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On the Identification of the Causal Effects of Audit Activity under Measurement and Selection Biases

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Abstract

We propose a causal analysis based on a linear Structural Equation Model (SEM) of the effect of audits on the compliance level of tax payers. Generally, when the audit rule is not based on randomization and we also have unobserved variables, it is very likely to have confounding and the causal effect estimate can be biased, if not detected by inspection of the graphical model related to the SEM and removed. In addition, both measurement bias and selection bias arise naturally in real situation of observed data, thus increasing the complexity of the problem to be solved. In this case, the classical causal effect identification techniques (backdoor, frontdoor and instrumental variable) cannot be directly applied. To solve the causal effect identification problem in such a context, we extend the effect restoration method for the measurement bias, according to the selection recoverability condition. The proposed method, combining the two techniques, can be used to obtain an estimate of the causal effect closer to the real one, compared to the previous estimation approach adopted in this field. Moreover, the methodology can be applied also in other contexts, on problems sharing the same causal structure, or having an equivalent one.

Keywords: causal effect; counterfactual; measurement bias; selection bias; tax audit.

1. Introduction

The problems of defining audit planning methodologies and assessing audit results have been extensively studied and a huge bibliography is available, starting from the pioneering work of [1]. The reader can refer to the introductory section "Audit rules in theory and practice" in [2].

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With reference to the planning of audit activities, the mathematical models are mainly inferred from game theory [3,4] and its application to tax compliance. The relationships between the probability of being audited, the audit costs, the taxation function and the fine function have been highlighted in the tax policy literature. The mathematical models used for the assessment of the audit effects are basically regression models that rely on cross-sectional data, panel data and time-series data [5,6,7]. Several models based on the *difference in differences* (DiD) methodology have been developed, but sometimes they provide uncertain results about the actual ability of the models to identify the causal effects of audits [8,9]. The golden rule to find the causal effects from audits is based on randomization. The taxpayers to be audited are picked at random, but there are audit rules that act differently. This is because, even though randomization usually allows to obtain a reliable estimate of tax evasion and tax audit effects, this methodology implies an expensive allocation of audit resources (in terms of specialized employees, data gathered/analysed and IT tools) which could instead be used to focus on cases with greater risk of evasion/avoidance, as happens in Italy. Moreover, when there are kind of selection bias as that of taxpayers who drop out from the reporting process, randomization could not be enough to identify causal effects [10]. In Italy, audit activity is carried out by the Italian Revenue Agency (IRA) adopting intervention methods based on specific systems of risk analysis and risk assessment of tax evasion/avoidance. These systems take into account the peculiarities which characterize each economic and territorial reality, in addition to the different macro-types of economic activity carried out by tax payers. The IRA mainly uses two kinds of tax audits: *soft inspection*, where inspectors access the taxpayer office/shop to check whether simple tax obligations are respected and *deep inspection*, where inspectors make use of several accounting and structural information. Both types of audit activity are carried out whenever circumstances require it. It is also necessary to point out that the selection of taxpayers to be audited, in order to allocate resources efficiently, is also carried out by promoting synergies with other national and international public authorities. Among the deep inspection methodology rules regarding the audience of small and medium−sized enterprises, one can find the *Studi di settore* (SdS) audit rule which, from a mathematical point of view, belongs to the Stackelberg non-cooperative models (*Principal-Agent* or *Leader-Follower*) with a cut-off audit rule. The IRA announced and committed to adopting an audit strategy where all (or a duly defined fraction of) taxpayers below the cut-off value are audited, while those above (or equal to) the cut-off value are not audited at all. As for the Italian context, here we mention two recent works to assess the effects of the audits where data sets gathered by the IRA have been used. The first is by Santoro and Fiorio [2], where a descriptive analysis of a data set has been carried out based on a regression model. The second is by D'Agosto and his colleagues [11], where a causal analysis based on a DiD model has been proposed. Some specific hypotheses have been proposed for the effects of the unobserved variables and the selection bias [10] of the outcome variable. The difficulty in identifying the total causal effect of audits lies in the joint presence of unobserved variables and conditioned variables (selection bias caused by the audit rule) that do not allow us to effortlessly eliminate the spurious effects between the treated variable and the result variable. As for the causal analysis based on directed graphs, the classical identification techniques based on backdoor, frontdoor or instrumental variables [12,13] cannot be directly applied. In this work, we offer a solution to bypass these difficulties. In accordance with this premise, the paper is organized as follows. Section 2 introduces the main features of the preceding works and the theoretical improvement of this new approach to the audit effect estimation. A descriptive taxpayer−IRA interaction model, which is the prerequisite for the mathematical model, is therein synthetized. The mathematical formulation of the causal graphical model specification based on a linear structural equation model ends the section. Section 3 presents the theoretical improvements of this work. The novelty here consists of a new framework that let us define and apply an asymptotical unbiased estimator of the causal audit effect on the taxpayer behaviour, even when selection bias and measurement bias are present. Some suggestions on the kind of proxies to choose for the model specification are reported. Section 4 proposes an example of the model specification, together with the estimation of the interventional and counterfactual effects (the *Average Treatment effects on the Treated*), using a synthetic data set. Subsection 4.1 is devoted to the concluding remarks.

