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Robust Geographically and Temporally Weighted Regression Using S-estimator in Criminal Case in East Java Province

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Abstract

Geographically weighted regression (GWR) is a model that can be used for data with spatial varying. Geographically and Temporally Weighted Regression (GTWR) is a development of the GWR model for data spatial and temporal varying. Parameter estimation in GTWR model uses weighted least square method which is very sensitive to outliers data. The outlier caused bias in parameter estimation, so it must be handled by robust GTWR (RGTWR). In this research, S-estimator was used to handle outliers and estimate an RGTWR. Both GTWR and RGTWR is used to build model crime rate in East Java 2011-2015. The Crime rate is used as a response variable and the percentage of poor people, population density, and human development index are used as explanatory variables. The best model in this research is RGTWR using S-estimator. RGTWR using S-estimator has a coefficient of determination equal to 98,2 meanwhile RMSE equal to 33.941 and MAD equal to 4.994.

Keywords: GWR; GTWR; S-estimator; RMSE; MAD; crime rate.

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1. Introduction

Geographically weighted regression (GWR) is a spatial statistical method to discover geographical variations in the relationship between response variables and different sets of covariates [7]. The GWR model uses a point approach that each parameter's value is estimated at location point. GWR model had a great interest in spatial data and successfully applied in spatial practical problems [6]. Beyoncé space, time is also an important dimension related to social and environmental cases [7]. The time aspect can provide valuable information on the dynamics of the underlying spatial processes. GWR model developed later by involving time aspects, which became known as geographically and temporally weighted regression (GTWR). GTWR model has been applied in economics. Widiyanti and his colleagues [15] using GTWR for modeling proportion of the poor district/cities in Central Java Province in 2014. In 2017, Sholihin and his colleagues [12] conducted a model for economic growth in Central Java using the GTWR approach. GTWR model in the estimation parameters uses weighted least squares method. Weighted least squares method is very sensitive to outlier data. According to Fotheringham and his colleagues [6], outlier effects can make models unsuitable for GWR. Fotheringham and his colleagues [6] suggested two methods to identify and handle outliers. The first method is to detect potential outliers and delete outliers from data. Another method is reweighting based on the standard error. Local outliers are not always global outliers [5] and not always wrong data [9]. Therefore, the outlier must be treated with care and not thrown away. Similar to GWR model, the outlier effect in GTWR model also makes the model is incompatible. Handling outliers in the GTWR model in this research by re-weighting. The name of the method is robust GTWR (RGTWR). RGTWR has been applied by Erda and his colleagues using M-estimator [4]. The other estimator that can be used for modeling robust GTWR is S-estimator. S-estimator has breakdown point of 50% [11]. In this research, the case study focused on crime rate in East Java 2011-2015. Crime is the act of someone who can be punished according to criminal law or other laws and regulations that apply in Indonesia. Crime in each region is often associated with economic, social and demographic factors. Several previous studied on criminality conducted by Simamora and Vita [13] in 2014 and Dona and Setiawan [3] in 2015. Simamora and Vita modeled factors that influence crime rates in East Java Province using the GWR approach. Dona and Setiawan also used spatial regression analysis for modeling factors that influence crime rates in East Java Province. Based on data released by the Indonesian Central Bureau of Statistics (BPS), crime rate in East Java relatively high. Crime rate in East Java Province from 2011 to 2015 has increased and decreased. East Java Province, including 5 provinces that are prone to crime [2]. However, several district/cities have crime rate are far from the other district/cities in East Java. So that is can be considered outliers. Therefore, RGTWR using S-estimator can be used for modeling crime rate in East Java in 2011-2015. Base on that background, the problem limitation focused on overcoming outliers in criminal data in East Java in 2011-2015 using the S-estimator. The purpose of this research is to look at the effectiveness of the RGTWR using S-estimator method in improving the GTWR model that contains outliers in criminal cases in East Java Provinces in 2011-2015.

