New Variable Step-Size NLMS Algorithm for Adaptive Noise Cancellation

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Abstract

Numerous Variable Step-Size Normalized Least Mean Square (VSS-NLMS) algorithms have been proposed to solve the problems of fast convergence rate and low value of steady-state misadjustment in the past three decades. In this paper we propose a new VSS-NLMS algorithm that employs the estimated power ratio of the adaptive filter to control the step-size update. The algorithm has a large step size in the initial stages to speed up the convergence rate, and then the step size is adjusted to maintain a low value of steady-state misadjustment. The performance of the proposed algorithm is evaluated for Adaptive Noise Canceller (ANC) using different speech signal with stationary and non-stationary noise added to it. The computer simulation highlights that the proposed algorithm has very fast convergence rate while maintaining a very low value of steady-state misadjustment.

Keywords: Least Mean Square; Adaptive noise canceller; variable step size.

1. Introduction

Effective voice communication has become very important in the world today. In audio applications noise from surrounding environment reduces quality of speech and audio signal. Adaptive Noise cancellation (ANC) has gained much attention as a technique to remove noise contained in speech signal and enhance the quality of speech and audio signal.
Remove the noise from the sound can be done by two approaches, the two microphone approach and the single microphone approach. In the two microphone approach there are primary and reference microphones. The primary microphone obtains speech corrupted by noise and the reference microphone obtains noise only which is correlated to the noise in the primary microphone. The basic concept was first introduced by Bernard Widrow et al in 1975 [1].

The LMS algorithm was developed by Widrow and Hoff in 1959. Its main advantage is low computational complexity, operational stability and ease of implementation, but this algorithm has some drawbacks such as slow convergence rate, high value of steady-state misadjustment, and using fixed step size. The value of the step size has trade-off between fast convergence rate and very small value of steady-state misadjustment. There has been a tremendous amount of research paper published on adaptive noise cancellation using variable step size algorithm [2-12].

In [2] the authors are the first to propose variable step size LMS algorithm, this algorithm uses the sign of the gradient component of the squared error to control the value of the step size. But the Algorithm has step size instability [3]. In [4] the authors use the squared error to control a variable step-size. The algorithm in [4] has larger step size when the estimation error is large and small step size when the estimation error is low. But the algorithm is sensitive to the input noise [5]. In [6] the authors propose an algorithm based on the use of estimated signal to noise ratio (SNR) to control the step size. The Algorithm has high mean squared error value when the noise interference is large. In [8] the authors propose a nonlinear function to compute the step size based on the estimated SNR. The algorithm gives low value of steady-state misadjustment. On the other hand the convergence rate is slow and the computational complexity is high. In [9] the authors propose to use the combination of two filters MF and SF for the adaptive noise cancellation. This algorithm has large delay in its response and high computational complexity. In [10] the author proposes to use Lorentzian function to compute the step size, which improved the convergence speed, especially in low SNR conditions. In [11] the author proposes a variable step size LMS algorithm using the Sigmoid function, the algorithm uses a large step size in the initial stage to speed up the convergence and then the step size is adjusted to a smaller value gradually during the convergence. But the algorithm has high value of steady-state misadjustment. In [12] the authors propose a variable step size LMS algorithm. This algorithm starts with high value for step size especially at the beginning signal occurrences, and then the value of the step size reduces gradually to reach the minimum fixed value. This minimum fixed value of the step size results a high value of steady-state misadjustment.

In this paper we propose a new variable step size LMS algorithm that gives fast convergence rate and low steady-state misadjustment values. The new variable step size is controlled by the ratio of the adaptive filter input signal power ($v_2^2(n)$) to the estimated output signal power of the adaptive filter ($\hat{y}_2^2(n)$). The algorithm has a large step size in the initial stage to speed up the convergence, and then the step size is adjusted to smaller value when approaching convergence.

This paper is organized as follows: In Section 2, we give the concept of adaptive noise cancellation. In section 3 we explain the details of the new algorithm. In Section 4, we give the simulation results of the proposed algorithm. Finally in section 5, the paper conclusion is given.
2. Concept of Adaptive Noise Canceller

Adaptive Noise Cancellation (ANC) is shown in Figure 1. It is used to remove the noise from sound. The ANC has two inputs called primary and reference inputs. The primary input \( d \) consists of the original speech signal \( s \) corrupted by the noise \( v_1 \). The input to the adaptive filter is the reference signal \( v_2 \) that is correlated with \( v_1 \) but uncorrelated with \( s \). The two signals \( v_1 \) and \( v_2 \) are correlated to the noise signal \( v \). The signal \( y \) is the adaptive filter output [1].

\[
e(n) = d(n) - y(n)
\]

\[
e(n) = s(n) + v_1(n) - y(n)
\]

Using a suitable adaptive algorithm, which main function is to minimize the power output \( e^2 \) [1].

