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Modulo 10 Error Detection Efficiency Analysis for Bank Routing Number

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Abstract

The Routing number is a nine-digit code used to identify specific banks and their transactions uniquely. Electronic Funds Transfer Routing Number Scheme is used to calculate the ninth digit of the routing number commonly known as the check digit. Fraud and insecurity cases associated with the banking sector necessitated the need to analyze the routing number scheme capabilities in error detection and correction. This paper discusses some of the significant limitations in the code as well as some of the code design shortfalls affecting the code's error detection and correction. It is then shown that there is a need for the design of a new code that improves the error detection of the routing number code.

Keywords: Error Detection; Error Correction; Twin Error; Adjacent Transposition Error.

1. Introduction

An error is a deviation from correctness or accuracy. Some of the common causes of these errors include but are not limited to human input error, noisy communication channels, and faulty telecommunication equipment. If a code word $u = 2547$ is sent through a communication channel and received as $v = 2537$ i,t implies $u \neq v$ thus an, error occurred in the third position [1].

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Error detection is the recognition of the occurrence of such an error while the rectification of such an error is called error correction. Modular arithmetic involves the computation of remainders generated through division [2]. Given a set of integers a, b, c, d, a is said to be congruent to b modulo c written $a \equiv b \ mod\ (c)$ if $c \mid (a - b)$ such that $(a - b) = cd$. Electronic Funds Transfer Routing Number Scheme uses modulo 10 for error detection. This scheme uses the check digit technique for error detection. A check digit is a form of redundancy added at the end of a code word for the purpose of error detection. The check digit is computed from the other digits or strings in the code [3]. The following is an illustration of how the Electronic Funds Transfer Routing Number Scheme works in the validation of bank routing numbers and in the calculation of the checked digit.

1.1 Validation and Check Digit Calculation Using the Electronic Funds Transfer Routing Number Scheme

The routing number is validated using the last digit of the nine-digit routing number. The last digit of the code is commonly referred to as the check digit in the electronic funds' transfer routing number scheme. Any routing number is considered valid if its last digit is equivalent to the routing number check digit.

$$check\ digit\ (a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$$

$where\ a_1 \dots a_9\ are\ integers\ between\ 0\ and\ 9$

The above-stated formula gives us how the routing number check digit is calculated [3].

Example: 1.1.1: Consider the validation of the routing number 2540 7022 6.

$$check\ digit\ (a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$$

$check\ digit = [7(2 + 0 + 2) + 3(5 + 7 + 2) + 9(4 + 0)]mod\ 10$ =106 $mod\ 10$ = **6**

This a valid routing number since the last digit match with the calculated check digit.

Transposition error occurs when a digit is reversed or interchanged. The error occurs in such a manner $hk \rightarrow kh$ or 89 → 98. Twin error occurs when two similar pairs of adjacent digits get interchanged with some other pair of similar adjacent figures that is $aa \rightarrow bb$. Jump Twin error is of the form $hck \rightarrow kch$ such that two digits of the code are interchanged and, as a result, the check digit and the checksum are wrong [4]. Phonetic error occurs when a digit is interchanged with another that is pronounced similarly. Silent error occurs when the checksum equation holds despite the sent and received codewords differing in some bit strings [5]. Numerous researchers have analyzed various check digit schemes, but none has gone into details to explain why adjacent transposition of digits that differ by 5 go undetected [6]. This paper analyzes the bank routing number in regards to transposition errors, twin errors and jump twin errors and points out the weaknesses in the formula that make the errors pass undetected. Despite the fact, the bank routing number remains useful it is vital to note that the inability of code to detect an error is detrimental to the code and it counters any strengths an algorithm possess [7]. Thus, any code algorithm should be able to detect 100% of the errors that occur.

2. Results and Discussion

Proposition 2.1: Routing number detects single digit errors

proof : Suppose $c_{sent} = a_1a_2a_3a_4a_5a_6a_7a_8a_9$ *where* a_9 *is a the check digit* and c is the sent routing number. It follows that *check digit* $(a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$ If a code $c_{received} = a_1a_2a_2a_4a_5a_6a_7a_8a_9$ as a result of a single error it follows that *check digit* $(a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_2 + a_6)]mod\ 10$. Thus, the check digit of the sent and received code will not be equal; hence, the single error will be detected.■

Proposition 2.2: Routing number 10 does not detect some single twin errors

Proof. By counterexample. A routing number is sent as 2540 70**11** 6 but is received as 2540 70**22** 6. In this case, the distance between the sent and received code is two.

$$d(C_1, C_2) = 2\ where\ C_1 = 2540\ 70\mathbf{11}\ 6 \text{ and } C_2 = 2540\ 70\mathbf{22}\ 6$$

It is clear that the checksum with the help of the check digit does not detect the errors in the sent routing number. Sent routing number 2540 70**11** 6.

$$check\ digit\ (a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$$

$check\ digit = [7(2 + 0 + 1) + 3(5 + 7 + 1) + 9(4 + 0)]mod\ 10$=96 mod 10 = **6**

Received routing number 2540 70**22** 6

$$check\ digit\ (a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$$

$check\ digit = [7(2 + 0 + 2) + 3(5 + 7 + 2) + 9(4 + 0)]mod\ 10$=106 mod 10 =**6**

The counterexample shows that modulo – 10 cannot indeed detect twin errors.