2. Material and Method

2.1. Geographically and Temporally Weighted Regression

Geographically and temporally weighted regression (GTWR) is a model to handle the heterogeneity in spatial and temporal simultaneously [14]. The GTWR model combines temporal and spatial information in the weighting matrix. GTWR model formula can be expressed as follows:

$$y_i = \beta_0(u_i, v_i, t_i) + \sum_{k=1}^{p} \beta_k(u_i, v_i, t_i)x_{ik} + \varepsilon_i \tag{1}$$

Where $\beta_k(u_i, v_i, t_i)$ is regression coefficient of the k explanatory variable at each space-time location-i. Similarly, the estimation $\beta_k(u_i, v_i, t_i)$ of can be expressed as follows:

$$\widehat{\boldsymbol{\beta}}(u_i, v_i, t_i) = [\boldsymbol{X^T W}(u_i, v_i, t_i)\boldsymbol{X}]^{-1}\boldsymbol{X^T W}(u_i, v_i, t_i)\boldsymbol{y} \tag{2}$$

Where $(u_i, v_i, t_i) = diag(w_{i1}, w_{i2}, \dots, w_{in})$ and $\boldsymbol{W}(u_i, v_i, t_i)$ is weighted matrix at the observed location (u_i, v_i) and time $t_i$with n is number of observations. The spatio-temporal distances $\left(d_{ij}^{ST}\right)$ is combining spatial distance $\left(d_{ij}^{S}\right)$ and temporal distance$\left(d_{ij}^{T}\right)$, then it is expressed as a linear combination between $\left(d_{ij}^{S}\right)$ and $\left(d_{ij}^{T}\right)$:

$$\left(d_{ij}^{ST}\right)^2 = \lambda\left(d_{ij}^{S}\right)^2 + \mu\left(d_{ij}^{T}\right)^2 \tag{3}$$

With

Spatial distance : (d_ij^S)^2=〖(u_i-u_j)〗^2+〖(v_i-v_j)〗^2

Temporal distance: (d_ij^T)^2=〖(t_i-t_j)〗^2

Where λ and μ are scale factors to balance the different effects used to measure the spatial and temporal distance in their respective metric systems [8].

Let be τ denote the parameter ratio μ/ λ and λ≠0 then equation obtained as follow:

$$\frac{\left(d_{ij}^{ST}\right)^2}{\lambda} = (u_i - u_j)^2 + (v_i - v_j)^2 + \tau(t_i - t_j)^2 \tag{4}$$

τ is to enlarge/reduce the temporal distance effect to match with spatial distance [8]. The parameter τ obtained from the minimum cross-validation (CV) criteria by initializing the initial value τ as written bellow [8]:

$$CV(\tau) = \sum_{i=1}^{n}[y_{=i} - \hat{y}_{\neq i}(\tau)]^2 \tag{5}$$

Furthermore, the estimator parameters λ and μ can be obtained by the iterative method based on the estimation results τ.

2.2. S-estimator

The S-estimator was first introduced by Rousseeuw and Yohai [11] with breakdown points that can reach up to 50%. The S-estimator minimizes the dispersion of residuals, defined $\widehat{\boldsymbol{\beta}}_{\boldsymbol{s}} = \mathbf{min}_{\boldsymbol{\beta}}\ \widehat{\boldsymbol{\sigma}}_{\boldsymbol{s}}\,(\boldsymbol{e_1}, \boldsymbol{e_2}, \dots, \boldsymbol{e_n})$ where dispersion $\boldsymbol{\hat{\sigma}_s(e_1, e_2, \dots, e_n)}$ is a solution

$$\boldsymbol{\frac{1}{n}\sum_{i=1}^{n} \rho\left(\frac{y_{i} - \sum_{i=1}^{n} x_{ij}\beta}{\hat{\sigma}_s}\right) = K} \tag{6}$$

where K is the right constant value to verify consistency with K=0.199 and ρ is an objective function that is used to find the weighting function in robust regression [4]. The objective function that can be used is the objective function of Tukey's bisquare [16]:

$$\boldsymbol{\rho(u_i) = \begin{cases} u_i\left[1 - \left(\frac{u_i}{c}\right)^2\right]^2 & , \quad |u_i| \le c \\ 0 & , \quad |u_i| > c \end{cases}} \tag{7}$$

with the weighting function are as follows:

$$\boldsymbol{W(u, c) = \begin{cases} \left[1 - \left(\frac{u_i}{c}\right)^2\right]^2 & , \quad |u_i| \le c \\ 0 & , \quad |u_i| > c \end{cases}} \tag{8}$$

Where c is tunning constant, with value of c = 1,547 and $\mathbf{u_i} = \mathbf{e_i}/\boldsymbol{\hat{\sigma}_s}$.

