An Alternative Proposed Method for Solution of Assignment Problem

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Abstract

The assignment problem is a particular type of linear programming problem. In this paper, we analyzed the standard and existing proposed methods. After studying these methods, we proposed a new alternative method for solving the assignment problem. We examined the newly proposed method by a couple of numerical examples and compare this result with the standard method. The main characteristic of this newly proposed method is that it constructed a very easy logical and arithmetical algorithm. Here we point out some advantages and limitations of the new proposed method. Programming code for the newly proposed method has been added in this paper.

Keywords: assignment problem; Hungarian method; linear programming problem; C++ programming code.

1. Introduction

In linear programming problem, assignment problem is introducing instantly after transportation problem [1]. The main aim of the assignment problem is to minimize total cost or time of several resources to an equal number of activities[1]. Assignment problem holds a condition that one resource can connect with only one activity[2].

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Applications of assignment problem are many and widespread because its uses just not only confined to assign jobs to worker or machines but also uses in personnel to offices, transportation, airbus to a destination, teachers to classrooms and so on[3,4]. Actually, assignment problem is one of the initial problems which is important in decision making [2]. Assignment problem was first discussed as the degenerate case of transportation problem in 1951 in the SCOOP symposium on linear inequalities and programming [5]. D.F. Votaw and A. Orden, in 1952 was first formulated assignment problem as a type of transportation problem [5] Harold Kuhn in 1955 was developed and published assignment problem[6]. He named the method as "Hungarian method" because his method was based on the methods of two Hungarian mathematicians: Dénes Kőnig and Jenő Egerváry [6]. In 1953 Dénes Konig provided an algorithm for linear assignment problem which was based on the translation of a paper of Jeno Egerváry [7].

2. Preliminaries

2.1. Mathematical representation of assignment problem

Generally, an assignment problem associated with a matrix or table, where the rows present the objects or manpower that we want to assign, and the column present the jobs or task that we want them assigned to. suppose we want to minimize the total cost or time of an assignment problem of n resources to m activities where each resource connects with one and only one job. Then the cost matrix (C_{ij}) is given below:

Activity

$$
\begin{array}{ccccc}
 & A_1 & A_2 & \ldots & A_n \\
\end{array}
$$

Resource
$$
\begin{array}{c}
R_1 \\
R_2 \\
\vdots \\
R_n
\end{array}
\begin{bmatrix}
C_{11} & C_{12} & \ldots & C_{1n} \\
C_{21} & C_{22} & \ldots & C_{1n} \\
\vdots & \vdots & \vdots & \vdots \\
C_{n1} & C_{n2} & \ldots & C_{nm}
\end{bmatrix}
\begin{array}{c}
1 \\
1 \\
\vdots \\
1
\end{array}
$$
Available

Required $\quad\quad 1 \quad\quad 1 \quad \ldots \quad 1$

The cost matrix is the same as that of a T.P except for that availability at each of the resource and the requirement at each of the destinations is unity. Let x_{ij} denote the assignment of ith resource to j^{th} activity such that

$$x_{ij} = \begin{cases} 1; & \text{if resource i is assigned to j} \\ 0; & \text{otherwise} \end{cases}$$

Then the mathematical formulation of the assignment problems is Minimize $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to the constraints $\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1; x_{ij} = 0 \text{ or } 1$

For all i = 1, 2… n & j = 1, 2…n

2.2. Classification of Assignment Problem

The assignment problem divided into two categories:

- Balanced Assignment Problem: When the number of rows and columns of assignment problem is equal then it is known as a balanced assignment problem.
- Unbalanced Assignment Problem: When the number of rows and columns of assignment problem is not equal then it is known as an unbalanced assignment problem.

In our newly proposed method, we consider only the balanced assignment problem

3. Algorithm of the new Proposed Method of solving the assignment problem

The steps of the proposed alternative method are discussed below:

Step-1: Find the smallest element of the matrix.

Step-2: Assign this smallest element with a box and eliminate the row and the column for this element.

Step-3: If there are two same smallest elements then find the sum of the row of those two elements respectively.

Assign the element whose sum is less.

Step-4: Repeat step-1, step-2 and step-3 for the remaining matrix and finally assign the final remaining element.

3.1. Numerical Examples

In this section, we examined our newly proposed method by considering two detailed numerical examples and compared our result with the standard method (Hungarian method).

Example 1

Assign the four tasks to four operators. The assigning costs are given in the table

Table 4.1.1(a): Assignment table of example 1

	1	2	3	4
A	20	28	19	13
B	15	30	31	28
C	40	21	20	17
D	21	28	26	12

Solution:

The smallest element of the matrix is 12. So, we assign this element with the box and eliminate the row and column of this element.

