declination ( $\delta$ ), and the angular distance from the cluster center that have been extracted for J, H, and K<sub>s</sub> (near infrared) region with radii of 30 arcmin for NGC 1545 and NGC 2099 open clusters.

The structure of this article is as follows: Section 2 reveals the data analysis. Section 3 shows the color magnitude diagram and isochrone fitting. Section 4 determines the cluster's distance to galactic center. Section 5 deals with the luminosity and mass functions, and Section 6 presents the dynamical state of these clusters. Conclusions are presented in Section 7.

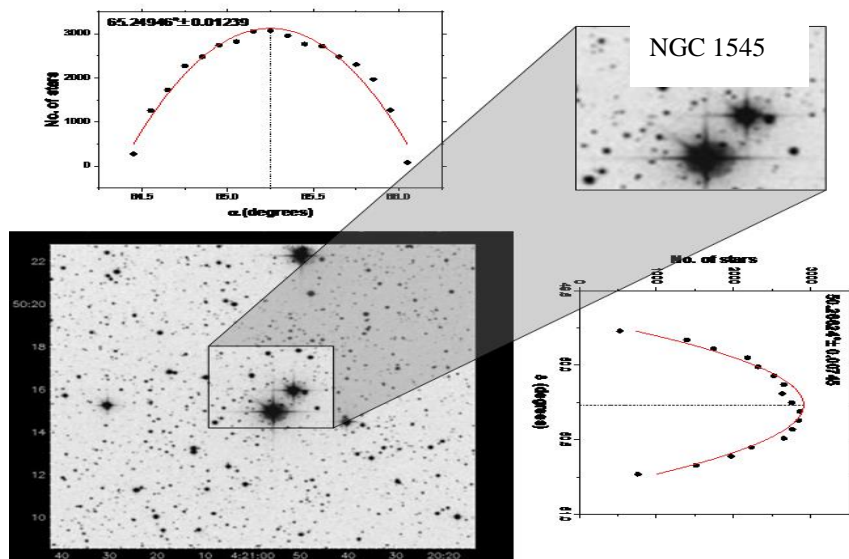
**Table 1:** The Fundamental Parameters of Two Young Open Clusters NGC 1545 and NGC 2099

Parameter	NGC 1545	NGC 2099	References
$\alpha$ (hh:mm:ss)	04 <sup>h</sup> 20 <sup>m</sup> 48 <sup>s</sup> .48 04 <sup>h</sup> 21 <sup>m</sup> 8 <sup>s</sup> .09	05 <sup>h</sup> 52 <sup>m</sup> 17 <sup>s</sup> .76 05 <sup>h</sup> 52 <sup>m</sup> 18 <sup>s</sup> .91	Cantat-Gaudin and his colleagues (2018) [2] Kharchenko (2016) [3]
$\delta$ (dd:mm:ss)	50°13'15".6 50°15'5".76	32°32'42".0 32°34'14".88	Cantat-Gaudin and his colleagues (2018) [2] Kharchenko (2016) [3]
$l$ (degree)	153°.366 153°.381	177°.642 177°.622	Cantat-Gaudin and his colleagues (2018) [2] Kharchenko (2016) [3]
$b$ (degree)	0°.145 0°.204	3°.086 3°.103	Cantat-Gaudin and his colleagues (2018) [2] Kharchenko (2016) [3]
Distance (pc)	863	1400	Kharchenko (2016) [3]
E (B-V) (mag)	0.354	0.350	Kharchenko (2016) [3]
Distance module ( $K_s$ ) (mag)	9.795	10.843	Kharchenko (2016) [3]
Distance module (J) (mag)	9.965	11.011	Kharchenko (2016) [3]
log (age) (yr)	8.81	8.55	Kharchenko (2016) [3]
$X_{\odot}$ (pc)	-640.5	-1434.8	Cantat-Gaudin and his colleagues (2018) [2]
$Y_{\odot}$ (pc)	321.21	59.1	Cantat-Gaudin and his colleagues (2018) [2]
$Z_{\odot}$ (pc)	1.8	77.4	Cantat-Gaudin and his colleagues (2018) [2]
$R_{gc}$ (pc)	8986.2	9775.0	Cantat-Gaudin and his colleagues (2018) [2]

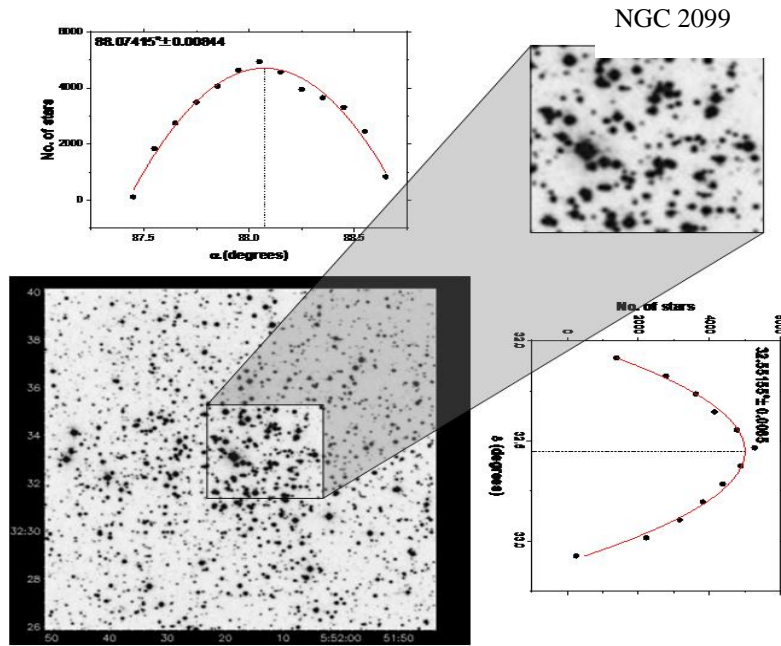
## 2. Data Analysis

### 2.1. Cluster Center Determination

We started the data analysis by redefining the location of the cluster center by applying a common procedure previously applied by several authors, e.g., [4-6], as follows: two perpendicular strips were cut along the declination and right ascension at the approximate center of the cluster, and histograms of the star counts were then created along each strip. The resulted histograms were then fitted with a Gaussian distribution function. The maximum value gives the position of the new cluster center, shown in Figures (1 a, 1 b) and listed in Table 2. The procedure was repeated with the GAIA DR2 data. Now centered on the new re-determined cluster center with an extraction circle with a radius of 30 arcmin.



**Figure 1 a:** The Gaussian fit showing the new center of highest density areas in  $\alpha= 04^{\text{h}}20^{\text{m}}59^{\text{s}}.87$ ,  $\delta= 50^{\circ}15'58''$ .46 of the image taken from LEDSA Digitized Sky Survey DSS, for open cluster NGC 1545.



**Figure 1 b:** The Gaussian fit showing the new center of highest density areas in  $\alpha= 05^{\text{h}}52^{\text{m}}17^{\text{s}}.8$ ,  $\delta= 32^{\circ}33'5''.58$  of the image taken from LEDSA Digitized Sky Survey DSS, for open cluster NGC 2099.