2.3. Estimation Robust GTWR (RGTWR) using S-estimator

RGTWR using S-estimator models by adding a weighting to the GTWR equation model as follows:

$$\rho(y_i) = \rho\left[\beta_0(u_i, v_i, t_i) + \sum_{k=1}^{p} \beta_k(u_i, v_i, t_i)x_{ik} + \varepsilon_i\right] \tag{9}$$

With$i = 1,2, \dots, n$; $x_i = x_{i1}, x_{i2}, \dots, x_{ik}$, the basic equation RGTWR S-estimator in obtaining the estimator parameters model uses the following equation:

$$\sum_{i=1}^{n} \rho(\varepsilon_i) = \sum_{i=1}^{n} \rho\left[\frac{y_i - X_i\beta(u_i, v_i, t_i)}{\sigma_S}\right] \tag{10}$$

The value of σ_s is calculated using equations like the following:

$$\hat{\sigma}_s = \begin{cases} \frac{median\,|e_i - median(e_i)|}{0.6745} & , iterasi = 1 \\ \sqrt{\frac{1}{nK}\sum_{i=1}^{n} \omega_i e_i^2} & , iterasi > 1 \end{cases} \tag{11}$$

Estimation parameters of RGTWR model using the S-estimator do by reweighted least squares method with the following equation:

$$\boldsymbol{\hat{\beta}}(u_i, v_i, t_i) = \left[\boldsymbol{X_i}^{\boldsymbol{T}}\boldsymbol{W_i X_i}\right]^{-1}\boldsymbol{X_i}^{\boldsymbol{T}}\boldsymbol{W_i y_i} \tag{12}$$

Estimation parameters RGTWR in equation (12) do by iteratively reweighted least square (IRLS). In the iteration, value of w_i will be change in each iteration.

Algorithm:

1. Calculate the value of $\hat{\boldsymbol{y}}_i$ with the following formula: $\hat{\boldsymbol{y}}_i = \boldsymbol{X}_i^T \, \hat{\boldsymbol{\beta}} \, (u_i, v_i, t)^0$

The values of $\hat{\boldsymbol{\beta}} \, (u_i, v_{i,}, t)^0$ in each location are obtained from the GTWR modeling that has been done before.

2. Calculate residual value $e_i = \boldsymbol{y}_i - \hat{\boldsymbol{y}}_i$
3. Calculate value, $u_i = \frac{e_i}{\hat{\sigma}_s}$

$$\hat{\sigma}_s = \begin{cases} \dfrac{median\,|e_i - median(e_i)|}{0.6745}, & iterasi = 1 \\ \sqrt{\dfrac{1}{nK}\sum_{i=1}^{n} \omega_i e_i^2} & , \ iterasi > 1 \end{cases}$$

4. Calculate the weight value (w_i) using Turkey's bisquare weighting function with the tuning constanta value c = 1.547.

$$w_i = \begin{cases} \begin{cases} \left[1 - \left(\dfrac{u_i}{1.547}\right)^2\right]^2, |u_i| \le 1.547 \\ 0, \qquad\qquad\qquad\quad , |u_i| > 1.547 \end{cases} & , iterasi = 1 \\ \dfrac{\rho(u_i)}{u_i^2} & , iterasi > 1 \end{cases}$$

5. Calculate $\widehat{\boldsymbol{\beta}_s} \, (u_i, v_i, t_j)^m = (\boldsymbol{X}_i' \boldsymbol{W}_1^{m-1} \boldsymbol{X}_i)^{-1} \boldsymbol{X}_i' \boldsymbol{W}_1^{m-1} \boldsymbol{y}_i$

6. Repeat steps 2-4 to obtain a convergent value of $\hat{\boldsymbol{\beta}}_s$

2.4. Data

The data used in this study was obtained from Central Bureau Statistics (BPS) of East Java from 2011-2015 [1]. The locations of the observation consist of 38 districts/cities in East Java Province.