Table 4.1.1(b): Solution table of example 1

	1	2	3	4
A	20	28	19	13
B	15	30	31	28
C	40	21	20	17
D	21	28	26	12

The reduced matrix is:

Table 4.1.1(c): Solution table of example 1

	1	2	3
A	20	28	19
B	15	30	31
C	40	21	20

The reduced matrix is:

Table 4.1.1(d): Solution table of example 1

	2	3
A	28	19
C	21	20

The total cost of assignment = A3+B1+C2+D4=19+15+21+12=67

The solution of this assignment problem using Hungarian method is 67.

Example 2

Assign the four tasks to four operators. The assigning costs are given in the table.

Table 4.1.2(a): Assignment table of example 2

	1	2	3	4
A	14	5	8	7
B	2	12	6	5
C	7	8	3	9
D	2	4	6	10

Solution:

The smallest element of the matrix is 2. So we assign this element with the box and eliminate the row and column of this element.

Table 4.1.2(b): Solution table of example 2

The reduced matrix is:

Table 4.1.2(c): Solution table of example 2

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The reduced matrix is:

Table 4.1.2(d): Solution table of example 2

The total cost of assignment = A2+B4+C3+D1=5+5+3+2=15

The solution of this assignment problem using Hungarian method is 15.

4. Results

The new alternative proposed method provides the same optimal result compared with the standard Hungarian method. Our newly proposed method represents a very easy logical and arithmetical algorithm. Therefore, it is easy to understand and hence requires fewer arithmetic calculations than other standard and existing methods. The limitation of the newly proposed method that it is only effective for $2n \times 2n$ matrix. Another limitation is that new proposed method does not give optimal result, it gives same result compared with standard method. In future work, it can be modified as a method that will reduce the limitation.

5. Conclusion

In this paper, we discussed the assignment problem and proposed a new alternative method of solving assignment problem which requires very easy procedure and consumes less time. We also compared the proposed method with the standard Hungarian method by illustrating two numerical examples which gave us the same optimal result. As the newly proposed method is very easy to understand it will be also easy to implement in the real field which will be very helpful for decision-makers.

Acknowledgement

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6. Appendix

C++ Programming Code for the Alternative Proposed Method

```cpp
#include<bits/stdc++.h>

using namespace std;

int rowsum[50];

int a[50][50];
```

```cpp
int b[50][50];

int main()

{

    int n;

    cin>>n;

    for(int i=1;i<=n;i++)

    {

        rowsum[i]=0;

        for(int j=1;j<=n;j++)

        {

            cin>>a[i][j];

            rowsum[i]+=a[i][j];

        }

    }

    int cost=0;

    while(1)

    {

        cout<<"-------------------\n";

        int mn = a[1][1];

        pair<int,int> mnindex = {1,1};

        for(int i=1;i<=n;i++)

        {
```

```
        for(int j=1;j<=n;j++)

        {

            cout<<a[i][j]<<" ";

            if(a[i][j]<mn || (a[i][j]==mn && rowsum[i]<rowsum[mnindex.first]))

            {

                mn = a[i][j];

                mnindex = {i,j};

            }

        }

        cout<<"\n";

    }

    cout<<"minimum value at "<<mn <<" at row = "<<mnindex.first<<" column ="<<mnindex.second<<"\n";

    cost+=mn;

    int bi=1;

    int bj=1;

    for(int i=1;i<=n;i++)

    {

        if(i==mnindex.first) continue;

        bj = 1;

        for(int j=1;j<=n;j++)

        {

            if(j==mnindex.second)continue;
```

```
                b[bi][bj++] = a[i][j];

        }

        bi++;

    }

    n-=1;

    for(int i=1;i<=n;i++)

    {

        rowsum[i]=0;

        for(int j=1;j<=n;j++)

        {

            a[i][j] = b[i][j];

            rowsum[i]+=a[i][j];

        }

    }

    if(n==0) break;

}

cout<<"------------------------\n\ntotal cost = "<<cost;

getchar();

getchar();

getchar();

}
```

7. Recommendations

Assignment problem is a special type of transportation problem which main objective is to minimize total cost or maximize total profit. Hungarian method is the most popular existing method for solving assignment problem. Till now many methods has been proposed to solve assignment problem. In this paper we also proposed an alternative new method which is very easy to calculate and consume less time. In our paper we included basic knowledge of assignment problem and we explain our new proposed method with two examples and compare with the result of existing method. We also discuss about the limitations of newly proposed method. Finally, we added a **C**++ programming code of our new proposed method.