**Table 2:** Estimated Center of NGC 1545 and NGC 2099

Parameter	NGC 1545	NGC 2099
$\alpha$ (hh:mm:ss)	04 <sup>h</sup> 20 <sup>m</sup> 59 <sup>s</sup> .87	05 <sup>h</sup> 52 <sup>m</sup> 17 <sup>s</sup> .8
$\delta$ (dd:mm:ss)	50° 15' 58".46	32° 33' 5".58
$l$ (degree)	153°.366	177°.642
$b$ (degree)	0°.145	3°.086

By comparing our results with that of Cantat-Gaudin and his colleagues (2018) [2] we noticed that:

- For NGC 1545 the calculated right ascension and the declination are both greater than that given by Cantat-Gaudin and his colleagues (2018) [2] by about 11<sup>s</sup>.39 and 2'42".86, respectively.
- For NGC 2099 the calculated right ascension and the declination are both greater than that given by Cantat-Gaudin and his colleagues (2018) [2] by about 0<sup>s</sup>.04 and 1'36".42, respectively.

**2.2. Radial Density Profile (RDP)**

The radial density profile (RDP) is the distribution of the mean surface density of stars in concentric rings as a function of radius from the cluster center outwards. We computed the density  $\rho(r)$  of each ring by dividing the

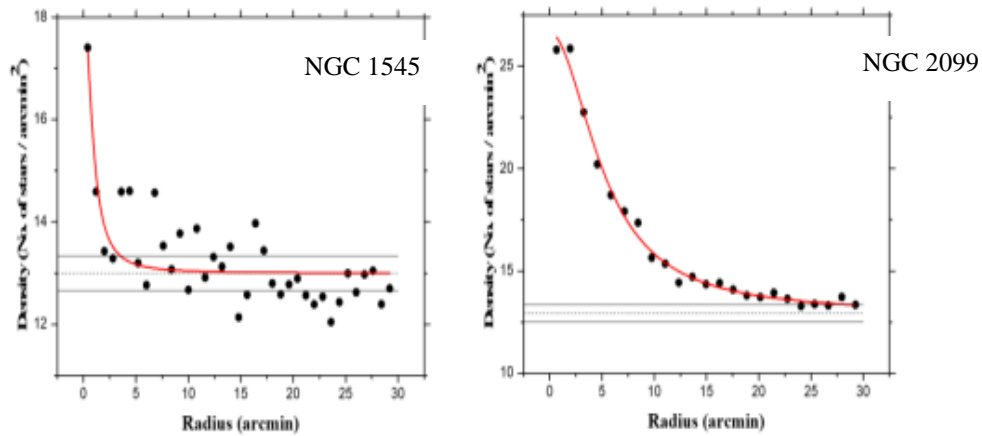
number of stars in the ring by its area (i.e.,  $N_i/A_i$ ). We then applied the empirical relation from [7] to parameterize the density function  $\rho(r)$  as:

$$\rho(r) = f_{bg} + \frac{f_0}{1 + \left(\frac{r}{r_{core}}\right)^2} \quad (1)$$

Where  $r_{core}$ ,  $f_0$ , and  $f_{bg}$  are the core radius, the central surface density, and the background surface density, respectively. The limiting radius of this cluster could reasonably be defined as the radius of the observed cluster area because the stellar background density shows a clear stability level at this border. This radius can be calculated if we consider the border of the background density level to be equal to  $\rho_b = f_{bg} + 3\sigma_{bg}$ , where  $\sigma_{bg}$  is the uncertainty of  $f_{bg}$ . If we add this border to Equation 1, the limiting radius  $r_{lim}$  of the cluster takes the following form, with its value given in Figure 2:

$$r_{lim} = r_{core} \sqrt{\frac{f_0}{3\sigma_{bg}} - 1} \quad (2)$$

The stellar concentration of a cluster is represented by the so-called cluster concentration parameter  $c$ , which is given by [8] as  $c = \log\left(\frac{r_{lim}}{r_{core}}\right)$ . The concentration parameter  $c$  and the limiting and core radius values are listed in Table 3.



**Figure 2:** The RDP of the NGC 1545 and NGC 2099 open clusters. The solid red lines represent the fitted density distribution and the dashed lines mark the background field density  $f_{bg}$ .

**Table 3:** RDP Parameters

Parameter	NGC 1545	NGC 2099
$f_{bg}$ (stars/arcmin <sup>2</sup> )	$12.9973 \pm 0.11428$	$12.921 \pm 0.14041$
$f_0$ (stars/arcmin <sup>2</sup> )	$5.10927 \pm 1.01588$	$13.68743 \pm 0.30316$

$r_{core}$ (arcmin)	$0.93954 \pm 0.28471$	$5.20076 \pm 0.21922$
$r_{lim}$ (arcmin)	$3.5 \pm 1.1$	$29.2 \pm 1.3$
$c$	$0.57 \pm 0.19$	$0.749 \pm 0.027$
$\rho_b$ (stars/arcmin <sup>2</sup> )	$13.34 \pm 0.11$	$13.34 \pm 0.14$

Nilakshi and his colleagues [9] concluded that the angular size of the coronal region is about 6 times the  $r_{core}$ . While Maciejewski and Niedzielski [10] reported that  $r_{lim}$  may vary for individual clusters between  $2r_{core}$  and  $7r_{core}$ . In our study, we noticed that the limiting radius is about  $0.57 \pm 0.19$  and  $0.749 \pm 0.027$  times core radius for NGC 1545 and NGC 2099, respectively. This is consistent with the results of Maciejewski and Niedzielski [10].

## 2.2. Membership of the cluster

To determine the cluster physical parameters such as the reddening, the age and the distance modulus, we have to separate the background stars (non-member stars) from the moving stars. We used techniques based on the parallax to determine the star cluster.

### 2.3.1. Vector Point Diagram (VPD)

The Vector Point Diagram (VPD) is a one of the first and widely used procedures that shows the correlation between the proper motion components pm of  $\alpha$  and pm of  $\delta$ . It is. The distribution of proper motion of stars VPD was proposed by Vasilevskis (Jones (1997)). Figure 3 represents a scattered plot. The blue points represent the member stars in the cluster, while the red points in the proper motion vector point diagram (VPD) represents the background stars (non-member stars).

### 2.3.2. Parallax

Figure 4 represents a histogram plot. The blue columns represent the star cluster, while the red columns represent the moving stars. To determine the star cluster, we select a subset of moving stars which is a subset of all the stars. Therefore, the star cluster resulted from a subset of moving stars that contains the highest column with outliers of it.

