Hall Effect and Temperature Distribution on Unsteady Micropolar Fluid Flow in a Moving Walls

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Abstract

Unsteady incompressible electrically conducting micropolar fluid between parallel plates where one plate is moving with constant velocity in the presence of transverse magnetic field studied. The effect of hall is taken in to consideration and it is assumed that the flow is also due to decaying pressure gradient with walls of the channel have different temperatures. The profile of velocities, microrotation and temperature studied numerically for various parameters Hartmann number, hall parameter and coupling number. The flow is simulated and the results are discussed graphically for various parameters influencing the flow.

Keywords: unsteady; micropolar fluid; Hall effect; migenatohydrodynamic.

1. Introduction

In the last several years considerable attention has been given to the study of the hydrodynamic thermal convection due to its important practical applications such as artificial fibers, the aerodynamic extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the boundary layer along a liquid film in condensation processes paper production etc.
Several authors including [1-6] have investigated analytically and numerically the effects of a transverse uniform magnetic field on the flow of viscous incompressible electrically conducting fluid between two infinite parallel plates.

The studies extended for Newtonian and diverse non-Newtonian fluids when the case one of the plates or both moving with uniform velocity have studied by may authors to mention few, Hassanien [7] studied two-dimensional unsteady viscous flow through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field when one of the plates is oscillating in its own plane. unsteady unidirectional flows of a fluid of second grade with sudden application of a constant pressure gradient or by the impulsive motion of one or two boundaries examined by Erdogan [8]. Further, Erdogan [9] studied unsteady flow over a plane walls due to one rigid boundary moved suddenly and one being free. Gupta [10] studied analytically hydromagnetic flow between two parallel non-conducting infinite planes one oscillating and the other fixed. Attia [11] studied effect of hall current on the velocity and temperature distributions of Couette flow with variable properties and uniform suction and injection when the fluid is acted upon by an exponential decaying pressure gradient and an external uniform magnetic field, also Attia [12] studied the problem of the unsteady state with heat transfer for fixed boundaries. Reference [13] numerically studied magnetohydrodynamic flow past an infinite continuous moving surface embedded in porous medium.

The theory of micropolar fluid formulated by Eringen [14] can be used to analyze the behavior of many of the fluids involved in technical processes and engineering applications, polymers, flows of exotic lubricants, animal bloods and real fluids with suspensions. An excellent review of micropolar fluids and their applications was given by [15]. Heat transfer in micropolar fluids is also important in various applications.

In this paper it is considered magnetohydrodynamic unsteady heat transfer flow of microplar fluid when one of the plates moving. The resulting governing equations are solved numerically to obtain the local similarity solutions. Graphical results for non-dimensional velocity, microrotation and temperature profiles presented for values of the parameters characterizing the flow.

2. Mathematical formulation and non-dimensionalization

2.1. Formulation of the problem

Consider an incompressible micropolar fluid flowing between two parallel plates distance h apart. Let x and y axes be chosen along and perpendicular to the walls respectively. Let the two plates are kept at two constant temperatures, $T_1$ for the lower plate and $T_2$ for the upper plate, with $T_2 > T_1$. The motion is assume due to the constant velocity, $U_0$, in the lower plate and an exponential decaying pressure gradient in the x-direction where initially both the fluid and plates are at rest. At time $t = 0$ a uniform suction from above and injection from below, with velocity $v_0$, are applied. Also magnetic field of uniform strength $B_0$ is applied perpendicular to the plate in the positive y-direction. The magnetic Reynolds number is taken to be very small enough so that the induced magnetic field can be neglected.

The current density given by the generalized equation of Ohm's law when the hall term is retained will have the
form:

\[ \vec{f} = \sigma [\vec{q}X\vec{B} + \eta(\vec{f}X\vec{B})] \]  

