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Measuring the Distance between Earth and Stars (A different approach)

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Abstract

In this paper, the distance between Earth and the Sun or other stars is measured. The objective is to use a different method to measure the distance between Earth and the stars. This approach for calculating the distance between the earth and stars heavily relies on trigonometry. Even though it's theoretical, it can be demonstrated practically, which I have done to some extent. Although the following method can be used to measure the distance from closer stars, planets, comets, and satellites, it may need more improvisations and recalibrations for measuring the distance of far-off stars.

Keywords: Astrometry [7]; Applied Mathematics; Trigonometry.

1. Introduction

Before the invention of radars [6] and radio telescope [8] astronomers and mathematicians had to depend heavily on Trigonometry to be more specific the parallax method [12] in determining the distances of celestial bodies. Earth's radius was also determined by trigonometry in different times by different people like Al-Biruni Reference [9] and Eratosthenes [10]. Where Eratosthenes used a shadow of a stick in different cities to determine earths radius, Al-Biruni used the elevation angle and a known height of a mountain to do so. In this paper, I am going to reverse engineer Al-Biruni's method [11]. Due to our technological advancements, we are now aware of earths radius and we can use this radius and the elevation angle of something distant to calculate its distance from earth's surface.

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2. The Theoretical Part

2.1 Derivation of the Main Equation

Finding a right-angle triangle with a base that is related to the radius of the Earth and determining an angle (theta) that connects the base and the triangle's distance (height) together can be used to derive the main equation for the theory.