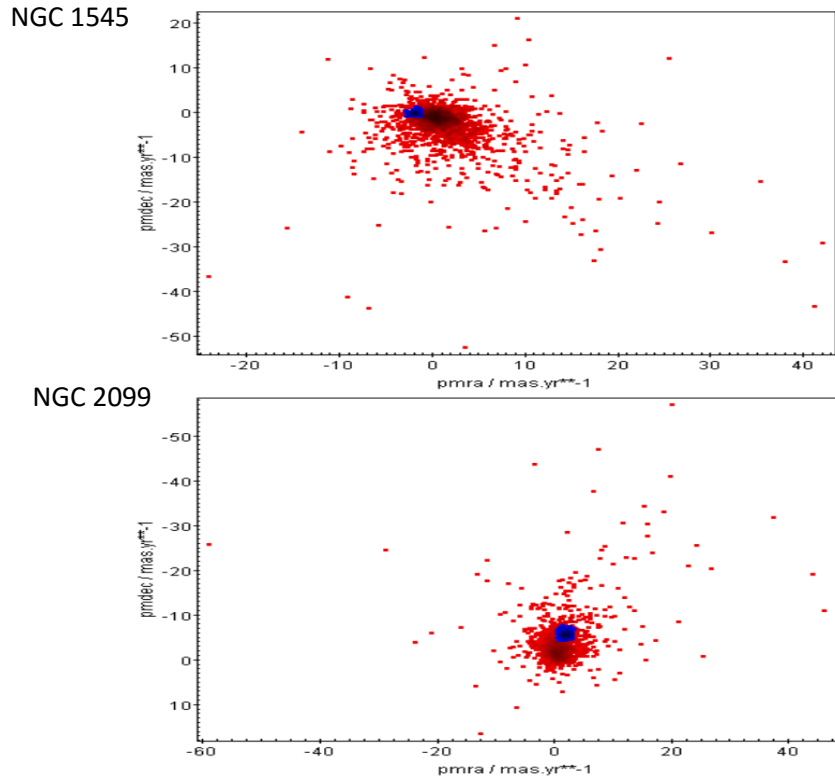
### 2.3.3. Color Magnitude Diagrams (CMDs)

#### 2.3.3.1. GAIA DR2 Bands

Figure 5a represents RP vs (BP – RP), while Figure 5b represents G vs (BP – RP). The blue points represent the member stars in the cluster, while the red points represent the background stars (non-member stars).

#### 2.3.3.2. 2MASS Bands

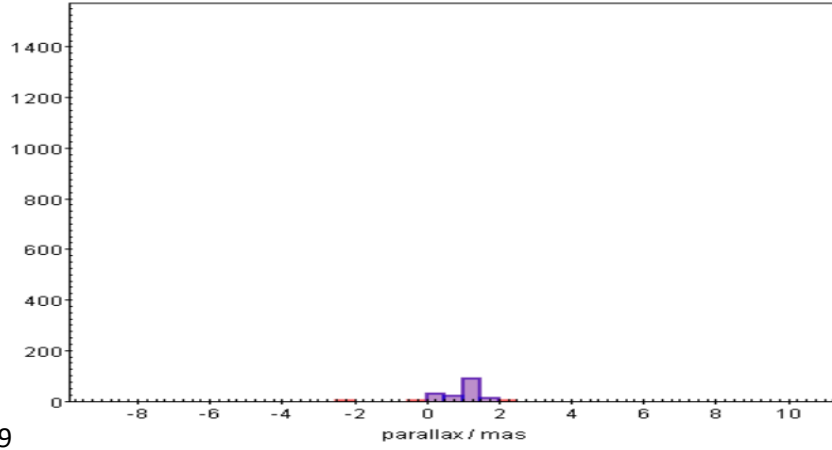
Figure 6a represents  $K_s$  vs  $J-K_s$ , while Figure 6b represents  $J$  vs  $J-H$ . The blue points represent the member stars in the cluster, while the red points represent the background stars (non-member stars).



**Figure 3:** VPDs of NGC 1545 and NGC 2099 open clusters.



NGC 1545



NGC 2099

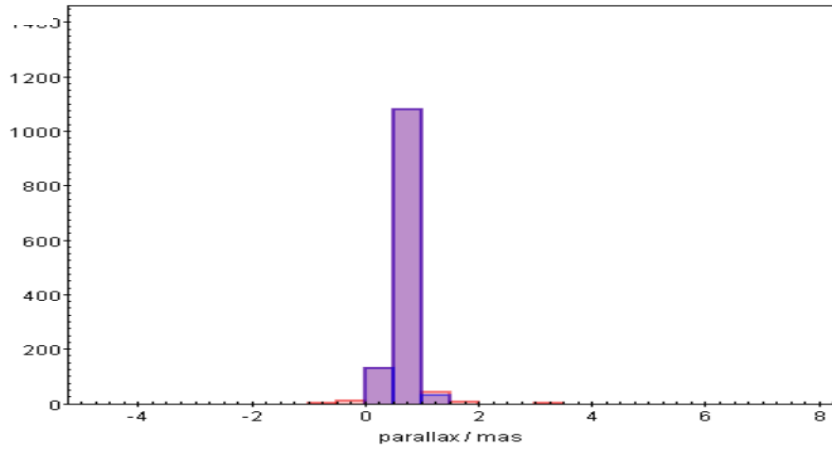


Figure 4: The parallax of NGC 1545 and NGC 2099 open clusters.