In this study, the crime rate was used as a response variable (y), The explanatory variables used in this study are shown in the table as follows:

Table 1: The explanatory variables

Variables	Information	Unit
X_1	Percentage of poor people	Percent
X_2	population density	Ratio
X_3	Unemployment rate	Ratio
X_4	human development index	Ratio

2.5. Data Analysis Procedure

The steps of analysis will be by the following:

1. Describing and exploring of crime rate data in East Java Province.
2. Testing the effects of spatial heterogeneity and time heterogeneity.
3. Analyzing GTWR model

   a. Determinating of spatial-temporal ratio parameters (τ)
   b. Determinating of spatial parameters (λ) and temporal parameters (μ)
   c. Determinating of optimum bandwidth (h_{ST})
   d. Calculating $\hat{\boldsymbol{\beta}}(u_i, v_i, t_j) = \left[\boldsymbol{X}'\boldsymbol{W}(u_i, v_i, t_j)\boldsymbol{X}\right]^{-1}\boldsymbol{X}'\boldsymbol{W}(u_i, v_i, t_j)\boldsymbol{y_i}$

4. Plotting of the residuals GTWR model to identify outliers.
5. Analyzing RGTWR S-estimator.
6. Calculating: R^2_{pseudo}, $Root\ Mean\ Square\ Error\ (RMSE)$, and $Mean\ Absolute\ Deviation\ (MAD)$.
7. Comparing model of GTWR and RGTWR S-estimator base on RMSE, MAD and R^2_{pseudo} each model.

3. Result and Discussion

3.1. Description of Data Crime Rate in East Java 2011-2015

East Java province consists of 38 districts/cities. The distribution of crime rates in East Java described in Figure 1. Based on the color degradation shown in Figure 1, in 2012-2013 there was a significant pattern change in Jember Regency where crime rates have decreased. In 2013, the pattern of distribution experienced a lot of changes, where there was a decrease in crime rates in Banyuwangi Regency, Probolinggo Regency, Ngawi Regency and increase in Jember Regency. In 2014, the pattern of distribution of crime rates also changed, namely a decrease in crime rates in Jember Regency, Bondowoso Regency, and Jombang Regency. In 2015, the crime rate of district areas was relatively uniform in the range of 20-100. However, urban areas have a relatively higher crime rate compared to district areas which are in the range of 92-380. In general, the crime rate in urban areas such as Malang City, Mojokerto City, Blitar City, Pasuruan City, Kediri City, and Surabaya City is always higher than other districts.