2. Materials and Methods

2.1 Previous works

The mathematical models used for the assessment of the audit effects are basically regression models that rely on cross-sectional data, panel data and time-series data [5,6,7], which are mainly descriptive tools. From a causal inference perspective, several models based on the DiD methodology have been also developed, but sometimes with uncertain results regarding the actual ability of the models to identify the causal effects [8,9]. As for the Italian taxation framework, two recent works have been designed for the analysis of the audit effects on taxpayer behaviour and data gathered by the IRA have been used to achieve their results. In the research of [2], which is focused on the effects of the SdS audit rule, a mathematical model that relates taxpayers' reporting behaviours to variables like the audit probability, tax rates, fine costs and concealment costs has been proposed and an empirical analysis to test the behavioural model has been carried out based on a *log*–*log* regression on the cross-sectional data for the year 2005. The information to estimate the audit probability and fine costs comes from the audits for the year 2000. The tax audit rule depends on the incongruence of the declared return with respect to the SdS estimate. The SdS estimate is based on free software that is downloadable from the IRA web site for each year t, which makes an estimate based on the structural and accounting data given by the taxpayers. The authors stressed that the specificity of the available data, which is truncated above the congruity level (a declared return x is "congruent" to the SdS estimate y if $x \geq y$), did not allow for a proper causal analysis and that their results must be interpreted only in a descriptive form. Furthermore, the regression model could be biased due to the correlation present within clusters of homogeneous activities, or because the use of regressors that are proxies of the concealment costs might be endogenous (Durbin–Wu–Hausman endogeneity tests have been performed). However, a number of theoretically relevant relations are confirmed by the analysis. For example, reports made by taxpayers seem to be positively associated to the firm's size. Furthermore, when taxpayers know that the probability to be audited decreases, they tend to report less. From a causal analysis perspective, also if aimed to evaluate the impact of a reform of tax audit rules implemented in 2006, a first approach was conducted by [14] in which the effects of a reform equivalent to an increase of the perceived audit probability have been investigated using a DiD regression model. The reform repealed a special audit exemption previously granted to businesses which adopted a stringent accounting standard. It has been shown that the tax reform increased the level of economic activity, as measured by the value of inventory, for the generality of businesses involved. However, an increase in profits and turnover was reported only by the subset of businesses which were more likely to perceive it as an increase in the probability of an audit. This result seems is in line with the predictions of the Allingham-Sandmo model [1]. In the work of D'Agosto and his colleagues [11], two kinds of tax audits were considered: the *soft inspection* (where inspectors access the taxpayer office/shop to check whether simple tax obligations are respected) and the *deep inspection* (where inspectors make use of several accounting and structural information available in the IRA databases, as for SdS audits). The first objective of the research is to check whether the tax audits of firms increase their tax compliance, while the second is to discover which type of audit produces the best result (soft, deep or a mix of both). The interest is also that of evaluating the enduring tax compliance, so the authors restrict their analysis to those firms enduring at least five years in their own market. Accordingly, firms that survive to a tax audit for some years are analysed. Therefore, their results are intended to be representative of those firms with a stable contribution to the production system. Individual firm activities are defined as a group of natural persons with an active VAT registration (including both employers and self-employed workers). A causal analysis based on a DiD model that relies on a *balanced panel* for the years 2004–2009 has been proposed. The panel data allow the authors to address the problem of identifying the effects that an audit at time *t* could have on the tax reporting at time *t*+1 and that over subsequent years. The control group has been selected using a *probit* model stratified on macro-sector business, (the quantiles of) turnover and regions with the input/output VAT (Value Added Tax) and the coherence/congruence to SdS among the explanatory variables. Although the DiD model could theoretically handle the effects of unobserved variables and selection bias on the outcome variable, their cumulative effects should be constant over time (necessary condition), and the authors report some empirical evidence of this. The analysis carried out by the authors is not only to allow for ex-post evaluation of the effectiveness of tax audits but also to guide the enforcement activity toward the kind of audit that will have a greater deterrent effect.

2.2 This contribution

In this work, we propose a model and a new methodology to identify the effects of audits on taxpayer behaviour in a context (the Italian taxation system) where the inspection activities are not based on a random selection of taxpayers (as done in the U.S.A. by the Internal Revenue Service, for example) but are based on a risk analysis that comes from processing the individual declared tax data [14,2,11]. However, it is well known that there are kind of selection bias, as that of taxpayers who drop out from the reporting process, where randomization of audits could not be enough to identify causal effects [10]. One of the most frequently used audit rules is the one based on SdS and, from here on, we will focus our attention on the Italian taxpayer audience which is subject to the SdS audit rules (small and medium−sized enterprises). The proposed methodology has been developed combining two recent techniques developed for the assessment of causal effects in a probabilistic graphical model: the *effect restoration* technique, which addresses the problem of measurement bias, and the *selection recoverability* technique, which can be effective in removing the selection bias. Now we will take a look at the set of variables necessary to set up our causal model for the identification of the audit effects and will return to these two topics in section 3.

The basic relationships among the fundamental variables that regulate the taxpayer–agency interaction process, which has been addressed in previous works, will also be done in the following sections, but without any optimization perspective. Instead, our point of view will be only focused on the probabilistic/causal associations between variables by adopting an SEM as the underlying statistical data generating model. To proceed with the details of the model specification, some aspects and variables found in the quoted paper of section 2.1 should be considered in the proposed model as well.

– The basic observed variables are the following: the tax rate, fine costs, revealed turnover, coherence/congruence to SdS, declared input/output VAT, regions (geographic zone), business activity, declared turnover, year, taxation regime, and business cycles.

– The basic unobserved variables are the following: real turnover, inspector behaviours, and taxpayer behaviours.

– There are at least two kinds of selection bias: one refers to the auditing rule, and the other refers to the taxpayers who decide to drop out from the tax reporting process.

– In case of the SdS audit rule, referring to the IRA praxis during the recent years of SdS application, the IRA announces and commits to an audit strategy where all the taxpayers with a positive gap (S) between the SdS estimate and the declared turnover that stay above a cut-off value (*Scut*) have a positive probability to be audited, while the remaining ones are not audited at all.

All the above information should be considered and characterized inside our model; although, to build a compact but completely specified graphical model, some *constraining hypotheses* are necessary:

– inspections for data reported at time *t* are communicated to the taxpayer and completed before the starting date of the tax reporting for time *t*+1. This avoids conducting the analysis based on different clusters of taxpayers, having different gaps between the time the audit is communicated and then completed;

– only two years are modelled, and so the direct/indirect effects on subsequent years *t*+2, *t*+3,…, etc. and the business cycle are not taken into account. This means that we cannot evaluate the enduring tax compliance, as the tax audit effect over years;

– the taxpayer may not report its turnover at time *t*+1, and we suppose that he/she takes this decision depending only on the real turnover value and on the audited turnover value at time *t* (obviously, there may be other reasons that do not depend on the audit activity);

– given the undeclared turnover unveiled from the audit activity X, the cumulative value of the unpaid tax and associated fine is a linear function of X, $\pi(1+\lambda)X$, where π is the mean tax rate and λ is the fine computed based on the value πX; we will make use of X, but not explicitly of π and λ;

– we consider only one taxation regime (we can stratify the population on it) and the attributes linked to the geographic zone and to the business activity are (almost) constant between *t* and *t*+1. So, for example, our analysis discriminate among different region localization (with its own peculiar land use, road networks, ecc.), but not among the same localization over time.

One of the consequences of this additional list is that, although the theoretical results are about the total causal effect (see section 3.1), it coincides with the direct effect of the audit at time *t* on the declared turnover at time *t*+1. Furthermore, we will refer to the variable X as the audit–result variable.

Fig. 1 shows a UML (Unified Modelling Language) activity diagram that synthetizes the two main steps of the taxpayer–Agency interaction we are going to model, which considers two generic reporting years, *t* and *t*+1, of the year–by–year tax reporting process. The first decision diamond refers to the Agency audit rule, which is known by the taxpayer, and is linked to the recovered/estimated turnover. This value is assessed by the taxpayer to make the decision depicted in the second diamond. Here, he/she could possibly drop out from the reporting process for legal or illegal reasons. For example, there may be some opportunity that can result in the taxpayer changing their business activity to a profitable one with a lower accounting burden (Santoro, 2016).