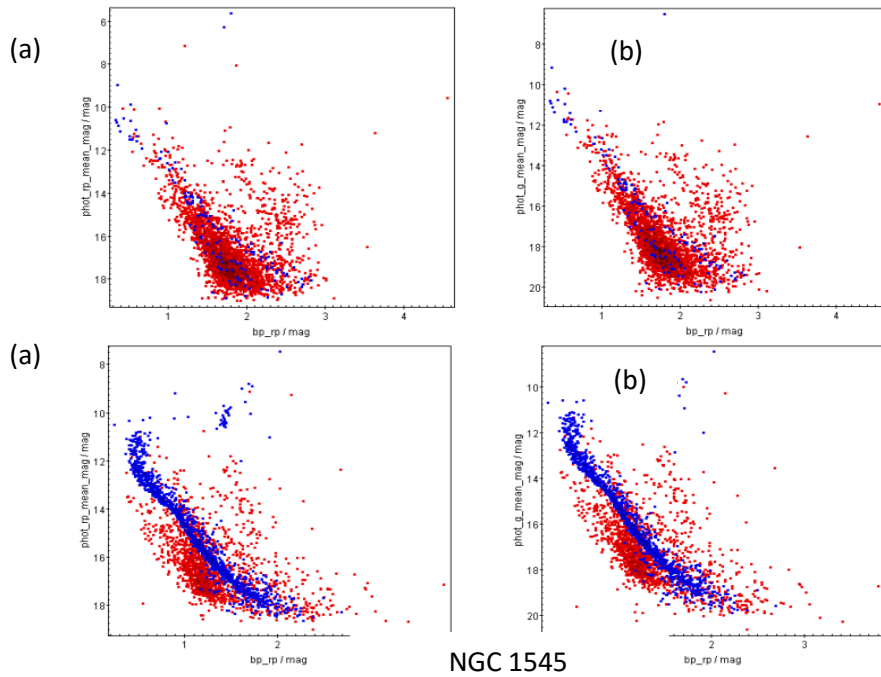
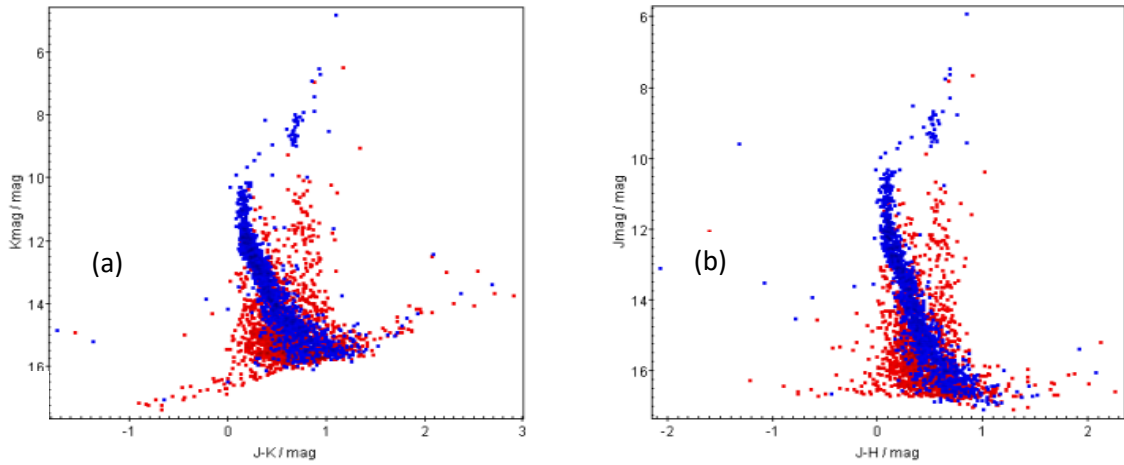
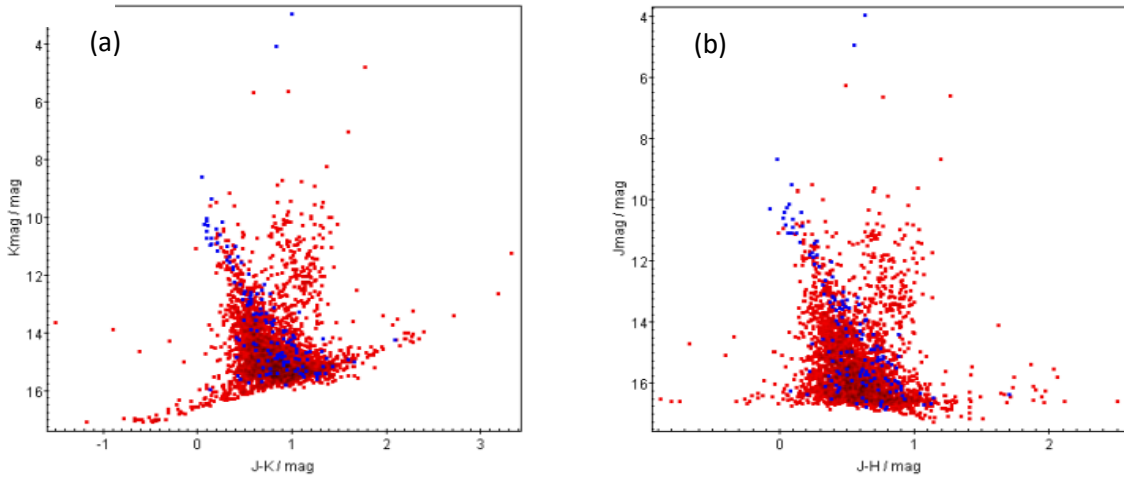


Figure 5: CMDs of NGC 1545 and NGC 2099 in GAIA DR2 Bands.

NGC 1545



NGC 2099

**Figure 6:** CMDs of NGC 1545 and NGC 2099 in 2MASS Bands.

### 3. CMDs and Isochrone Fitting

The main photometric parameters (e.g. age, reddening, and distance modulus) can be determined by using the CMDs fitting with the MESA Isochrones and Stellar Tracks (MIST) for synthetic photometry, with solar metallicity of  $Z = 0.018$ . In our study, we constructed CMDs for (RP, BP-RP & G, BP-RP) and ( $K_s$ , J- $K_s$  & J, J-H) of the NGC 1545 and NGC 2099 open clusters. The fitting of isochrones with the observed CMDs of NGC 1545 and NGC 2099 is shown in Figures 7 and 8.

#### 3.1. CMDs in GAIA DR2 Bands

We cleaned the observed CMDs (RP, BP-RP and G, BP-RP) of NGC 1545 and NGC 2099 of field star contamination and then fitted them with several isochrones of different ages. The best isochrone was then visually matched to the observed CMD. This was then used to give the absolute distance modulus and color excess. Figure 7 shows these CMDs superposed with the selected isochrone, which has an age of  $\log(\text{age}) = 7.95$  and  $8.69$  [yrs], respectively, assuming solar metallicity.

**Table 4:** The physical parameters of the cluster NGC 1545 and NGC 2099 in GAIA DR2 Bands

Parameter	NGC 1545	NGC 2099
Color excess at RP E(BP-RP) (mag)	0.09	-0.08
Color excess at G E(BP-RP) (mag)	0.08	-0.08
Apparent Distance modulus at RP (m-M) <sub>RP</sub> (mag)	9.6	10.6
Apparent Distance modulus at G (m-M) <sub>G</sub> (mag)	10.4	10.6
Absolute Distance modulus at RP (m-M) <sub>0</sub> (mag)	9.4437	10.739
Absolute Distance modulus at G (m-M) <sub>0</sub> (mag)	10.216	10.784
Average of Distance modulus (m-M) <sub>0</sub> (mag)	9.8301	10.761
Distance (d) (pc)	924.73 ± 0.3864	1419.9 ± 0.0223
d <sub>L</sub> (pc)	1.882	24.108

### 3.2. CMDs in 2MASS Bands

We cleaned the observed CMDs (K<sub>s</sub>, J-K<sub>s</sub> and J, J-H) of NGC 1545 and NGC 2099 of field star contamination and then fitted them with several isochrones of different ages.

The best isochrone was then visually matched to the observed CMD. This was then used to give the absolute distance modulus and color excess.

Figure 8 shows these CMDs superposed with the selected isochrone, which has an age of log(age) = 7.95 and 8.69 [yrs], respectively, assuming solar metallicity.

**Table 5:** The physical parameters of the cluster NGC 1545 and NGC 2099 in 2MASS Bands

Parameter	NGC 1545	NGC 2099
Color excess at $K_s$ $E(J-K_s)$ (mag)	0.00205	-0.09
Color excess at J $E(J-H)$ (mag)	0.001	-0.09
Apparent Distance modulus at $K_s$ $(m-M)_{K_s}$ (mag)	9.5	10.6
Apparent Distance modulus at J $(m-M)_J$ (mag)	9.5	10.6
Absolute Distance modulus at $K_s$ $(m-M)_0$ (mag)	9.49968	10.6142
Absolute Distance modulus at J $(m-M)_0$ (mag)	9.49937	10.6569
Average of Distance modulus $(m-M)_0$ (mag)	9.49952	10.6356
Distance (d) (pc)	$924.73 \pm 0.3864$	$1419.10 \pm 0.0223$
$d_L$ (pc)	1.616	22.752