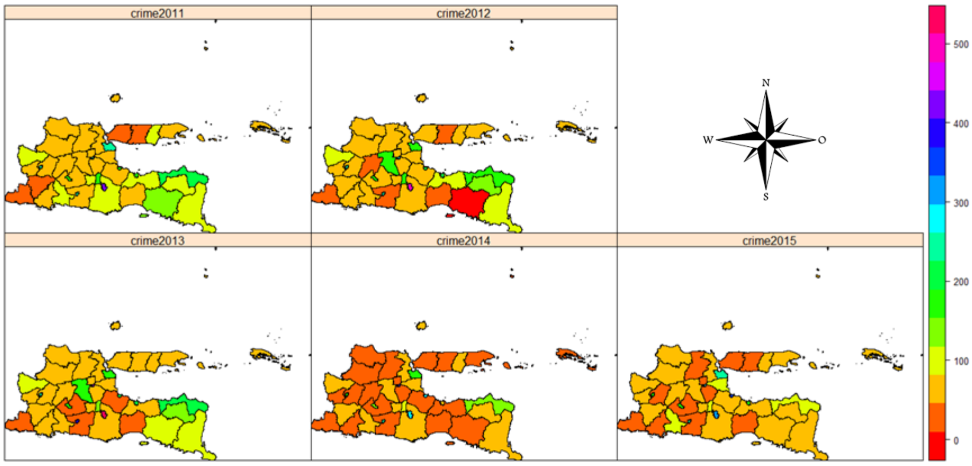

Figure 1: Distribution of crime rate in East Java 2011-2015

Boxplot of crime rate in East Java in Figure 2 showed that the diversity of crime rate data in 2011-2013 was greater than in 2014 and 2015. This different diversity showed that the difference between time on criminality data in East Java in 2011-2015. The distribution of crime rate data each year tends to be asymmetrical, this can be seen from the median value of the data that is sloping downwards. In Figure 2, it can also be seen that there is an outlier data each year, where from 2011-2015 outlier data is available in urban areas in East Java. However, the crime rate in Malang City has always been detected as an outlier during the 2011-2015 period. In general, the crime rate of urban areas in East Java shows higher results than the district areas. This causes different lifestyles and habits between cities and districts. Religious factors also influence differences in crime rates in urban and district areas, with a lack of religion also being one of the triggers for the crime. District areas are dominated by rural communities, where religious life is still inherent in community life. In addition to these factors, many other factors that cause crime in urban areas are much higher. These factors include economic factors, the availability of jobs and the diversity of urban society.

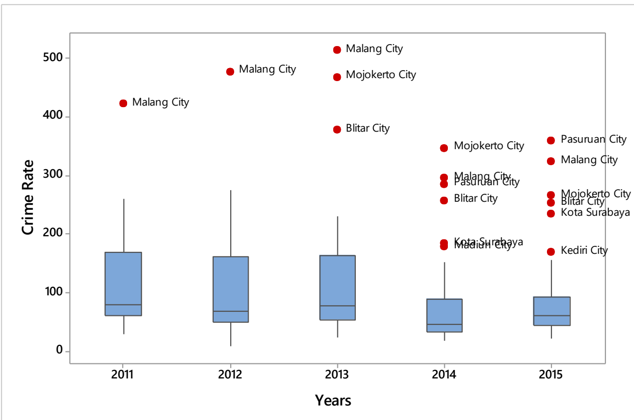

Figure 2: Boxplot of crime rate in East Java 2011-2015

The linear relationship between each explanatory variable and the response variable is seen using Pearson correlation. Table 2 showed that the percentage of poor people (X_1) with crime rate (Y) has a negative linear relationship. The higher the percentage of poor people in a location, the crime rate will decrease in that location. Variable population density (X_2), Unemployment rate (X_3), and human development index (HDI) (X_4) have a positive linear relationship with the crime rate (Y) in the East Java Province. The higher the population density, Unemployment rate, and the human development index at the location, the higher the crime rate at that location.

Table 2: Correlation between response and explanatory variables

Variables	*Pearson* correlation	*p-value*
Y with X_1	-0.547	0.000
Y with X_2	0.813	0.000
Y with X_3	0.496	0.000
Y with X_4	0.622	0.000

.**Table 3:** VIF value on each explanatory of variables

Years	X_1	X_3	X_3	X_4
2011	3.200	2.742	2.627	7.172
2012	3.262	2.732	1.822	5.590
2013	3.617	2.874	1.620	5.649
2014	3.307	2.642	1.620	5.257
2015	3.164	2.580	1.560	5.286
2011-2015	3.210	2.608	1.566	5.226

Test the assumption of multicollinearity can be done by looking at the VIF (Variance Inflation Factor) [10]. VIF is defined as a multicollinearity measurement of a variable with other explanatory variables in the analysis, and it is connected directly to the regression coefficient variant associated with this explanatory variable [10]. VIF value presented in Table 3. When VIF values greater than 10 (VIF> 10), it was indicated that the occurrence of multicollinearity. Based on Table 3, the VIF value obtained by each explanatory variable is less than 10. This showed that between explanatory variables does not occur multicollinearity.