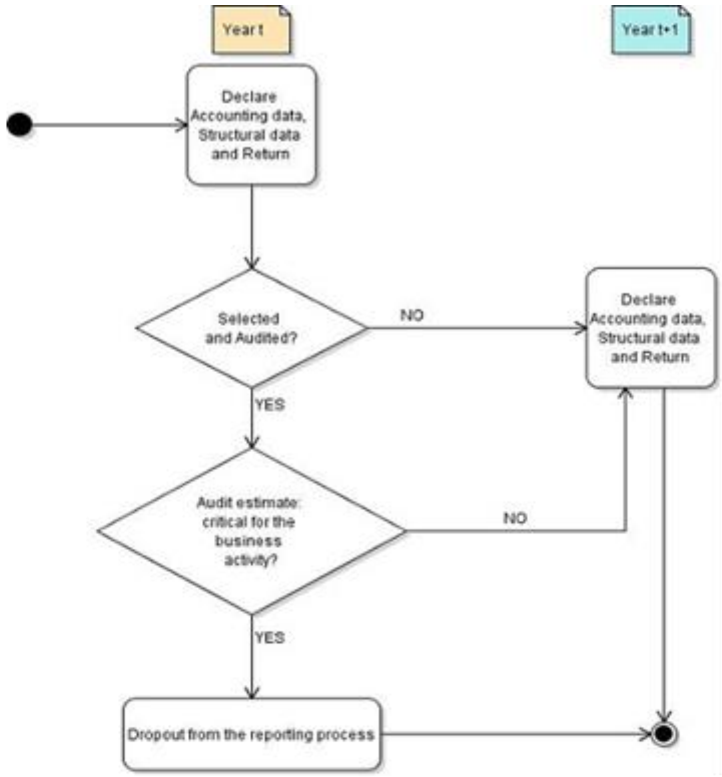

Figure 1: The modelled taxpayer behavioural flow–chart.

The list of variables used within the proposed model (Model_A) that can be found inside the causal graph of Fig. 2.a are shown in Figure 3.

[W]: the output Value Added Tax (VAT). This is a *proxy* variable for the true turnover, which is not explicitly used by the tax audit rule for this model.

[Z_1]: the declared turnover inside the SdS modules at time *t*.

[Z_2]: the SdS estimated turnover at time *t*.

[G]: the geographic zone or region of the taxpayer business activity.

[L]: the IRA local office that conducts the audit activity.

[B]: the business or macro–sector activity code.

[**Q**]: a vector of additional (respect to W and Z) declared information at time *t*.

[X]: the undeclared turnover resulting from the audit activity (if S>*Scut*) and is *Null* otherwise (the value is not present in the database).

[S_2]: the variable for those taxpayers who decide to declare ($S_2 \geq S_2cut$) or not declare ($S_2 < S_2cut$) their returns at time *t*+1 (conditioned variable).

[S]: the difference between Z_2 and Z_1 used by the audit rule S>*Scut* (conditioned variable).

[U]: the actual turnover obtained by the tax payer at time *t* (not observed).

[U_2]: the difference between the actual turnover at time *t*+1 and the actual turnover at time *t* (not observed).

[Y]: the difference between the declared turnover at time *t*+1 and the declared turnover at time *t*.

Figure 3: List of variables for Model_A.

Finally, a list of the most relevant objectives addressed with this work can be summarized as follows:

- to define a minimal set of variable capable to model the tax interaction process synthetically described in Fig. 1;
- to extend, for a linear SEM, a well-known effect restoration theorem which addresses the measurement bias so that it can also deal with selection bias;
- to propose a causal graphical model for the identification of the audit effects on taxpayer behaviours to which the new theorem can be applied;
- to give some necessary properties that the proxy variables of the unobserved true taxpayer turnover must

fulfil so that the previous result can be stated; and

– to make some parallel analysis between the previous modelling approach on this subject and the proposed model.

2.3 Mathematical and probabilistic graphical tools

In the proposed model, we consider a recursive Gaussian linear SEM with a set of variables $\{\mathbf{V},\Xi\}$, which are described as follows:

$$\boldsymbol{\Xi} \sim \mathrm{N}(\boldsymbol{\mu}_\Xi,\boldsymbol{\Sigma}_{\Xi\Xi}),\ [\boldsymbol{\Sigma}_{\Xi\Xi}]_{ij} = 0\ \forall i\neq j \tag{1.a}$$

$$\mathbf{V} = \mathbf{CV}+\boldsymbol{\Xi} \ \Rightarrow\ \mathbf{V} = (\mathbf{I}-\mathbf{C})^{-1}\boldsymbol{\Xi},\ \ \boldsymbol{\mu}_\mathrm{V} =(\mathbf{I}-\mathbf{C})^{-1}\boldsymbol{\mu}_\Xi \tag{1.b}$$

$$\boldsymbol{\Sigma}_\mathrm{VV} = (\mathbf{I}-\mathbf{C})^{-1}\boldsymbol{\Sigma}_{\Xi\Xi}(\mathbf{I}-\mathbf{C})^{-\mathrm{T}},\ \ \boldsymbol{\Sigma}_\mathrm{VV}^{-1} = (\mathbf{I}-\mathbf{C})^\mathrm{T}\boldsymbol{\Sigma}_{\Xi\Xi}^{-1}(\mathbf{I}-\mathbf{C}) \tag{1.c}$$

where, $\boldsymbol{\Xi} \sim \mathrm{N}(\boldsymbol{\mu}_\Xi,\boldsymbol{\Sigma}_{\Xi\Xi})$, says that $\boldsymbol{\Xi}$ is a Normal (Gaussian) multivariate random variable with mean $\boldsymbol{\mu}_\Xi$ and covariance matrix $\boldsymbol{\Sigma}_{\Xi\Xi}$ (positive definite, so invertible), $[\boldsymbol{\Sigma}_{\Xi\Xi}]_{ij}$ is the element of matrix $\boldsymbol{\Sigma}_{\Xi\Xi}$ in position (i,j); $\mathbf{V}$ is an ordered vector of endogenous/exogenous random variables (observable or not); $\boldsymbol{\Xi}$ is the vector of exogenous Gaussian random variables (the stochastic disturbances); $(\mathbf{I}-\mathbf{C})^{-\mathrm{T}}$ is a shorthand for $((\mathbf{I}-\mathbf{C})^{-1})^\mathrm{T}$ and $\mathbf{C}$ an upper triangular matrix of structural coefficients whose non-zero elements c_{ij} $(i<j)$ represent the direct causal effect of V_j on V_i, which imply that $(\mathbf{I}-\mathbf{C})$ is invertible. The matrix $(\mathbf{I}-\mathbf{C})^{-1}$ conveys the information about the covariates and causal effects, as explicitly set in the (1.c) formulas, and the element in the position corresponding to (Y, X) in that matrix is exactly the total causal effect of X on Y. See, among others, [15,16,17,18].