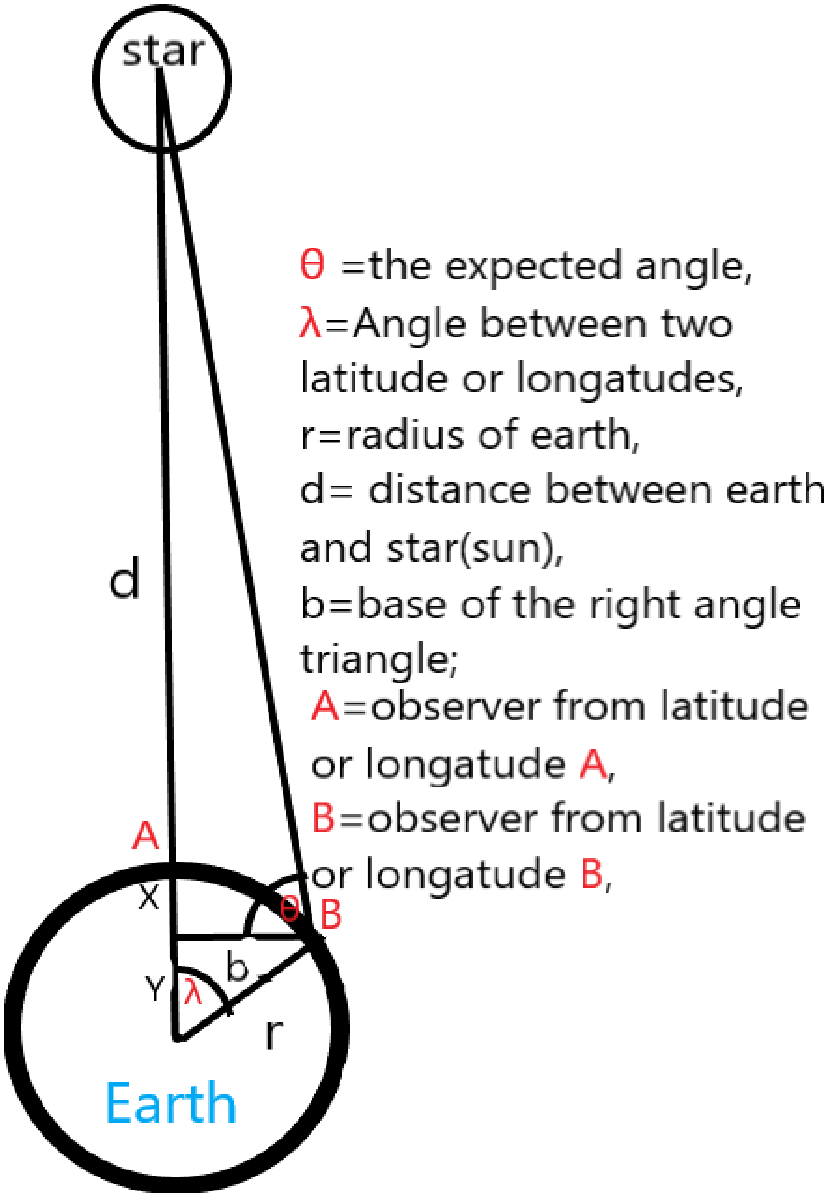

Figure 1: Depiction of the theoretical part

Disclaimer: The a and b points in Figure 1's description must have the same latitude or longitude; if they do, the other two must be different (for example, a = 80◦n, 50◦e and b = 80◦n, 40◦w, where both points have the same latitude but different longitudes).If two observers from two different locations (let's say a and b) look at the sun at the same time in figure 1, they will obtain two different positions or altitudes of the sun, which will be ∠a and

∠b. These two angles will form the base of our right-angled triangle, which is made up of the radius of the earth, and another angle, which is extremely small, will converge in the object being observed. Figure 1 illustrates how ∠b forms an angle with the base of a right-angled triangle. Thus, we can obtain that [2].

$$tan\theta = \frac{d + X}{b}$$

2.2 The Variable Elements

Certain components of equation 1 change depending on the circumstance. For example, $\angle\theta$, the height of the right triangle, and its base. To predict their value under various conditions, separate equations are required.

2.2.1 Base of the right triangle

Figure 1 shows that the angle between a and b ($\angle\lambda$), related to the base of the right-angled triangle. Figure 1 shows that a second right-angled triangle is created by combining the base of the first right-angled triangle (b) and the radius of the earth, as well as $\angle\lambda$. A trigonometric function between the earth's radius (a constant value) and the base of the first triangle (b) is then included in the equation [4]that we derive from there. It is: -.

$$cos\lambda = \frac{r^2 + r^2 - b^2}{2r^2}$$

or, $2r^2 cos\lambda = 2r^2 - b^2$

or, $b^2 = 2r^2 - 2r^2 cos\lambda$

or, $b^2 = 2r^2(1 - cos\lambda)$

or, $b^2 = 2r^2\left(2\sin^2\frac{\lambda}{2}\right)$

$$b = 2r^{\frac{1}{2}}\left(\sin^{\frac{\lambda}{2}}\quad\right)$$

For real life scenarios on the other hand, the earth is not a perfect sphere therefore radius in two points of earth won't be the same. For instance, in figure 2, the observer 'A' (distance from the center of the earth $r_{1)}$ is in the pole and observer 'B' (distance from the center of the earth r_2) is somewhere in near the equator, therefore we can say that $r_2>r_1$. Moreover, it is important to take in account the elevation of the observation center from sea level. Therefore, we can define, r_2 **= Distance from earths center + elevation of the observation center from sea level**. Therefore, our equation [3] [1] can be rewritten as,

$$b = 2.(r1.r2)^{\frac{1}{2}}\left(\sin\quad\frac{\lambda}{2}\right)$$

2.2.2 Angle theta

From figure 1 we can see that we can't determine $\angle\theta$ by just observation though it's linked with ∠B, ∠X and ∠λ. (figure 2). It is evident in the figure-2 that ∠λ is directly connected with ∠A therefore ∠θ. As example if ∠λ is 90◦ and if ∠ A is 90◦ than its evident that ∠B will converge to 0◦, on other hand if ∠A stays the same and we only change the ∠λ to 45◦ than ∠B changes to 45◦. ∠B is the main variable which is related to ∠θ. In figure 2 we can see that ∠θ is created when ∠ B meets the base of right triangle(b). So, ∠B plays the most important part the formation of ∠θ. theta is also depended on ∠X the difference between right angle and ∠A. Here is an equation which determines the value ∠θ

$$\angle\theta = (\angle\lambda + \angle B) - \angle X$$

Additionally, we must understand that ∠X =∠90◦ -∠A. Figure 2 shows that the value of $\angle\theta$, rises as the values of ∠B and $\angle\lambda$. rise. Additionally, when ∠ X rises, $\angle\theta$ falls for the same reason.