Within the framework of the above-noted assumptions, the appropriate conservation equations can be described by the following equations:

\[ \rho \frac{\partial u}{\partial t} + \rho v_0 \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + (\mu + \kappa) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial v_1}{\partial y} - \frac{\sigma B_0^2}{1 + \delta_h} (u + B_h w) \]  

\[ \rho \frac{\partial w}{\partial t} + \rho v_0 \frac{\partial w}{\partial y} = (\mu + \kappa) \frac{\partial^2 w}{\partial y^2} + \kappa \frac{\partial v_1}{\partial y} + \frac{\sigma B_0^2}{1 + \delta_h} (w - B_h u) \]  

\[ \rho j \frac{\partial v_1}{\partial t} + \rho j v_0 \frac{\partial v_1}{\partial y} = -2\kappa v_1 - \frac{\partial w}{\partial y} + \gamma \frac{\partial^2 v_1}{\partial y^2} \]  

\[ \rho j \frac{\partial v_2}{\partial t} + \rho j v_0 \frac{\partial v_2}{\partial y} = -2\kappa v_2 - \frac{\partial w}{\partial y} + \gamma \frac{\partial^2 v_2}{\partial y^2} \]  

\[ \rho C_p \frac{\partial T}{\partial t} + \rho v_0 \frac{\partial T}{\partial y} = k_f \frac{\partial^2 T}{\partial y^2} + (2\mu + \kappa) \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] + \gamma \left[ \left( \frac{\partial v_1}{\partial y} \right)^2 + \left( \frac{\partial v_2}{\partial y} \right)^2 \right] \]  

\[ + \frac{\sigma B_0^2}{1 + \delta_h} \left( u^2 + w^2 \right) \]  

Initial conditions:

\[ u(y, 0) = w(y, 0) = v_1(y, 0) = v_2(y, 0) = T(y, 0) = 0 \]  

the boundary conditions:

\[ u(0, t) = w(0, t) = v_1(0, y) = v_2(0, t) = 0, \quad T(0, t) = T_1, \quad t > 0 \]  

\[ u(h, t) = w(h, t) = v_1(h, y) = v_2(h, t) = 0, \quad T(h, t) = T_2, \quad t > 0 \]  

where \( \rho \) is density, \( u \) and \( w \) are fluid velocity components of \( \vec{q} \), \( v_1 \) and \( v_2 \) microrotation components, \( \mu \) the dynamic viscosity, \( \kappa \) the gyroviscosity, \( \sigma \) the electrical conductivity of the fluid, \( \vec{f} \) is current density, \( \eta \) is hall factor, \( B_0 \) magnetic induction, \( B_h = \sigma \eta B_0 \) is a hall parameter, \( j \) the microinertia, \( \gamma \) is material constant, \( C_p \) the specific heat, \( \kappa_f \) is thermal conductivity of the fluid.

The exponentially decaying pressure gradient is assumed of the form, \( \frac{dp}{dx} = f e^{-at} \), where \( f \) and \( a \) are constants.

### 2.2. Non-dimensionalization

Introduce the non-dimensional variables through

\[ \hat{u} = \frac{u}{u_0}, \quad \hat{w} = \frac{w}{w_0}, \quad \hat{v}_1 = \frac{v_{1h}}{v_0}, \quad \hat{v}_2 = \frac{v_{2h}}{v_0}, \]
\[ \theta = \frac{T_1 - T_2}{T_1 - T_4}, \quad \hat{t} = \frac{\tau_0}{k}, \quad \hat{\varphi} = \frac{\gamma}{k}, \quad \hat{\Gamma} = \frac{\kappa}{\rho u_0^2} \]  

(10)

Substituting equation (10) into equations (2-6) after dropping the hat we get the following non-dimensional equations

\[
\frac{\partial u}{\partial t} + \frac{R}{Re} \frac{\partial u}{\partial y} = \Gamma e^{-\alpha t} + \frac{1}{Re (1-N)} \frac{\partial^2 u}{\partial y^2} + \frac{N}{Re (1-N)} \frac{\partial v_1}{\partial y} - \frac{H_a^2}{Re (1+B_h)} (u + B_h w) \tag{11}
\]

\[
\frac{\partial w}{\partial t} + \frac{R}{Re} \frac{\partial w}{\partial y} = \frac{1}{Re (1-N)} \frac{\partial^2 u}{\partial y^2} + \frac{N}{Re (1-N)} \frac{\partial v_1}{\partial y} - \frac{H_a^2}{Re (1+B_h)} (w - B_h u) \tag{12}
\]

\[
\frac{\partial v_1}{\partial t} + \frac{R}{Re} \frac{\partial v_1}{\partial y} = \frac{N}{Re (1-N)} \left( -2 v_1 - \frac{\partial w}{\partial y} \right) + \frac{2-N}{M^2 Re j} \frac{\partial^2 v_1}{\partial y^2} \tag{13}
\]