Let X, Y, and Z be the Gaussian univariate random variables and $\mathbf{T}$ be Gaussian multivariate. Then, we have the following well-known results:

$$\beta_\mathrm{YX} = \mathrm{Var[X]}^{-1}\mathrm{Cov[Y,X]} = \sigma_\mathrm{YX} /\sigma_\mathrm{XX} \tag{2.a}$$

$$\beta_\mathrm{YX.Z} = \mathrm{Var[X|Z]}^{-1}\mathrm{Cov[Y,X|Z]} = \sigma_\mathrm{YX.Z} /\sigma_\mathrm{XX.Z} \tag{2.b}$$

$$\sigma_\mathrm{YX.Z} = \mathrm{Cov[Y,X]} - \mathrm{Cov[Y,Z]Var[Z]}^{-1}\mathrm{Cov[Z,X]} = \sigma_\mathrm{YX} - \sigma_\mathrm{YZ}\sigma_\mathrm{ZX}/\sigma_\mathrm{ZZ} \tag{2.c}$$

$$\sigma_{\mathrm{YX}.\mathbf{T}} = \sigma_\mathrm{YX} - \boldsymbol{\Sigma}_{\mathrm{Y}\mathbf{T}}\boldsymbol{\Sigma}_{\mathbf{TT}}^{-1}\boldsymbol{\Sigma}_{\mathbf{T}\mathrm{Z}} \tag{2.d}$$

$$\sigma_{\mathrm{YX.Z}\mathbf{T}} = \sigma_{\mathrm{YX}.\mathbf{T}} - \sigma_{\mathrm{YZ}.\mathbf{T}}\sigma_{\mathrm{ZX}.\mathbf{T}}/\sigma_{\mathrm{ZZ}.\mathbf{T}} \tag{2.e}$$

where $\mathrm{Cov[Y,X]} = \sigma_\mathrm{YX}$, $\mathrm{Var[X]} = \sigma_\mathrm{XX}$, $\mathrm{Var[X]}^{-1} = 1/\sigma_\mathrm{XX}$, Var[X|Z] and Cov[Y,X|Z] are, respectively, the variance of X and the covariance of X and Y conditioned on the variable Z; β_YX is the X's regression coefficient obtained regressing Y on X; $\beta_\mathrm{YX.Z}$ is the X's regression coefficient obtained regressing Y on X and Z**;** $\boldsymbol{\Sigma}_{\mathrm{Y}\mathbf{T}}$ and $\boldsymbol{\Sigma}_{\mathbf{T}\mathrm{Z}}$ are a row vector and a column vector. One can see the upper triangular matrix $\mathbf{C}$ as a *weighted adjacency matrix* of a *direct acyclic graph* (DAG), which is another mathematical and probabilistic graphical model [19] associated with our linear SEM. A DAG G is defined by a pair of sets $G = (\mathbf{N},\mathbf{A})$, where $\mathbf{N} = \mathbf{V}$ is the set of nodes, $\mathbf{A} = \{(\mathrm{N}_i,\mathrm{N}_j)\ \forall c_{ij}\in\mathbf{C}{:}c_{ij}\neq 0\}$ is the set of oriented arcs (here the arc direction goes from N_j to N_i), and c_{ij} are the arc weights. The ordering of the $\mathbf{V}$ components induces a topological (partial) order of the nodes in G.

For an introduction on graphs, their mathematical and graphical implementations and examples of their applications, the reader can refer, among others, to [20]. For the graph-theoretic terminology used in this paper, the concepts of blocking a path and the *d*–separation between a couple of nodes (i.e. random variables), we refer readers to [12]. In that context, there is a 1:1 relationship between the SEM and the DAG such that they are interchangeable.

3. Results

3.1 Effect restoration with selection bias

Despite the simplifications reported in section 2.2, the graphical model in Fig. 2.a is not suitable for the direct use of the classic identification techniques of the total effect [12,13], including the *back-door* criterion, *front-door* criterion and *instrumental variable* criterion (conditioned or not). This is due to the presence of the unobserved variables U and U_2 and the conditioned variables S and S_2, which induce confounding and selection bias in the total causal effect going from X to Y.

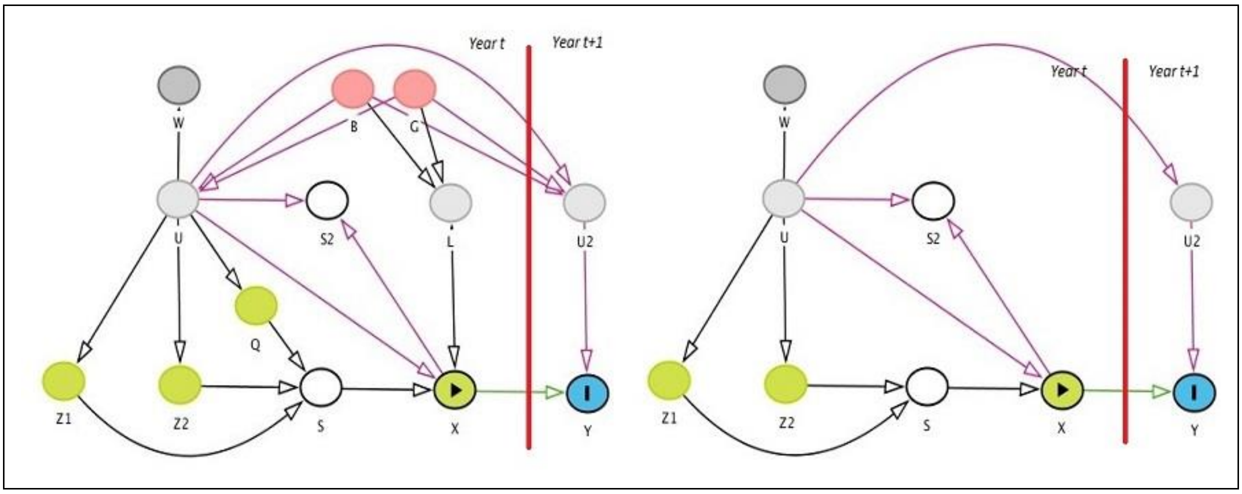

Figure 2: The Model_A causal DAG:

(a) on the left the integral version, and (b) the simplified version stratified on B and G.