Squaring (1):

\[
e^2 = s^2 + (v_1 - y)^2 + 2s(v_1 - y)
\]

By taking the expectation of (2):

\[
E[e^2] = E[s^2] + E[(v_1 - y)^2] + 2E[s(v_1 - y)]
\]

Since \( s \) is uncorrelated with \((v_1, y)\):

\[
E[e^2] = E[s^2] + E[(v_1 - y)^2]
\]
When the filter minimize the output power $E[e^2]$, the signal power $E[s^2]$ will remain unaffected [1].

$$E_{\text{min}}[e^2] = E[s^2] + E_{\text{min}}[(v1 - y)^2]$$  \hspace{1cm} (5)

From (5) When $E[e^2]$ is minimized, $E[(v1-y)^2]$ is therefore also minimized, and the output $e = s$ when $y=v1$. So minimizing output power causes $e = s$, this mean the output signal to be noise free [1].

3. Proposed Algorithm

In the proposed algorithm the variable step size value is controlled by the ratio of the adaptive filter input signal power ($v^2(n)$) and the estimated output signal power of the adaptive filter ($y^2(n)$). This ratio is referred as PR(n). We constrain the variable step size with predetermined maximum and minimum values to ensure that the algorithm will be stable. The algorithm starts with a zero initial value for the filter weights $w(n)$, this yields small value for filter output signal $y(n)$. Consequently, the value of the power ratio $PR(n)$ is high, which pushes the algorithm to use large step size value $\mu(n)$. On the other hand, at steady state the values of $v2(n)$ and $y(n)$ are comparable, then the ratio $PR(n)$ approaches a low value. Low value $PR(n)$ in terms pushes the algorithm to use smaller value for the step size value $\mu(n)$ in order to maintain low value to maintain low value of steady-state misadjustment.

The new step size function:

$$\mu(n) = \begin{cases} 
\mu_{\text{max}} & F(n) > \mu_{\text{max}} \\
\mu(n) & \mu_{\text{min}} > F(n) > \mu_{\text{max}} \\
\mu_{\text{min}} & F(n) < \mu_{\text{min}} 
\end{cases}$$  \hspace{1cm} (6)

Where,

$$F(n) = A \times PR^3(n) + B$$  \hspace{1cm} (7)

$$A = \frac{\mu_{\text{max}} - \mu_{\text{min}}}{PR_{\text{max}}^3 - PR_{\text{min}}^3}$$  \hspace{1cm} (8)

$$B = \mu_{\text{max}} - A \times PR_{\text{max}}^3$$  \hspace{1cm} (9)

In the proposed algorithm the value of $PR_{\text{min}}$, $\mu_{\text{min}}$ and $\mu_{\text{max}}$ are constants, and the value $PR_{\text{max}}$ depend on the correlation between $v1(n)$ and $v2(n)$. The value of $PR_{\text{max}}$ will be between 2 and 10, if the correlation is high then $PR_{\text{max}} = 2$, if the correlation is low then $PR_{\text{max}} = 10$. The values of $\mu_{\text{min}}$, $\mu_{\text{max}}$ and $PR_{\text{min}}$ are chosen to achieve fast initial convergence and small misadjustment value.

For the estimation of $PR(n)$, we compute the power of the noise $P_N(n)$, and filter output signal power $P_Y(n)$. The power $P_Y(n)$ is calculated as in (11) using the output of the adaptive filter $y(n)$. The power of $P_N(n)$ is calculated as in (12) using the input of the adaptive filter $v2(n)$.

\[
PR(n) = \text{abs}\left(1 - \frac{P_N(n)}{P_Y(n)}\right)
\]

(10)

Where

\[
P_Y(n) = \sum_{i=0}^{H} y^2(n - i)
\]

(11)

\[
P_N(n) = \sum_{i=0}^{H} v^2(n - i)
\]

(12)

Where \( H \) is the number of samples used to estimate \( P_Y(n) \) and \( P_N(n) \).

The weight update equation is similar to Normalized Least Mean Square (NLMS) algorithm as in [13], where \( \mu \) is replaced by \( \mu(n) \), the new step size in equation (6).

\[
w(n+1) = w(n) + \frac{\mu}{\epsilon + \|v2(n)\|^2} e(n)v2(n)
\]

(13)

Where \( w \) is the vector of adaptive filter weights, \( n \) is the iteration number, \( \epsilon \) is a small positive number, introduced to avoid a data over-flow error when \( \|v2(n)\| \) becomes too small as in [13], and \( \mu(n) \) is the adaptation step size of the NLMS.

4. Simulation Results

In this paper we select three different variable step size algorithms to compare with our proposed algorithm. First, the algorithm in [8], which has low value of steady-state misadjustment, but it has slow convergence rate.

Second, the algorithm in [11], it provides fast convergence rate, but it has high value of steady-state misadjustment. The Third algorithm in [12] provides fast convergence rate and computational complexity is less, but it has high value of steady-state misadjustment.