3.2. Heterogeneity Spatial Test

Heterogeneity spatial analysis to detect whether there is heterogeneity or not because of spatial influence. This spatial heterogeneity occurs because of the different characteristics of an area and other regions. Heterogeneity testing of variance using the Breusch-Pagan statistical test. It was carried out to determine whether there was diversity by spatial influence. Tests are conducted annually and simultaneously on 38 districts/cities in the East Java Province from 2011 to 2015. The results of the Breusch-Pagan Test are presented in Table 4. The test results in Table 4 showed that there is spatial heterogeneity in crime rates in East Java in the period 2011-2015. This conclusion is obtained from seeing the resulting p-value of 1,688 x 10-11 and significant at level 5%.

Therefore, the application of spatial regression using GTWR is appropriate to be used to modeling crime rate in East Java in the period 2011-2015.

Table 4: Breusch Pagan test in East Java

Years	Breusch Pagan value	p-value
2011	10.421	0.0339**
2012	14.377	0.006185**
2013	20.615	0.0003775**
2014	27.368	1.675×10^{-5}**
2015	9.674	0.04629**
2011-2015	56.357	1.688×10^{-11}**

(**) significant at level 5 %, (*) significant at level 10%

3.3. GTWR and RGTWR Models Analysis

GTWR modeling involves spatial and temporal elements, where heterogeneity spatial occurs in every location and every time. This condition allows different models to be produced in each location and time. The first step of the GTWR model is determining the kernel function.

The kernel function used in this study is the kernel exponential function. The next step is the determination of a distance matrix using the exponential kernel function. In GTWR model, the distance matrix is formed based on the interaction between spatial distance and temporal distance. Therefore, in the formation of a distance matrix parameters are needed to balance spatial distance and temporal distance.

This balancing parameter is included because there are differences in units of spatial distance and temporal distance. Balancing parameters consist of spatial distance parameter (λ), temporal distance parameter (μ) and parameter ratio (τ). The parameter values are shown in Table 5.

Table 5: The balancing parameters value GTWR models

h_S	h_{ST}	λ	μ	τ
0.136	0.108	2.206	0.015	0.007

Outliers on the GTWR model showed in the boxplot Figure 3. Generally, the error of the GTWR model is detected as outliers. The Outliers are most common in 2012 and 2013 and Malang City is the location most often regarded as an outlier.