For the effect identification where a latent confounder is present, there are approaches based on latent factor analysis [21,22,23] that are generalized to the multi-factor latent variable solving algorithm, as shown in [24] and [25], where the last work also considers cases when selection bias is detected. Here, to identify the total causal effect in a causal model with the concomitant presence of measurement bias (of latent confounding variables) and selection bias, we propose an extension of the method called *effect restoration* that does not use the external unbiased sample study for the proxy error mechanism (see Theorem 2 in [26]) in a way that the condition of *selection recoverability* of [27] can also be met. Recently, [28,29] further developed selection recoverability for sets of treated variables **X** and response variables **Y**. The context is then the one of measurement bias with an unobserved confounder for which observed proxies are available. Selection bias is also present, which differentially affects the selection of subjects in the sample. In the area of linear SEM with

Gaussian errors, we mention the definitions of the total causal effect and the *selection back-door* criterion for the identifiability and *selection–recoverability* of the total effects [27]. In the following, suppose that the selection mechanism uses a truncated Gaussian variable. For example, $S \leq s$ means that the unit is in the sample, while $S > s$ means that it is not (below, we let S' be the truncated variable, and $S' = \{s': s' \leq s, s \in S\}$). Here, the selection–backdoor criterion is slightly extended in a way that allows the use of a linear regression to get the causal total effect via intervention.

Definition 1 (total causal effect)

In a recursive linear SEM with $X, Y \in \mathbf{V}$, the following are equivalent definitions of τ_{YX}, which is the total causal effect of X on Y:

- $\tau_{YX} = \sum_{i=1..k} p_i$, where k is the number of directed paths between X and Y and p_i is the product of the structural coefficient on the *i-th* directed path;
- $\tau_{YX} = E[Y|do(X=x+1)] - E[Y|do(X=x)]$; and
- $\tau_{YX} = \partial E[Y|do(X=x)]/\partial x$.

where $do(X=x)$ denote the action of intervening on X, fixing $X=x$ for all the population of subjects [12,13] and removing all the influences on X, in contrast to $X=x$ which denote the sub–population naturally having value $X=x$.

Theorem 1 (selection–backdoor criterion)

Let {X, Y} and $\mathbf{Z} = \{\mathbf{Z}^+ \cup \mathbf{Z}^-\}$ be disjoint subsets of $\mathbf{V}$ in the recursive Gaussian linear SEM with DAG G such that $\mathbf{Z}^+$ contains all non-descendants of X and $\mathbf{Z}^-$ the descendants of X. Let $S, \mathbf{M}, \mathbf{T} \subseteq \mathbf{V}$, where S is the selection mechanism variable, $\mathbf{M}$ are the variables collected under selection bias and $\mathbf{T}$ are the variables collected in the population-level. If the set $\mathbf{Z}$ of variables satisfies the following conditions:

a) $\mathbf{Z}^+$ blocks all backdoor paths from X to Y;
b) X and $\mathbf{Z}^+$ block all paths between $\mathbf{Z}^-$ and Y, namely ($\mathbf{Z}^- \perp\!\!\!\perp Y | X, \mathbf{Z}^+$);
c) X and $\mathbf{Z}$ block all paths between S and Y, namely ($S \perp\!\!\!\perp Y | X, \mathbf{Z}$); and
d) $\mathbf{Z} \cup \{X, Y\} \subseteq \mathbf{M}$, $\mathbf{Z} \subseteq \mathbf{T}$,

then $\mathbf{Z}$ is said to satisfy the *selection backdoor criterion* relative to (X,Y), we have $Pr[y|do(X=x)] = \int_{\mathbf{z}} (Pr[y \mid X = x, \mathbf{Z} = \mathbf{z}, S \leq s] Pr[\mathbf{Z} = \mathbf{z}])$ and the total causal effect of X on Y is the linear regression coefficient $\beta_{YX.S\mathbf{Z}}$.

Proof ($\tau_{YX} = \beta_{YX.S\mathbf{Z}}$)

The equivalence $Pr[y|do(\mathrm{X}=x)] = \int_{\mathbf{z}}(Pr[y \mid \mathrm{X}=x,\mathbf{Z}=\mathbf{z},\mathrm{S}\le s]Pr[\mathbf{Z}=\mathbf{z}])$ can be found in (Bareinboim, Tian & Pearl, 2014), so we give only the proof of $\tau_{YX} = \beta_{YX.S\mathbf{Z}}$.

The last equation $\tau_{YX} = \beta_{YX.S\mathbf{Z}}$ comes from the equation $E[Y=y|S=s,X=x,\mathbf{Z}=\mathbf{z}] = \beta_0+\beta_{YX.S\mathbf{Z}}x+\beta_{YS.X\mathbf{Z}}s+\boldsymbol{\beta}_{Y\mathbf{Z}.SX}^{\mathrm{T}}\mathbf{z}$ that is valid for any multivariate Gaussian variable, the equivalence $\phi(Y \mid X<\alpha) = \frac{1}{\int_{-\infty}^{\alpha}\phi_X(x)\partial x}\int_{-\infty}^{\alpha}\phi(Y \mid X=x)\phi_X(x)\partial x$ (with $\phi(.)$ the Gaussian density function) and $\tau_{YX} = \partial E[Y|do(X=x))]/\partial x$, which is the definition of the total effect for a linear SEM. Therefore, we can write the equation $E[Y|do(X=x))] =$

$$\int_y y\int_{\mathbf{z}}(Pr[y \mid \mathrm{X}=x,\mathbf{Z}=\mathbf{z},\mathrm{S}\le s]Pr[\mathbf{Z}=\mathbf{z}]) =$$

$$\int_y y\int_{\mathbf{z}}\left(\frac{1}{\int_{s'}Pr[\mathrm{S}=s']}\int_{s'}Pr[\mathrm{Y}=y|\mathrm{S}=s',\mathrm{X}=x,\mathbf{Z}=\mathbf{z}]Pr[\mathrm{S}=s']Pr[\mathbf{Z}=\mathbf{z}]\right) =$$

$$\int_{\mathbf{z}}Pr[\mathbf{Z}=\mathbf{z}]\frac{1}{\int_{s'}Pr[\mathrm{S}=s']}\int_{s'}Pr[\mathrm{S}=s']\int_y yPr[\mathrm{Y}=y|\mathrm{S}=s',\mathrm{X}=x,\mathbf{Z}=\mathbf{z}] =$$

$$\int_{\mathbf{z}}Pr[\mathbf{Z}=\mathbf{z}]\frac{1}{\int_{s'}Pr[\mathrm{S}=s']}\int_{s'}Pr[\mathrm{S}=s']\mathrm{E}[\mathrm{Y}=y|\mathrm{S}=s',\mathrm{X}=x,\mathbf{Z}=\mathbf{z}] =$$

$$\int_{\mathbf{z}}Pr[\mathbf{Z}=\mathbf{z}]\frac{1}{\int_{s'}Pr[\mathrm{S}=s']}\int_{s'}Pr[\mathrm{S}=s'](\beta_0+\beta_{YX.S\mathbf{Z}}x+\beta_{YS.X\mathbf{Z}}s'+\boldsymbol{\beta}_{Y\mathbf{Z}.SX}^{\mathrm{T}}\mathbf{z}) = \beta_0+\beta_{YX.S\mathbf{Z}}x+\beta_{YS.X\mathbf{Z}}\mu_{s'}+\boldsymbol{\beta}_{Y\mathbf{Z}.SX}^{\mathrm{T}}\mathbf{z},$$

where $\boldsymbol{\beta}_{Y\mathbf{Z}.SX}$ is a vector, $\mu_{s'}$ is the mean of the truncated variable S' and the need for the intercept β_0 is because we deal with generic Gaussian variables that are not necessarily centred at zero (see Section 2.3 in [30]). Finally, we have $\partial E[Y|do(X=x))]/\partial x = \beta_{YX.S\mathbf{Z}}$.♦