Proposition 2.3: Routing number 10 does not detect some jump twin errors

Proof. By counterexample. A routing number is sent as 2540 70**22** 6 but is received as 2**22**0 70**54** 6. In this case, the distance between the sent and received code is four. $d(C_1, C_2) = 4\ where\ C_1 = 2540\ 70\mathbf{22}\ 6$ and $C_2 = 2\mathbf{22}0\ 70\mathbf{54}\ 6$ It is clear that the checksum with the help of the check digit does not detect the errors in the sent routing number.

Sent routing number 2540 70**22** 6

.$check\ digit\ (a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$

$check\ digit = [7(2 + 0 + 2) + 3(5 + 7 + 2) + 9(4 + 0)]mod\ 10$ =106 mod 10 = **6**

Received routing number 2**22**0 70**54** 6

$$check\ digit\ (a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$$

$check\ digit = [7(2 + 0 + 5) + 3(2 + 7 + 4) + 9(2 + 0)]mod$ 10=106 mod 10 =**6**

The counterexample shows that modulo – 10 cannot indeed detect jump twin errors.

Proposition 2.4: Routing number 10 does not detect some silent errors

Proof. By counterexample. A routing number is sent as 25**4**0 7**0**22 6 but is received as 25**0**0 7**4**22 6. In this case, the distance between the sent and received code is two.

$$d(C_1, C_2) = 2\ where\ C_1 = 25\mathbf{4}0\ 7\mathbf{0}22\ 6 \text{ and } C_2 = 25\mathbf{0}0\ 7\mathbf{4}22\ 6$$

It is clear that the checksum with the help of the check digit does not detect the errors in the sent routing number. This is in line with the work of Gallian [5] who showed that using a prime modulus is more effective in error detection since even though a composite modulus detects all single digit errors it, however, allows some twin error and transposition errors to pass undetected. A good example of adjacent transposition of digits that go undetected is those that differ by 5 [3].

Theorem 2.5: Suppose an error detecting scheme with an even modulus detects all single-position errors. Then for every a_i and a_j there is a transposition error involving positions i and j that cannot be detected.

Proof. Let the modulus be $2m$ where $m \in Z$. For all single errors to be detected it, is necessary that the mapping σ_i be permutations. In order to detect all transposition errors involving positions i and j it is necessary that $\sigma_i(a) + \sigma_i(b) \neq \sigma_i(b) + \sigma_i(a)$ for all $a \neq b$ in Z_{2m}. It then follows the mapping

$\sigma_i(x) = \sigma_j(x) - \sigma_i(x)$ must be a permutation of Z_{2m}. But summing the elements of Z_{2m} modulo 2m we have $m = m + 0 + (1 + 2m - 1) + (2 + 2m - 2 + \cdots + (m - 1 + m + 1)$.

Thus,

$$m = \sum x - \sum \sigma(x) = \sum\left(\sigma_j(x) - \sigma_i(x)\right) = \sum\left(\sigma_j(x)\right) - \sum\left(\sigma_i(x)\right) = m - m = 0.$$

hence the proof. ■

Example 2.1.1

Consider a sent conventional routing number mod ten $u = 1011\mathbf{49}352$ which is received as $v = 1011\mathbf{94}52$.

Sent routing number $1011\mathbf{49}352$.

$$check\ digit\ (a_9) = [7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8) + 9(a_3 + a_6)]mod\ 10$$

$check\ digit = [7(1+1+3)+3(0+4+5)+9(1+9)]mod\ 10$ =152 mod 10 = **2**

Received routing number $1011\mathbf{94}352$

$$check\ digit\ (a_9) = [7(a_1+a_4+a_7)+3(a_2+a_5+a_8)+9(a_3+a_6)]mod\ 10$$

$check\ digit = [7(1+1+3)+3(0+9+5)+9(1+4)]mod\ 10$=122 mod 10 =2

Hence the transposition of digits 49 cannot be detected. Thus, adjacent transposition of digits that go undetected are those that differ by five. In our case four and nine differ by five.

3. Conclusion

In Routing number mod ten some transposition errors, twin and Jump twin errors go undetected. Since modulo – 10 is a composite number it raised some concerns since according to Gallian (1991), a composite modulus was not as effective as a prime modulus would be in error detection. It is evident that modulo – 10 is not a field since a given ring Z_n is a field if and only if n is a prime. It is observed that it might not be possible to devise a modulus 10 scheme that detects all transposition and single digit errors. As observed, there is a need to design and develop a new algorithm that improves the error detection capability of the routing number code. Such a code is likely to use a prime modulus in order to overcome the challenges in error detection of a composite modulus.

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