\[
\frac{\partial v_2}{\partial t} + \frac{R}{Re} \frac{\partial v_2}{\partial y} = \frac{N}{Re (1-N)} \left( -2 v_2 - \frac{\partial w}{\partial y} \right) + \frac{2-N}{M^2 Re j} \frac{\partial^2 v_2}{\partial y^2} \tag{14}
\]

\[
\frac{\partial \theta}{\partial t} + \frac{R}{Re} \frac{\partial \theta}{\partial y} = \frac{1}{Pr Re} \frac{\partial^2 \theta}{\partial y^2} + 2-N \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v_1}{\partial y} \right)^2 + \left( \frac{\partial v_2}{\partial y} \right)^2 \right] + \frac{(2-N) Re Ec}{M^2} \left[ \left( \frac{\partial v_1}{\partial y} \right)^2 + \left( \frac{\partial v_2}{\partial y} \right)^2 \right] + \frac{H_a^2}{Re (1+B_h)} \right] (u^2 + w^2) \tag{15}
\]

Where \( R = \frac{\rho u_0 h}{\mu} \) (suction parameter), \( Re = \frac{\rho u_0 h}{\mu} \) (Reynolds number), \( N = \frac{k}{\mu} \) (coupling number), \( H_a^2 = \frac{\sigma M^2 h^2}{\mu} \) (Hartmann number), \( a_j = \frac{j}{h^2} \) (micro-inertia density parameter), \( M = \frac{k^2(2\mu + k)}{\gamma(\mu + k)} \) (micropolar parameter), \( \text{pr} = \frac{\mu C_p}{k_f} \) (Prandtl number), \( Ec = \frac{\mu^2}{\rho^2 C_p h^2(T_2 - T_1)} \) (Eckert number).

Initial conditions:

\[ u(y, 0) = w(y, 0) = v_1(y, 0) = v_2(y, 0) = 0(y, 0) = 0 \]  

(16)

the boundary conditions:

\[ u(0, t) = w(0, t) = v_1(0, y) = v_2(0, t) = 0, \quad 0(0, t) = 0, \quad t > 0 \]  

(17)

\[ u(1, t) = 1, \quad w(1, t) = v_1(1, y) = v_2(1, t) = 0, \quad 0(1, t) = 1, \quad t > 0 \]  

(18)

3. Results and discussion

To study the velocity, microrotation and the temperature we fix the parameters \( \text{pr} = 0.72, \ Re=1, \ Ec = 0.4, \ \Gamma = 2, \ \alpha = 2, \ R= -2. \) Figure 1 and figure 2 shows the profile of velocities components, microrotations and temperature
for $N=0.5$, $H_a = 2$ and $B_h = 2$.

It can be clearly observed from figure 1 the velocity $u$ increases towards the lower plate as it is expected. The velocity increase due to the influence of the decaying pressure gradient with time up till a maximum value and then constant due to the motion of the lower plate. Figure 2 shows the velocity component $w$ this component velocity is due to the assumption $B_h ≠ 0$ otherwise vanishes as it can be seen from figure 8. The microrotation component $\nu_1$ shows the nature of the micropolar fluid, that is the symmetric effect with the center Eringen [13].

**Figure 1:** The profile of velocity ($u$) for $N=0.5$, $H_a = 2$ and $B_h = 20.5$

**Figure 2:** The profile of velocity ($w$) for $N=0.5$, $H_a = 2$ and $B_h = 2$
Figure 3: The profile of microrotation ($v_1$) for $N=0.5, Ha = 2$ and $B_h = 2$.

In figure 4 it can be seen the profile of microrotation component $v_2$ through time. Figure 5 shows the distribution of temperature from its maximum at the lower boundary. Figure 6 shows the distribution of temperature at different time levels and after sufficiently long time the whole domain will be filled homogeneously.