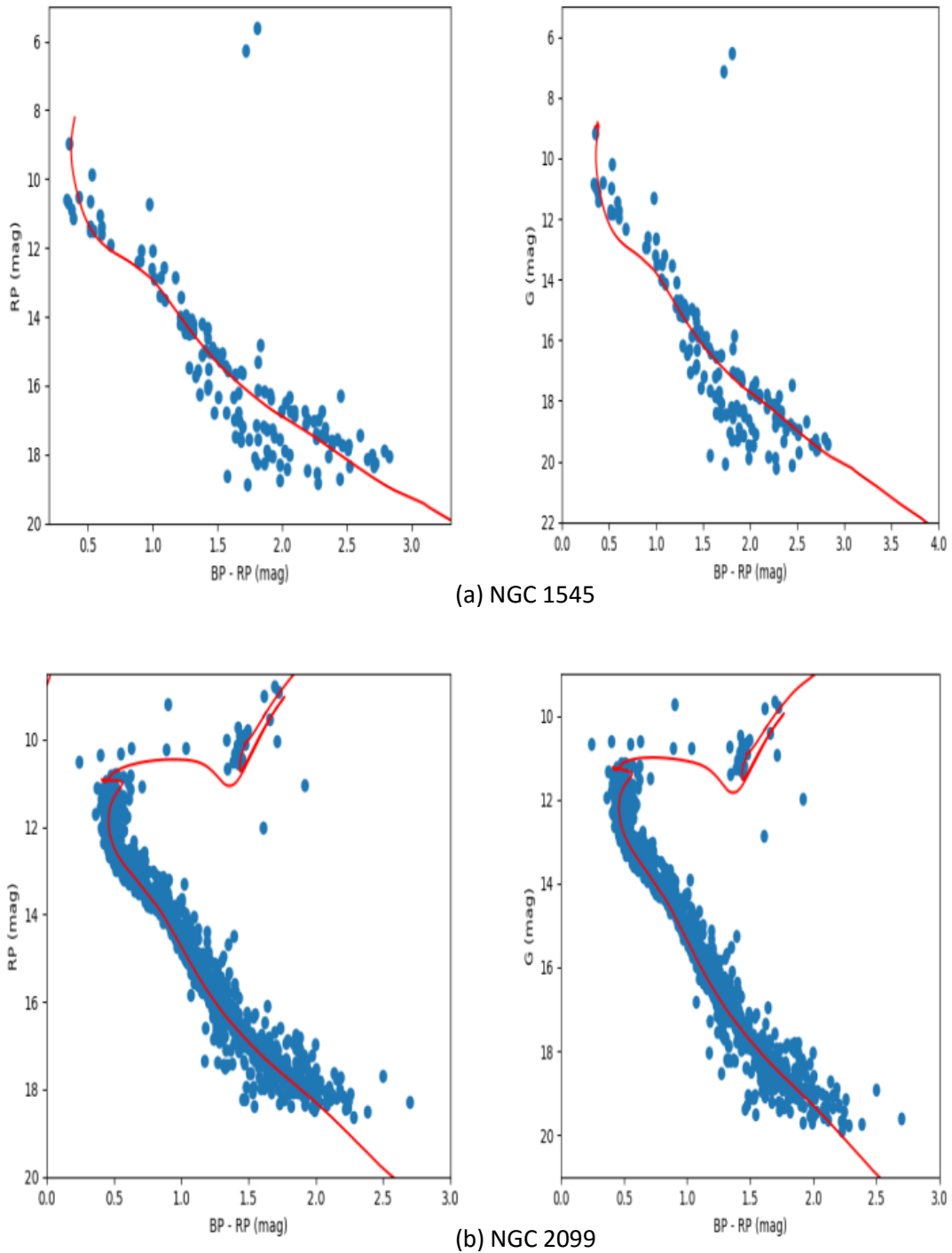
By comparing our results of  $\log(\text{age})$ ,  $(m - M)_{K_s}$  and  $(m - M)_J$ , with that obtained by Kharchenko (2016) [3] as listed in Table 1, we notice that:

- For NGC 1545: Our  $\log(\text{age})$  is less than that of Kharchenko (2016) [3] by about 0.86. Our results on the distance modulus at  $K_s$  and J is less than that of Kharchenko (2016) [3] by about 0.30 and 0.47, respectively.

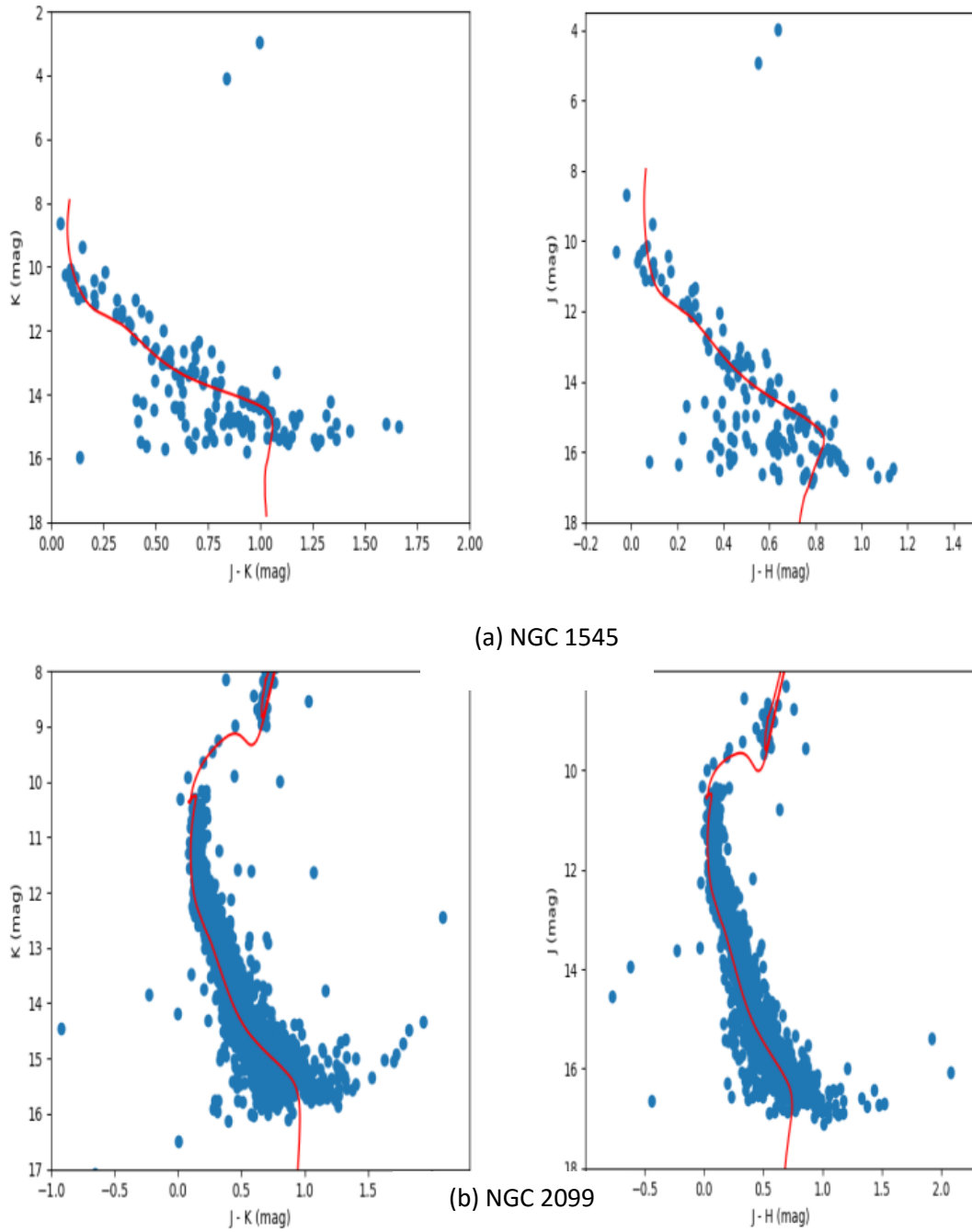
- For NGC 2099: Our  $\log(\text{age})$  is greater than that of Kharchenko (2016) [3] by about 0.14.