Now we can state our customized version of the Kuroki and Pearl theorem, which is also selection-recoverable if all the listed conditions are met.

Theorem S-KP

For a recursive Gaussian linear SEM with variables $\mathbf{V} = \{Y,X,S,W,U,Z_1,Z_2\}$, where S is the selection mechanism, U is unobserved, W, Z_1, and Z_2 are proxies for the U observed at the population level and α_{WU} is the structural coefficient of the arc (U,W), let U = $\mathbf{Z}^+$ and $\mathbf{Z}^- = \{\varnothing\}$ in the preceding theorem. If the following conditions are met:

1) U satisfies the *selection–backdoor* criterion relative to (X,Y) given S;
2) {X,U} *d*–separate Y, Z, W, and S from each other, where Z can be Z_1 or Z_2; and
3) $X \perp\!\!\!\perp W|\{U,S\}$, $S \perp\!\!\!\perp W|U$, $\sigma_{XW.S} = \sigma_{XU.S}\alpha_{WU}$ and $\sigma_{YW.S} = \sigma_{YU.S}\alpha_{WU}$,

then the total effect of X on Y is identifiable, selection–recoverable and is given by

$$\tau_{\mathrm{YX}} = \frac{\sigma_{XY.S}\sigma_{WZ.XS} - \sigma_{YZ.XS}\sigma_{WX.S}}{\sigma_{XX.S}\sigma_{WZ.XS}} \tag{3}$$

where Z in (3) can be Z_1 or Z_2 and X⫫W|{U,S} means that X is stochastically independent of W conditioned on U and S.

Proof

From the selection–backdoor criterion, let be $\tau_{\mathrm{YX}} = \beta_{YX.U} = \beta_{YX.US} = \dfrac{\sigma_{YX.US}}{\sigma_{XX.US}} =$

$\left(\sigma_{YX.S} - \dfrac{\sigma_{XU.S}\sigma_{UY.S}}{\sigma_{UU.S}}\right) \Big/ \left(\sigma_{XX.S} - \dfrac{\sigma^2_{XU.S}}{\sigma_{UU.S}}\right)$, which is both identifiable and selection–recoverable and

equivalent to $\tau_{\mathrm{YX}} = \left(\sigma_{YX.S} - \dfrac{\alpha^2_{WU}\sigma_{XU.S}\sigma_{UY.S}}{\alpha^2_{WU}\sigma_{UU.S}}\right) \Big/ \left(\sigma_{XX.S} - \dfrac{\alpha^2_{WU}\sigma^2_{XU.S}}{\alpha^2_{WU}\sigma_{UU.S}}\right) =$

$\left(\sigma_{YX.S} - \dfrac{\sigma_{XW.S}\sigma_{YW.S}}{\alpha^2_{WU}\sigma_{UU.S}}\right) \Big/ \left(\sigma_{XX.S} - \dfrac{\sigma^2_{XW.S}}{\alpha^2_{WU}\sigma_{UU.S}}\right)$ (4).

The last equivalence comes directly from the last equations in Theorem S–KP.3. Using the properties in 2.c), Theorem S–KP.2) and Theorem S–KP.3), we obtain the following relationships: $\alpha_{WU} = \beta_{WU} = \beta_{WU.S}$,

$\sigma_{YW.XS} = \sigma_{WU.XS}\sigma_{YU.XS} / \sigma_{UU.XS}$, $\sigma_{WZ.XS} = \sigma_{WU.XS}\sigma_{ZU.XS} / \sigma_{UU.XS}$, $\sigma_{YZ.XS} = \sigma_{YU.XS}\sigma_{ZU.XS} / \sigma_{UU.XS}$, and

$\sigma_{UU.XS} = \sigma_{UU.S} - \sigma^2_{UX.S} / \sigma_{XX.S} \Rightarrow \sigma_{YW.XS}\sigma_{WZ.XS} / \sigma_{YZ.XS} = \beta^2_{WU.XS}\sigma_{UU.XS} =$

$\alpha^2_{WU}\sigma_{UU.XS} = \alpha^2_{WU}\sigma_{UU.S} - \alpha^2_{WU}\sigma^2_{UX.S} / \sigma_{XX.S} = \alpha^2_{WU}\sigma_{UU.S} - \sigma^2_{XW.S} / \sigma_{XX.S} \Rightarrow$

$\alpha^2_{WU}\sigma_{UU.S} = \sigma_{YW.XS}\sigma_{WZ.XS} / \sigma_{YZ.XS} + \sigma^2_{XW.S} / \sigma_{XX.S}$ (5).

By substituting (5) in (4) and having $\sigma_{YW.XS} = \sigma_{YW.S} - \sigma_{XY.S}\sigma_{XW.S} / \sigma_{XX.S}$, we finally obtain $\tau_{\mathrm{YX}} = \dfrac{\sigma_{XY.S}\sigma_{WZ.XS} - \sigma_{YZ.XS}\sigma_{WX.S}}{\sigma_{XX.S}\sigma_{WZ.XS}}$. ♦

Some remarks on this result. The proposed one has been proved to be a consistent estimator of the causal effect of audits in a taxation system where selection bias arises from both the audit rule and the dropout of taxpayers from the reporting process (at least in the Italian context). Among the three requirements that let us identify the causal effects of audits, two concern properties of the observed proxy variables we choose for the unobserved variables. This has practical relevance in leading the analyst towards the duly defined proxies to place inside the model, even in the presence of selection bias. Condition 2) and the first equation of 3) contain the following requests: X⫫W|{U,S} and Z⫫W|{U,X}. The variable W represents an indicator independent [31] from Z that is not directly nor indirectly used by the auditing rule (there is not a causal path or another kind of open path from W to X that is not separable by U given S), also if it is correlated to the audit variable (treatment), the conditioned S variables or the other proxies $\{\mathbf{Q}, Z_1, Z_2\}$. Therefore, we summarize these necessary requisites for the proxy variables in the following proposition.