Figure 4: The profile of microrotation ($v_2$) for $N=0.5, Ha = 2$ and $B_h = 2$. 
It is seen from figures 7 that with increase in Hartmann number the variation of distribution of velocity $u$ decreases whereas the induced velocity component $w$ increases and also showing that when the Hartmann number is zero this component of velocity vanishes as depicted in figure 8. Figure 9 shows the effect of Hartmann number on the microrotation component, increasing the value of $Ha$ will increase in microrotation $\nu_1$ showing the reverse flow with symmetric effect about the center. The effect of $Ha$ on microrotation $\nu_2$ shows it pushed towards the upper plate as the value of $Ha$ increases. The temperature has an increasing effect as it can be seen from figure 11 when $Ha$ increased.
Figure 7: Effect of Hartman numbers ($Ha$) on velocity ($u$), for $N = 0.5$, $t = 2$ and $B_h = 2$

Figure 8: Effect of Hartman numbers ($Ha$) on induced velocity ($w$), for $N = 0.5$, $t = 2$ and $B_h = 2$
Figure 9: Effect of Hartman numbers ($H_a$) on microrotation ($v_1$), for $N = 0.5$, $t = 2$ and $B_h = 2$.

Figure 10: Effect of Hartman numbers ($H_a$) on induced velocity ($w$), for $N = 0.5$, $t = 2$ and $B_h = 2$. 

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The hall parameter, $B_h$, effect on velocity components shown on figure 12 and figure 13, it is seen that by increasing $B_h$ the flow can be enhanced. However, the effect of $B_h$ on velocity component $w$ shown in figure 13
with increase in $B_h$ the velocity $w$ decreases when $0 < B_h < 1$ and further showing that if $B_h = 0$ no flow in this direction i.e. $w = 0$.

![Figure 13: Effect of Hall parameter ($B_h$) on velocity ($w$), for $N = 0.5$, $t = 2$ and $H_a = 2$.](image)

From figure 14 it can be seen microrotation velocity $v_1$ vanishes when $B_h = 0$. The effect of $B_h$ on microrotation $v_2$ studied in figure 15 shows increase in $B_h$ increase $v_2$, it is also noted that the flow tilted

![Figure 14: Effect of Hall parameter ($B_h$) on microrotation ($v_1$) for $N = 0.5$, $t = 2$ and $H_a = 2$.](image)
towards the lower plate. The temperature decrease with increase in the parameter $B_h$ as it is depicted in figure 16.

![Figure 15: Effect of Hall parameter ($B_h$) on mirrorotation ($\nu_2$), for $N = 0.5$, $t = 2$ and $H_a = 2$](image1)

![Figure 16: Effect of Hall parameter ($B_h$) on temperature ($\theta$), for $N = 0.5$, $t = 2$ and $H_a = 2$](image2)

In equation (2) when $\kappa = 0$ and hence in equation (11) when $N=0$ the equations (11) and (13) are decuple. It can
be seen from figure 19 and figure 20 when \( N = 0 \) or (i.e., as \( \kappa \to 0 \)) there is no microrotation and hence the fluid is Newtonian. Increasing the coupling number \( N \) both the main velocity \( u \) and the component velocity \( w \) decreases. The microrotation \( \nu_1 \) and \( \nu_2 \) both increases with coupling number increases. The temperature increases with increase in the value of \( N \).

**Figure 17:** Effect of coupling number \( (N) \) on velocity \( (u) \), for \( B_h = 2, t = 2 \) and \( H_a = 2 \)

**Figure 18:** Effect of coupling number \( (N) \) on induced velocity \( (w) \), for \( B_h = 2, t = 2 \) and \( H_a = 2 \)
Figure 19: Effect of coupling number (N) on microrotation ($v_1$) for $B_h = 2$, $t = 2$ and $H_a = 2$

Figure 20: Effect of coupling number (N) on microrotation ($v_2$) for $B_h = 2$, $t = 2$ and $H_a = 2$
4. Conclusion

In this paper the unsteady micropolar fluid flow between plates when one plate is moving and the other is fixed studied. The transverse magnetic field applied perpendicular to the plates, it is further flow is due to exponential decaying pressure gradient and the Hall Effect taken in to account. The resulting non-dimensional system of equations solved numerically. It is noted that inclusion the hall parameter will enhanced the flow. The more the fluid is micropolar the fluid motion decelerates as compared to the Newtonian fluid flow. The temperature decreases with increase in the hall parameter $B_h$ and increasing with coupling number or the Hartmann number increases for fixed time level. The article focused only the one dimensional unsteady flow it is hoped that the findings of this investigation will contribute its part for the higher dimensional flows.

References