Our distance modulus at  $K_s$  and J is less than that of Kharchenko (2016) [3] by about 0.23 and 0.36, respectively. Our calculation indicates that the cluster distance for NGC 1545 and NGC 2099 is equal to  $794.153 \pm 0.0002$  pc and  $1340.02 \pm 0.0213$  pc, respectively. By comparing our results of distances with those listed by Kharchenko (2016) [3] catalogs, we notice that our calculated distance for NGC 1545 is smaller than

theirs by about 68.847 pc. While for NGC 2099, our calculated distance is less than theirs by about 59.98 pc.



**Figure 7:** [RP, (BP-RP) and G, (BP-RP)] Isochrone (solid red curve) with age of  $\log(\text{age}) = 7.95$  and  $8.69$  is fitted to the observed CMDs of NGC 1545 and NGC 2099.



**Figure 8:**  $[K_s, (J-K_s)$  and  $J, (J-H)]$  Isochrone (solid red curve) with age of  $\log(\text{age}) = 7.95$  and  $8.69$  is fitted to the observed CMDs of NGC 1545 and NGC 2099.

#### 4. Distance of the Cluster from the Galactic center

The cluster's distance from the Galactic center  $R_{gc}$  is determined by using the cluster-Sun distance, as well as the projected distance to the Galactic plane ( $X_{\odot}$  and  $Y_{\odot}$ ), and the distance from the Galactic plane  $Z_{\odot}$ . Based on Tadross [11], our calculations for  $R_{gc}$ ,  $X_{\odot}$ ,  $Y_{\odot}$  and  $Z_{\odot}$  are represented by:

**Table 6:** Cluster's distance to the Galactic center in GAIA DR2 Bands

Parameter	NGC 1545	NGC 2099
$X_{\odot}$ (kpc)	-0.82603945	-1.41654904
$Y_{\odot}$ (kpc)	0.41566684	0.06056126
$Z_{\odot}$ (kpc)	0.00233905	0.07640135
$R_{gc}$ (kpc)	9.3352984	9.91702827

**Table 7:** Cluster's distance to the Galactic center in 2MASS Bands

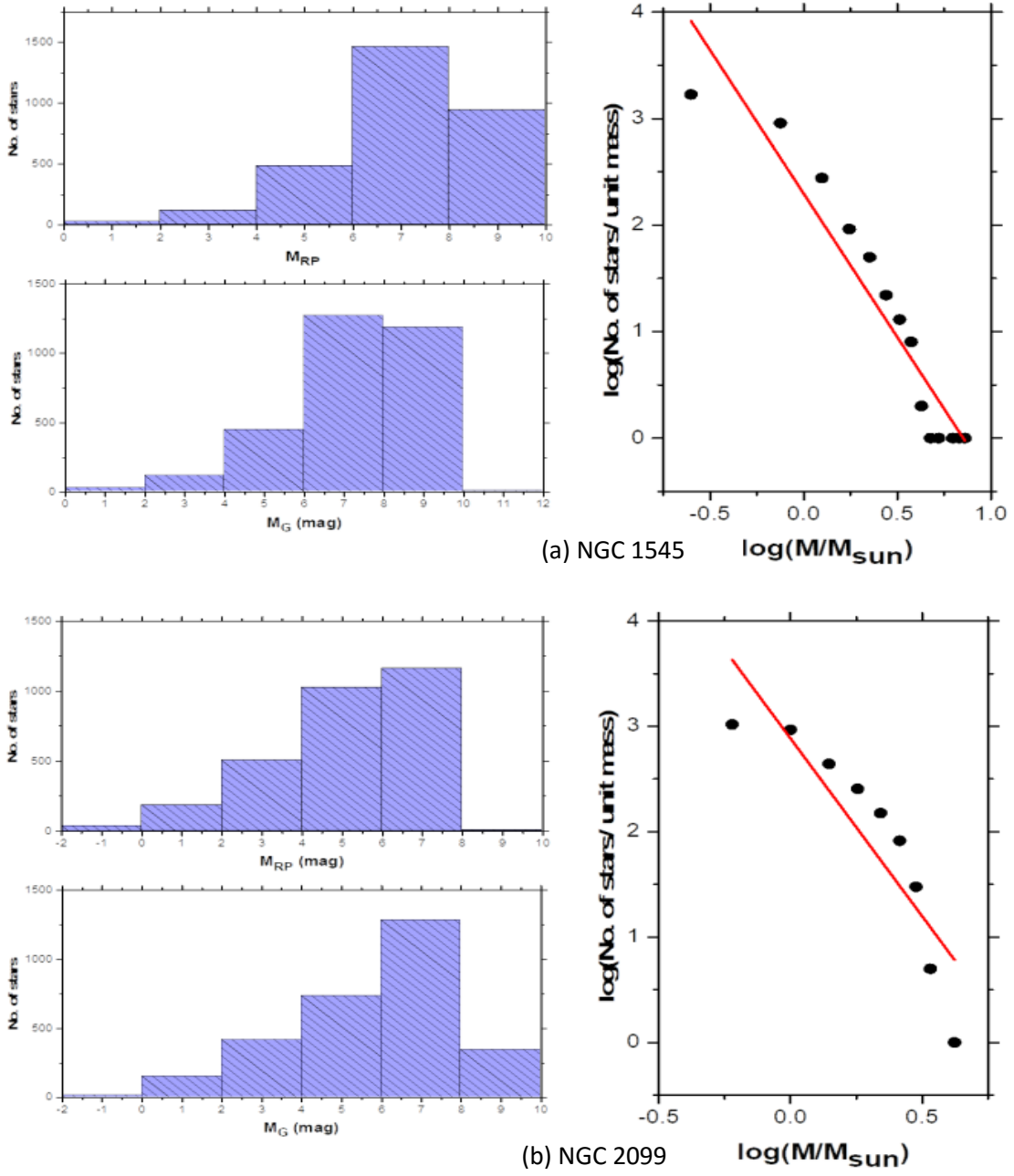
Parameter	NGC 1545	NGC 2099
$X_{\odot}$ (kpc)	-0.70948927	-1.33681611
$Y_{\odot}$ (kpc)	0.35678562	0.05761972
$Z_{\odot}$ (kpc)	0.00282612	0.07250001
$R_{gc}$ (kpc)	9.21639824	9.83725203

## 5. Mass Function and Luminosity

The luminosity function LF is described in this study as the total number of stars as a function of absolute magnitudes in a certain region of surface area. The main characteristics of studying the open clusters are to study the mass function MF, which represents the mass distribution (i.e., histogram of stellar masses) of a population of stars with respect to their assumed initial mass (the mass they were formed before with). The Initial Mass Function IMF is defined in terms of a power law as follows:

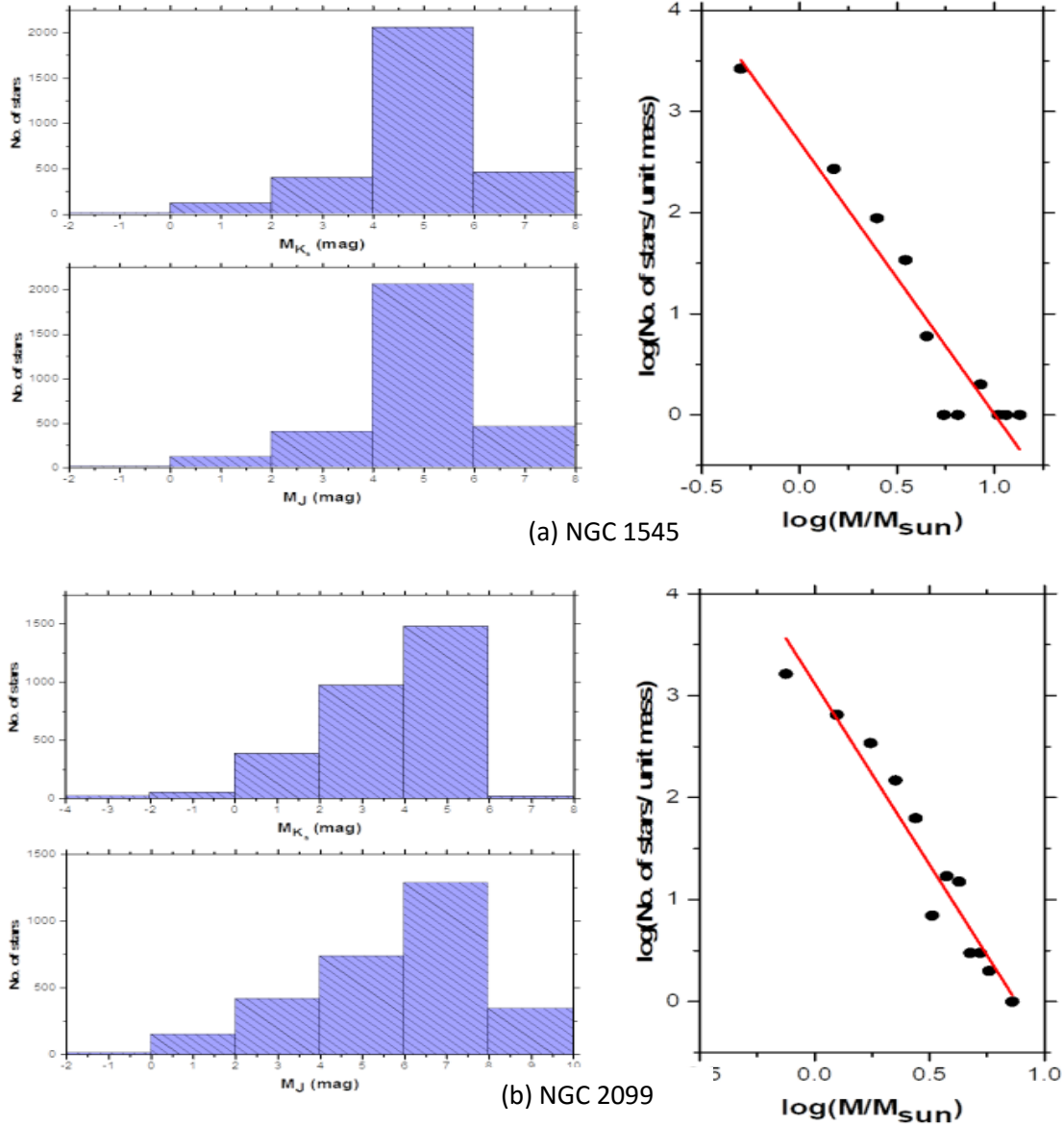
$$\frac{dN}{dM} \propto M^{-\alpha} \quad (3)$$

Where,  $\frac{dN}{dM}$  is the number of stars on mass interval ( $M: M + dM$ ), and  $F$  is a dimensionless exponent. From Salpeter [12], the IMF for massive stars ( $>1 M_{\odot}$ ) has been well examined and established i.e.  $\alpha = 2.35$ . The steep slope of the IMF shows that the number of low-mass stars is greater than the high-mass stars. MF correlated with LF by the so-called mass-luminosity relation MLR. To determine the cluster LF, we counted the observed stars in the form of absolute magnitude after applying the distance modulus. LF of those two open clusters NGC 1545 and NGC 2099 are constructed as shown in Figure 9 and 10.



**Figure 9:** The distribution of the LFs and MFs of NGC 1545 and NGC 2099 in GAIA DR2 Bands.





**Figure 10:** The distribution of the LFs and MFs of NGC 1545 and NGC 2099 in 2MASS Bands.

In this study, the MF will be estimated based on the MESA isochrones. The relation is a polynomial function of second order, which means:

- For NGC 1545:

- In GAIA DR2 Band:

$M/M_{\odot} = (4.034419043) - (0.7347578927) M_G + (0.03509029284) M_G^2$ . From the polynomials, we can get the total estimated mass  $M_c$  as  $1997.85842 M_{\odot}$

- In 2MASS Band:

$M/M_{\odot} = (3.781056578) - (1.053717086) M_{K_s} + (0.07749375171) M_{K_s}^2$  From the polynomials, we can get the total estimated mass  $M_c$  as  $1847.36496M_{\odot}$

- For NGC 2099:

- In GAIA DR2 Band:

$M/M_{\odot} = (3.049989594) - (0.4464340646) M_G + (0.01771642556) M_G^2$ . From the polynomials, we can get the total estimated mass  $M_c$  as  $3289.40412M_{\odot}$

- In 2MASS Band:

$M/M_{\odot} = (2.70861591) - (0.5824344956) M_{K_s} + (0.03373319858) M_{K_s}^2$ . From the polynomials, we can get the total estimated mass  $M_c$  as  $3317.5277M_{\odot}$

Figures 9 a, 9 b, 10 a and 10 b (right panel), show the MFs of these open clusters. The slopes are:  $-2.69397 \pm 0.24234$  (in GAIA DR2 Bands)  $-2.69142 \pm 0.26602$  (in 2MASS Bands) and  $-3.37104 \pm 0.65729$  (in GAIA DR2 Bands)  $-3.55212 \pm 0.28216$  (in 2MASS Bands) for NGC 1545 and NGC 2099. Our results are in agreement with Salpeter [12]. By using our total estimated mass  $M_c$  and the relation 4 given by Jeffries and his colleagues [14], our results give a tidal radius  $r_t$  of about 18.39 pc (in GAIA DR2 Bands), while 17.91 pc (in 2MASS Bands) and 21.71 pc (in GAIA DR2 Bands), while 21.77 pc (in 2MASS Bands) for NGC 1545 and NGC 2099, respectively.