Proposition 1

For the identification of the causal effects in our model, it is necessary for the analyst to choose the proxies among at least two groups of variables, say Groups A and B, which are *d*−separable by the unobserved variable U, conditioned on S, and have the following properties.

Group A: these proxies should be directly or indirectly used inside the auditing rule (are direct or indirect causes).

Group B: these proxies should not be directly nor indirectly used inside the auditing rule (are not direct or indirect causes).

3.2 Audit causal effect with selection bias

In section 2.2, some peculiarities of the model limiting its applicability have been listed. Now we are going to list what the model can address. Our commitment has been to identify the direct causal effect of the audit activity on the taxpayer's declared turnover and, to do this, we need to gather and represent some information. The remaining most relevant hypotheses are as follows.

- The only unobserved variables are the true turnover at time *t* and *t*+1, which can be seen as a synthesis representing both the technical and the economic efficiency of the taxpayer, along with the Agency's professional auditor and technical characteristics.
- Inside the SdS audit rule, one could consider other declared variables, as represented by the vector **Q**. We require that each set of proxy variables for the true taxpayer return, for which we use letters Z, W and Q, are mutually disjoint ($\{\mathbf{Z} \cap \mathbf{W}\} = \{\mathbf{Z} \cap \mathbf{Q}\} = \{\mathbf{Q} \cap \mathbf{W}\} = \{\varnothing\}$).
- Graphically, the DAG in Fig. 2 does not report the stochastic disturbance variables.

By reading the DAG (Fig 2.a), it is clear that the two variables B and G are confounders of the causal effect between the audit and tax return increase. This explicitly results in the need to let the authors choose these variables as "controlling variables" in the preceding quoted models (section 2.1). A possible correlation among the stochastic disturbance of the two variables (B and G) does not change too much in this context. What emerges from the DAG and from the quoted works is that, although it is not known, there are other specific causes related to the real taxpayer turnover that, apart from the ones collected and summarized by the business activity code and the geographic location of the activity, affect the increase or decrease of the reported turnover, as the arc (U, U_2) tells us. This is a further example where the hypothesis regarding the relationships and constraints of the variables can help the analyst to discover the possible weaknesses or threats in the effects of controlling for some of them. Consequently, to the fact that conditioning or stratifying on a variable is a decision that lies on both causal and statistical matters, we suppose that there is the opportunity to stratify the data on variables B and G. Therefore, the hypothesis of theorem S−KP are met by the model in Fig. 2.b and the identification result for the causal effect easily follows. We summarize our consideration in the proposition below.

Proposition 2

Model_A, which is stratified on the variables B and G, satisfies the requisites for the application of theorem S–KP, with S = $\{S,S_2\}$, and the causal direct effect of audits on taxpayers' behaviours is identifiable and selection–recoverable in each stratum. By making use of this result, it is possible to calculate the interventional causal effect for the linear system, but one can also compute a counterfactual causal effect ($Y_{X=x}|E=e$) by means of the general formula reported in Pearl (2009) page 389, where the assignment E = e is generic evidence:

$$E[Y_{X=x}|E=e] = E[Y|E=e] + \tau_{YX}(x - E[X|E=e]) \tag{6}$$

This computation will be possible whenever the quantity E[Y|do(X=x)] is identifiable and we are going to apply this formula in the next section.

4. Discussion

To see how the formula of the total causal effect works, we give a simple example of its application to a test model with self-generated synthetic data. For the proposed model of the taxpayer–inspector interaction, in equation (7), we show the linear SEM that has been used to generate 1000 random variables with a PC, which is coherent with the hypothesis listed in section 2.2 and the DAG in Fig. 2.b. Stochastic disturbances are reported, when needed. Here, again, the tax rate, tax regime, fine costs and business cycle are not explicitly included. So, we deal with a two–years model, where variables Z_1, Z_2, W, U, S, S_2 and X refer to the generic year t and variables U_2 and I refer to the year t+1. The data have been generated in a stratum where B (business activity) and G (geographic zone) have been fixed, resulting in a sufficiently homogeneous subpopulation of taxpayers. Here, for the sake of simplicity, the unobserved variable L has been embedded in the error term of X, X does not explicitly use Z_2 and S_2 does not explicitly use U (their coefficients are 0):

$$\begin{cases} Z_1 = \alpha U + \Xi_{Z1} \\ Z_2 = \beta U + \Xi_{Z2} \\ X = \theta(U - Z_1) + \Xi_X \\ W = \gamma U + \Xi_W \\ S = Z_2 - Z_1 \\ S_2 = X \\ U = \Xi_U \\ U_2 = \varphi U + \Xi_{U2} \\ Y = \alpha U_2 + incY \end{cases} \tag{7}$$

where all the $\Xi_{.}$ s are standardized Normal random variables, except for Ξ_U, which is N(100,20^2), and Ξ_{U2}, which is $N(0,\varphi^2)$. For the sake of simplicity, the variable X in Table 1 will be used with a slightly different implementation. $IncY = IncY(X)$ refers to the value that we add to the declared value of U_2 for audited taxpayers who report their return at time *t*+1, so representing the effect of the audit we are looking for. One can reason that the constant *Scut* used in S is linked to the standard deviation σ for S. For the SdS estimate inside Z_2, it has not been replicated the linear regression based on the structural and accounting variables, but a simpler linear formulation has been adopted based on the true return value that lets S be activated when the gap between the declared and the estimated return is over a fixed cut-off value (*Scut*). The system of equations is coherent with the general mathematical framework shown in formulas (1) a–c. In this test model, it has been possible to generate the values for the unobserved variables (in the real word) and to check the effectiveness of the estimated total effect under our hypotheses and model settings (see the DAG in Fig. 2.b). In Table 1, the parameters used inside the model along with the conditions satisfied by S and S_2 are listed (we used the ternary operator syntax (cond) ? if_true : if_false). Table 2 shows the relevant covariances for the causal effect computation that have been obtained from the self-generated data. From the generated population of 1000 taxpayers, almost half or 507 of the taxpayers have S > *Scut* = 5 and are audited, while 139 of them dropout from the reporting process. For the test, we consider a situation where the natural increase of the true return value for not being audited is not constant between time *t* and *t*+1 but is proportional to the true return at time *t*. This feature with the audit selection rule and the dropout rule of taxpayers actually affect the reported turnover values in a way that could bias a DiD causal effect estimate.

Table 1: Model_A constant parameters and conditions used for the test.