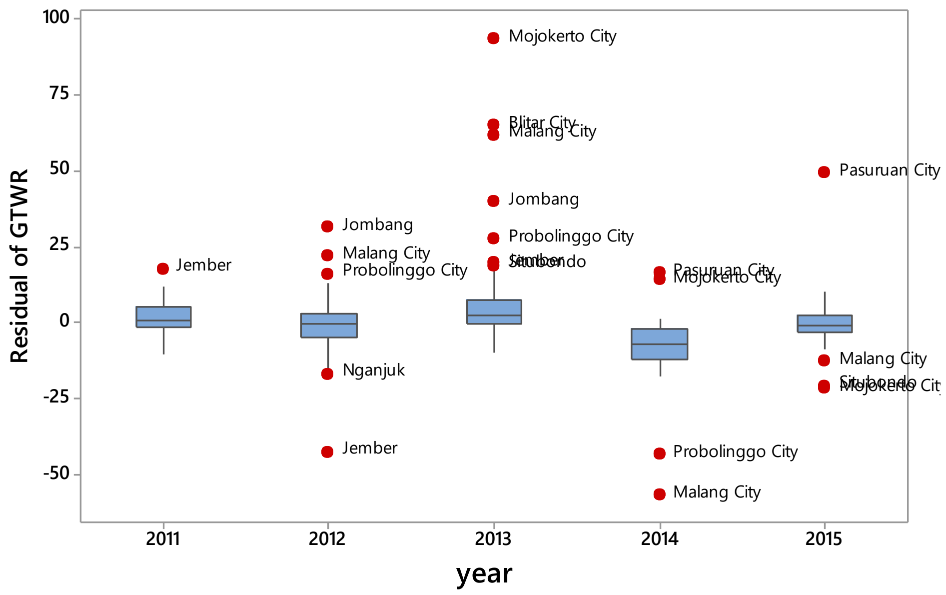

Figure 3: Residual of GTWR model

Outliers detected in the GTWR model cause the resulting model to be less precise in estimating the actual value. Therefore, the GTWR model needs to be improved so that the resulting model is better at estimating the actual value. In this research, the establishment of the GTWR model uses robust GTWR S-estimator (RGTWR). The RGTWR model was obtained through an iterative process by utilizing the estimated GTWR parameters as initial values. Iterations are carried out in each location and each time, so the resulting iterations can be different in each location and time. The iteration carried out aims to obtain convergent parameter estimates, where the convergent level used is 0.001.

3.4. Comparison GTWR and RGTWR Models

Summary of the estimation parameters of the GTWR and RGTWR models includes the minimum, maximum and range presented in Table 6. Based on Table 6, the RGTWR model changes the range of parameter estimation generated by the GTWR model. The parameter estimation change can also be seen from the median value generated by both models. The parameter estimation of the variable X_1 indicates an increase, while the parameter estimation of the variables X_3 and X_4 indicates a decrease in the median value. RGTWR also changes the direction of the parameter estimation from positive to negative. RGTWR also changes the direction of the parameter estimation from positive to negative as in the estimation parameter of the variables X_1 of 2012 in Tulungagung Regency. In general, changes in the estimated value of parameters, range, and median in the RGTWR model indicate that the robust model influences the GTWR model.

Comparison of the absolute mean error of the models presented in Table 7. Base on Table 7, the RGTWR model relatively shows a smaller value compared GTWR model. This indicates that the RGTWR model provides a better guess than GTWR model.

Table 6: Summary parameters estimation

Model	Parameter estimation	Min	Max	Range	Median
GTWR	$\widehat{\beta_0}$	-0.161	3.040	3.201	0.014
	$\widehat{\beta_1}$	-14.787	22.176	36.962	1.832
	$\widehat{\beta_2}$	-1.154	0.067	1.221	0.037
	$\widehat{\beta_3}$	-114.746	23.312	138.058	2.341
	$\widehat{\beta_4}$	-1.851	9.084	10.935	0.378
RGTWR	$\widehat{\beta_0}$	-0.174	3.370	3.544	0.017
	$\widehat{\beta_1}$	-15.714	22.332	38.046	2.156
	$\widehat{\beta_2}$	-1.191	0.080	1.271	0.037
	$\widehat{\beta_3}$	-120.849	23.994	144.843	2.028
	$\widehat{\beta_4}$	-1.840	9.084	10.924	0.215

Table 7: Comparison of absolute mean model errors

Year	Absolute mean model errors ($\lvert\overline{e_i}\rvert$)	
	GTWR	RGTWR
2011	4.418	2.478
2012	7.847	4.238
2013	11.860	9.349
2014	9.969	5.151
2015	5.555	1.751
2011-2015	7.930	4.594

Comparison of the goodness models based on the RMSE value, MAD and R^2_{pseudo} presented in Table 8. In Table 8 can be seen that the RGTWR S-estimator model gives better results than the GTWR model. This can be seen from the decrease in RMSE and MAD and the increase in R^2_{pseudo} produced by the RGTWR model. The decrease in RMSE and MAD shows that the resulting errors are smaller so that the estimated value of the resulting response variable approaches the actual value.

Table 8: Comparison of the goodness models

Model	RMSE	MAD	R^2_{pseudo}
GTWR	37.382	7.930	0.975
RGTWR	33.941	4.994	0.982

4. Conclusion

The RGTWR using s-estimator model effectively improved the GTWR model which contained outliers in criminal cases in East Java Province in 2011-2015. RGTWR model can reduce the absolute mean of errors generated by the GTWR model and also change the estimation parameters generated by the GTWR model. Besides that, comparison of the goodness of the resulting model also shows a decrease in the value of RMSE and MAD produced by the RGTWR model. Thus, the RGTWR model can provide a guess value that is closer to the actual value of the data and that shows that the resulting by RGTWR model is more presentative.

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