2.2.3 The X and Y factor

This term needs to be introduced in figure 1. Only imaginary figures can use the following terms (such as the sun's altitude in 90 ◦, which never happens). Thus, the primary formula will be:

$$tan\theta = \frac{d + X}{b}$$

or, $d = b.\ tan\theta - X$

or, $d = b.\ tan\theta$

As the X factor is irrelevant [2] in real life case, we can omit it and go forward with our last equation

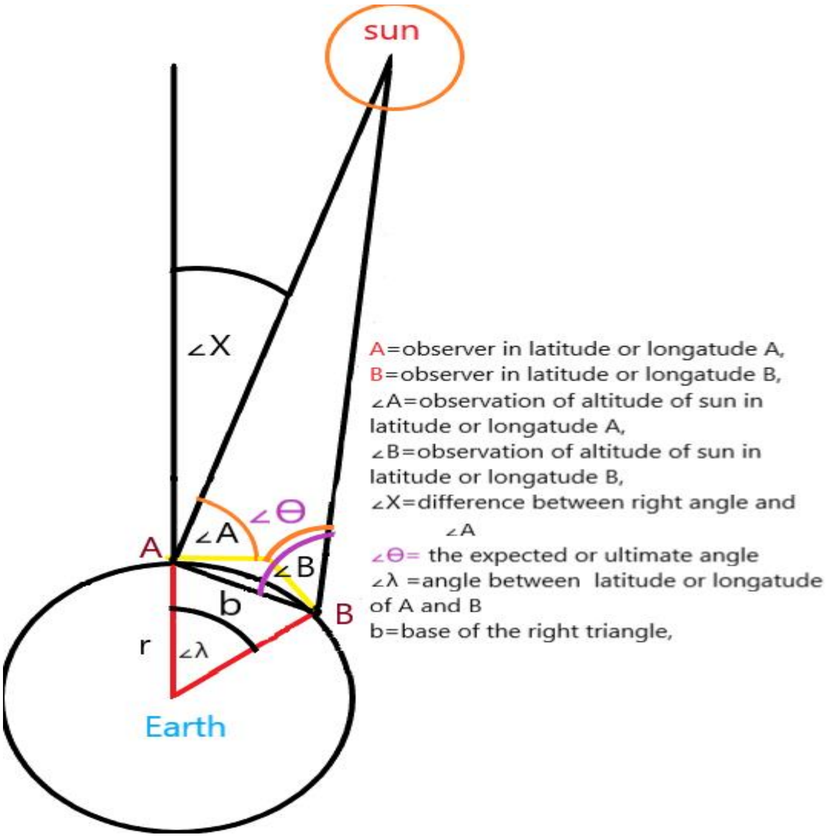

Figure2: (not to measure)

3. Execution

Here, we are going to try to determine the precision of $\angle\theta$, Using Sun's celestial data): -

$$\theta = \tan^{-1}\left(\frac{d}{b}\right)$$

Or,$\theta = \tan^{-1}\left(\frac{150{,}140{,}000}{6378.1}\right)$

Or,$\theta = 89.99756602$ ⍰

As I don't have enough data, I am using the earths radius as the base of the triangle and the distance between earth and sun as d. From the results it is quite evident that to measure the $\angle\theta$, we must make a precise measure of the angle (at least for 6 decimals)

4. Constraints

Due to the lack of experimentation and data this theory holds some limitations mainly in taking a precise measurement of the desire angle and calculating the base. But in the 21st century these limitations [5] can be solved by a few experimentations. Some of the constraints that may hinder the precise results of these theory according to me are:

1) Determining the r_1 and $r_{2,}$ as determining r_1 is dependent heavily by the observations center location and elevation getting an absolute or near absolute value can be a daunting task. As determining r_1 and r_2 is directly connected with the base of the triangle, slight deviations can hinder a precise result.
2) Determining the desired angle holds challenges as well. Observing a celestial body from different time zones can create problems due to earths uneven shape. Moreover, Demography (limiting the POV of the observer) around the observer and weather issues are stakeholders as well in observing the celestial body.
3) Determining an angle after 6 decimals needs a precise machine. Even though we have precise machines for the parallax methods, the machines might need to recalibrate for this approach.

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