All simulations in this paper are done using MATLAB 8.1. Many numerical experiments are done to compare the performance of the different ANC algorithms. The simulations of the proposed ANC are carried out using a speech signal sampled at a sampling frequency of 8 kHz. The number of bits per sample is 8 bits and the total number of samples is 40,000 samples or 5 seconds for real time. In all simulations, the following values for the parameters are used for all the algorithms: \( L=20, H=20, N=100, P=1000, R=20, \mu_{\text{max}}=0.8, \mu_{\text{min}}=0.001 \). For algorithm [8], \( \mu_{\text{min}}=0.005, \text{SNR}_{\text{max}}=10 \text{ dB}, \text{SNR}_{\text{min}}=-20 \text{ dB} \). For algorithm [10], \( \gamma=0.005 \) and \( \sigma=0.0002 \). For algorithm [11], \( \beta=0.1 \) and \( \alpha=10 \). For algorithm [12], \( \mu=0.98 \). For proposed algorithm \( \text{PR}_{\text{min}}=0.5, \text{PR}_{\text{max}}=5, A=0.0064 \) and \( B=0.0002 \). For all algorithms the values of the parameters are chosen to obtain the best performance.

We represent the noise source by \( v(n) \), to produce noise source to the primary and reference input we will use two filters which have different impulse responses. The filters will produce \( v1(n) \) and \( v2(n) \) signals, these two
signals are correlated to the noise signal v(n). The MATLAB filters coefficient for v1(n) and v2(n):

\[
ma = [1, -0.2, 0.4, -0.2]
\]

(14)

\[
v1(n) = \text{filter}(ma,1.2,v(n))
\]

(15)

\[
ma = [1.1, -0.3, 0.4, -0.3]
\]

(16)

\[
v2(n) = \text{filter}(ma,1,v(n))
\]

(17)

We describe the performance of the adaptive noise canceller in terms of its misadjustment (M) as in [8], and Mean-Square Error (MSE) as in [14], where the MSE is given by:

\[
\text{MSE}(n) = (S(n) - e(n))^2
\]

(18)

And the misadjustment M is given by:

\[
M = \frac{\text{EMSE}_{ss}}{\text{MSE}_{min}}
\]

(19)

Where \(\text{MSE}_{min}\) equals the power of the original speech signal given by [8]:

\[
\text{MSE}_{min} = \left(\frac{1}{K-P}\right) \sum_{n=P}^{K-1} s^2(n)
\]

(20)

The Steady State Excess Mean-Square Error (EMSE_{ss}) given by [8]:

\[
\text{EMSE}_{ss} = \left(\frac{1}{K-P}\right) \sum_{n=P}^{K-1} \text{EMSE}(n)
\]

(21)

Where K is the total number of samples of the speech signal, and P is the number of samples at which the algorithm reaches steady state [8].

\[
\text{EMSE}(n) = \left(\frac{1}{R}\right) \sum_{j=1}^{R} \text{EMS}(n-j)
\]

(22)

Where R is the number of samples used to estimate EMSE. Using R to smooth the plot of EMSE [8].

4.1. Stationary Noise

In the first simulation the noise is assumed to be a zero mean white Gaussian noise with variance (\(\sigma_g^2 = 0.001\)) and original speech signal is the sound of a male speech signal. Figure 2 shows that the proposed algorithm has faster convergence rate with a small value of steady-state misadjustment. Table 1 shows that the performance of the proposed algorithm outperforms the others algorithms.
**TABLE 1**: Comparison the EMSEss and M for the four algorithms with stationary noise.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>EMSEss (dB)</th>
<th>M %</th>
</tr>
</thead>
<tbody>
<tr>
<td>[8]</td>
<td>-22.9</td>
<td>5.7</td>
</tr>
<tr>
<td>[12]</td>
<td>-19.3</td>
<td>13.29</td>
</tr>
<tr>
<td>Proposed</td>
<td>-39.1</td>
<td>0.14</td>
</tr>
</tbody>
</table>

![Graph (a)](image1)

![Graph (b)](image2)

![Graph (c)](image3)

![Graph (d)](image4)
4.2. Nonstationary Noise

In this section, we assume the noise source to be the sound of a car in full throttle download from [15] and we take the first five seconds from the sound. The original speech signal is the sound of a male speech. Figure 3 shows that the proposed algorithm has a faster convergence rate with a small value of steady-state misadjustment in the presented nonstationary noise signals. Table 2 shows how the performance of the proposed algorithm outperforms other algorithms used in the compression.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>EMSEss (dB)</th>
<th>M %</th>
</tr>
</thead>
<tbody>
<tr>
<td>[8]</td>
<td>-27</td>
<td>4.85</td>
</tr>
<tr>
<td>[12]</td>
<td>-28</td>
<td>3.8</td>
</tr>
<tr>
<td>Proposed</td>
<td>-35.8</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Table 2: Comparison of EMSEss and M for the four algorithms with nonstationary noise.
5. Conclusion

In this paper, we propose a new variable step size NLMS algorithm. This algorithm updates a coefficient by a time varying step size which is controlled by the ratio of the adaptive filter input signal power to the estimated output signal power of the adaptive filter. Computer simulations show that the proposed algorithm has superior performance when compared to [8,11,12] algorithms in the stationary and non-stationary environments. The results demonstrate that the proposed algorithm has significant improvements in the speed of convergence rate while maintaining a small value of steady-state misadjustment, especially when the correlation between v1(n) and v2(n) is more than 80%. Our future work is to implement the proposed algorithm using field programmable gate array.

References