$$r_t = 1.46 \sqrt[3]{M_c} \quad (4)$$

**Table 8:** The derived parameters

Parameter	NGC 1545	NGC 2099
Slope ( $\alpha$ ) (in GAIA DR2 Bands)	$-2.69397 \pm 0.24234$	$-3.37104 \pm 0.65729$
Slope ( $\alpha$ ) (in 2MASS Bands)	$-2.69142 \pm 0.26602$	$-3.55212 \pm 0.28216$
$M_c$ (in GAIA DR2 Bands)	$1997.85842M_{\odot}$	$3289.40412M_{\odot}$
$M_c$ (in 2MASS Bands)	$1847.36496M_{\odot}$	$3317.5277M_{\odot}$
$r_t$ (pc) (in GAIA DR2 Bands)	18.39	21.71
$r_t$ (pc) (in 2MASS Bands)	17.91	21.77

## 6. Dynamical State

To describe the dynamical state of the open clusters, we calculated the dynamical evolution parameter (i.e.,  $\tau = T_{\text{age}} / T_{\text{relax}}$ ), where  $T_{\text{relax}}$  is the dynamical relaxation time, which is defined as the time the cluster requires to reach stability (Maxwellian state). During that time, the low mass star in a cluster possesses the largest random velocity, occupying a larger volume than the high mass [15]. If  $\tau \gg 1$ , then the cluster may be called dynamically relaxed, and vice versa. Mathematically, the relaxation time has the form [16]

$$T_{\text{relax}} = \frac{8.9 \times 10^5 \sqrt{N} R_h^{\frac{3}{2}}}{\langle m \rangle^{\frac{1}{2}} \log(0.4 N)} \quad (5)$$

Where  $R_h$  is the radius containing half of the cluster mass,  $N$  is the number of cluster members, and  $\langle m \rangle$  is the average mass of the cluster stars. Table 9 presents the numerical values of these parameters for both NGC 1545 and NGC 2099. As seen from Table 8, the dynamical evolution parameters (i.e.,  $\tau$ ) for both clusters NGC 1545 and NGC 2099 36 (in GAIA DR2 Bands) or 43.48 (in 2MASS Bands) and 10.43 (in GAIA DR2 Bands) or 11.40 (in 2MASS Bands), respectively. Thus NGC 1545 and NGC 2099 can be considered as a dynamically relaxed cluster.

**Table 9:** The Dynamical State Parameters of NGC 1545 and NGC 2099

Parameter	NGC 1545	NGC 2099
No. of members (N)	3059	2932
$T_{\text{age}}$ (log) yrs	7.95	8.69
$\langle m \rangle$ (in GAIA DR2 Bands)	$0.65311 \pm 0.55143 M_{\odot}$	$1.1219 \pm 0.55662 M_{\odot}$
$\langle m \rangle$ (in 2MASS Bands)	$0.60391 \pm 0.73287 M_{\odot}$	$1.13149 \pm 0.64902 M_{\odot}$
$R_h$ (pc) (in GAIA DR2 Bands)	$0.253 \pm 0.077$	$2.147 \pm 0.09$
$R_h$ (pc) (in 2MASS Bands)	$0.217 \pm 0.066$	$2.026 \pm 0.086$
$T_{\text{relax}}$ (Myr) (in GAIA DR2 Bands)	2.51	46.64
$T_{\text{relax}}$ (Myr) (in 2MASS Bands)	2.07	42.57
$\tau$ (in GAIA DR2 Bands)	36	10.43
$\tau$ (in 2MASS Bands)	43.48	11.40

## 7. Dissection and Conclusion

The core aim of this study is to find the key photometric parameters as a first study of the open clusters NGC 1545 and NGC 2099.

Our calculations are based on the BP, RP and G and near infrared region  $JHK_s$  using the 2MASS catalog and the GAIA DR2 survey. Our calculation results are summarized in the following points:

- In this study, we have re-calculated the centers of the clusters. For NGC 1545, the right ascension and declination is found to be greater than that given by Cantat-Gaudin and his colleagues (2018) [2] by about  $11^s.39$  and  $2'42''.86$ , respectively. For NGC 2099 both right ascension and declination are found to be greater than that given by Cantat-Gaudin and his colleagues (2018) [2] by about  $0^s.04$  and  $1'36''.42$ , respectively.
- We have determined the number of cluster members are 3059 and 2932 for NGC 1545 and NGC 2099, respectively.
- We have constructed the RDP, showing that the limiting radius for NGC 1545 is  $3.5 \pm 1.1$  arcmin and  $29.2 \pm 1.3$  arcmin for NGC 2099.
- The construction of the CMD of solar metallicity ( $Z = 0.018$ ) enables us to calculate certain photometric parameters for those clusters, such as the distance modulus at RP ( $m - M$ )<sub>RP</sub> = 9.6 mag and 10.6 mag, and distance modulus is at G ( $m - M$ )<sub>G</sub> = 10.4 mag and 10.6 mag, while distance modulus at  $K_s$  ( $m - M$ ) <sub>$K_s$</sub>  = 9.5 mag and 10.6 mag, and distance modulus at J ( $m - M$ )<sub>J</sub> = 9.5 mag and 10.6 mag, which indicates a distance of about  $924.73 \pm 0.3864$  pc (in GAIA DR2 Bands) or  $794.153 \pm 0.0002$  pc (in 2MASS Bands) and  $1419.9 \pm 0.0223$  pc (in GAIA DR2 Bands) or  $1340.02 \pm 0.0213$  pc (in 2MASS Bands) for NGC 1545 and NGC 2099, respectively. The reddening at RP are E (BP- RP) = 0.09 mag, but at G are E (BP- RP) = 0.08 mag for NGC 1545 and the reddening at RP are E (BP- RP) = -0.08 mag, but at G are E (BP- RP) = -0.08 mag for NGC 2099 in GAIA DR2 Bands, while the reddening at  $K_s$  are E (J-  $K_s$ ) = 0.00205 mag, but at J are E (J- H) = 0.001 mag for NGC 1545 and the reddening at  $K_s$  are E (J-  $K_s$ ) = -0.09 mag, but at J are E (J- H) = -0.09 mag for NGC 2099 and in 2MASS Bands.
- We have determined the LF and MF by applying the MLR for LF, which shows a steady increase towards low luminosity stars from high luminous ones. In contrast, the value of MF slopes was about  $-2.69397 \pm 0.24234$  (in GAIA DR2 Bands) or  $-2.69142 \pm 0.26602$  (in 2MASS Bands) and  $-3.37104 \pm 0.65729$  (in GAIA DR2 Bands) or  $-3.55212 \pm 0.28216$  (in 2MASS Bands) for NGC 1545 and NGC 2099 open clusters, respectively, which were found to be close to values reported by Salpeter.
- The total mass calculated for these two open clusters is around  $1997.85842M_{\odot}$  (in GAIA DR2 Bands) are  $1847.36496M_{\odot}$  (in 2MASS Bands) and  $3289.40412M_{\odot}$  (in GAIA DR2 Bands) or  $3317.5277M_{\odot}$  (in 2MASS Bands) for NGC 1545 and NGC 2099 respectively.
- Finally, we have calculated the dynamical evolution parameter  $\tau$  for both clusters; form our calculations, we can see that the NGC 1545and NGC 2099 can be considered as a dynamically relaxed cluster.

## **Acknowledgements**

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