Parameters and conditions		
α	0.7	Fraction of the mean declared true return
β	0.75	Fraction of the mean estimated true return
γ	0.2	Fraction of the mean declared true return as VAT
Scut	5	Cut-off value used inside the S condition
S_2cut	100	Cut-off value used inside the S_2 condition
φ	0.1	Fraction of the mean true reports at time *t* that represent the mean increase in the returns between time *t* and *t*+1
θ	0.8	Fraction of the difference between the mean true reports and the mean declared reports that can be disclosed by the Agency's inspectors
S_2	$(S_2 < S_2cut)\ ?\ S_2 : Null$	Condition that can result in the taxpayer to not reporting its return at time *t*+1
S	$(S > Scut)\ ?\ S : Null$	Condition to audit a taxpayer
X	$\theta(U-Z_1)\,\xi_X$	The audited return value (ξ_X is a multiplicative factor, here uniformly distributed in [0,1])
Y	$(S \neq Null \text{ AND } S_2 \neq Null)\ ?\ Y+IncY : Y$	The audit effect ($IncY(X) = \varphi X$)

Table 2: covariances computed from the sample data set.

$\sigma_{YZ1.SS2}$	29.74999	$\sigma_{WZ1.XSS2}$	31.49838
$\sigma_{XW.SS2}$	4.13691	$\sigma_{WZ2.XSS2}$	42.32349
$\sigma_{YX.SS2}$	31.49838	$\sigma_{YZ1.XSS2}$	43.55000
$\sigma_{WZ1.SS2}$	44.31033	$\sigma_{YZ2.XSS2}$	14.62220
$\sigma_{XZ1.SS2}$	14.45368		
$\sigma_{XX.SS2}$	30.09477		
$\sigma_{YZ2.SS2}$	30.76047		
$\sigma_{WZ2.SS2}$	45.61461		
$\sigma_{XZ2.SS2}$	15.01946		

This aggregated statistical information and the covariance equations in (2.d) and (2.e) can be used to compute the causal effect τ_{YX}. For example, to find the covariances on the right of Table 2, we used the ones on the left, as follows:

$$\sigma_{WZ.XSS_2} = \sigma_{WZ.SS_2} - \sigma_{WX.SS_2}\sigma_{XZ.SS_2} / \sigma_{XX.SS_2}.$$

Now, the causal effect τ_{YX} can be easily computed from (3) using Z_1 or Z_2 with a very small difference due to the randomness of the generated sample data (they converge asymptotically):

$$\tau_{YX} = \frac{\sigma_{XY.SS_2}\sigma_{WZ_1.XSS_2} - \sigma_{YZ_1.XSS_2}\sigma_{WX.SS_2}}{\sigma_{XX.SS_2}\sigma_{WZ_1.XSS_2}} = \mathbf{0.99915};$$

$$\tau_{YX} = \frac{\sigma_{XY.SS_2}\sigma_{WZ_2.XSS_2} - \sigma_{YZ_2.XSS_2}\sigma_{WX.SS_2}}{\sigma_{XX.SS_2}\sigma_{WZ_2.XSS_2}} = \mathbf{0.99917}.$$

Therefore, for our example, we can conclude that an increase of one Euro in the audited turnover X at time *t* cause an increase in the declared turnover at time *t*+1 of 1 Euro. We also report some statistics of Y and X (both observable) and *incY* (not observable) together with the results of the regression of Y on X ($Y \sim \beta_0 + \beta X + \Xi$) in the stratum {S≠*Null*,S_2≠*Null*}. This to compare such values with the preceding ones and stress the relative weight that may have controlling for the correct variables in a regression framework.

For our purposes, the evaluation of the counterfactual causal effect requires the identification of the subpopulation of audited taxpayers who file their returns at time *t*+1, and we see what happens if their declared turnover is the true turnover (X_i=0). The problem seems to arise naturally in the following statement form: see what has happened with the audited taxpayers and what would have happened if their declared turnover was the real one (or they would not have been audited).

Table 3: regression of Y on X in the stratum {S≠*Null*,S_2≠*Null*} – synthetic output.

mean(Y|S,S_2)	16.24139
mean(*incY*|S,S_2)	8.84445
mean(X|S,S_2)	8.84445
mean(*incY*|S,S_2)/*mean*(X|S,S_2)	**1.00000**
mean(Y|S,S_2)/*mean*(X|S,S_2)	1.83634

	Value	*p–value*	*Standard Error*
β_0	6.98444	1.3279E-187	0.11931
β	**1.04664**	1.9246E-253	0.01146

The equation (6) that is to be solved is as follows:

$E[Y_{X=0}\,|S\neq Null,S_2\neq Null] =$

$E[Y|S\neq Null,S_2\neq Null] + \tau_{YX}(0 - E[X|S\neq Null,S_2\neq Null]) =$

$16.24139 + \mathbf{0.99917}(0-8.84445) = 16.24139\ \ -8,83711.$

This result allows us to conclude that, on average, the audit activity has increased the declared turnover at time *t*+1 by 8.8 Euros. This kind of analysis does have relevance for supporting the tax policy decisions of a revenue agency (Santoro, 2016). Moreover, the computed counterfactual causal effect $\tau_{YX}(E[X|S\neq Null,S_2\neq Null])$ coincides with the *Average Treatment effects on the Treated* (ATT), this can be seen after some algebra starting from the formula $E_{x\in X|S\neq Null,S2\neq Null}[E[Y_{X=x}-Y_{X=0}\,|S\neq Null,S_2\neq Null]]$, and it refers to the effects on all the taxpayers below the cut-off value *Scut* if one of the following two conditions is met by the auditing rule:

– all the taxpayers below the cut-off value *Scut* are audited;

– the subset of taxpayers below the cut-off value *Scut* to be audited are picked at random.

Note also that, given the computed variables' covariances, the way we obtain the total effect from equation (3) results in a unique value. On the other hand, with a statistics-based approach like the DiD methodology, one needs to use a matching algorithm (propensity score or other techniques) to choose a "good" control group. This allows the analyst to proceed in a sort of trial and error process by means of the significance tests or goodness of fit indicators to assess the soundness of the result, also if these indicators could not be directly linked to suitably causal properties and if they are correlated to the specific observed sample data.

4.1 Concluding remarks

In this paper, an asymptotically unbiased estimator has been proposed for the problem of finding the causal effect of audits on the taxpayers' behaviours in the Italian context. The estimator is mainly based on the extension of a result presented in [26]. It has been shown that this formula differs from the regression approach proposed in previous works on the problem and we expect that the proposed method can be used to obtain an improved causal effect estimate, closer to the real one. Furthermore, the methodology can be also applied in other fields, with problems sharing an equivalent graphical causal structure. The next steps will be directed towards applying this class of estimators to a real-world data set and removing some simplifications: extending the two−year model to a m−year model (m>2), including the direct/indirect effects over the years, and considering audits finished before the tax declaration for year t+1 based on declared data of a year preceding t (t−2 or t−3, for example).

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