Electro-Magnetic Ship Propulsion Stability under Gusts

M.Y. Abdollahzadeh Jamalabadi, Jae Hyun Park

Abstract

The purpose of this study is to analytically investigate the effect of Stuart number as well as magnetic and electrical angular frequency on the velocity distribution in a magneto-hydro-dynamic pump. Results show that as Stuart number approaches zero the velocity profile becomes similar to that of fully developed flow in a pipe. Furthermore, for high Stuart number there is a frequency limit for stability of fluid flow in certain direction of flow. This stability frequency is depending on geometric parameters of channel. Furthermore stability frequency of electro-magnetic field is independent of gusts frequency and fluid thermo-physical properties.

Keywords: gusts; stability frequency; Stuart number; transient flow; duct flow

1. Introduction

Electromagnetic propulsion (EMP) is the system accelerating fluid by using electrical and magnetic fields. When an electric current flows through a conductor in a magnetic field, a Lorentz force pushes the conductor in a direction perpendicular to the conductor and the magnetic field. In spite of electric motors, the electrical energy used for EMP is not used to produce rotational energy for motion. The laws were known in the nineteenth century from the work of Hartmann on electromagnetic pumps in 1918. EMP and its applications for seagoing ships and submarines (without the aid of either propellers or paddles) have been investigated since at least 1958 when Warren Rice filed a patent explaining the technology in US 2997013[1].
The collection consists of a water channel open at both ends extending longitudinally through or attached to the ship, a means for producing magnetic field throughout the water channel, electrodes at each side of the channel and source of power to send direct current through the channel at right angles to magnetic flux in accordance with Lorentz force (see Figure 1). The Yamato 1, experimental MHD propulsion craft, is propelled by two MHD thrusters (without any moving parts), a liquid helium-cooled superconductor (cooled in order to maintain its zero-resistance property), the seawater as the electrically conducting fluid, and can travel at 15 km/h (8 knots) [2].

MHD pump is topic of many researches for simulation [3], fabrication [4], and experimental study [5] in recent years. It has many industrial applications in the nuclear magnetic resonances [6], micro-fluidics [7], sensors [8], actuators [9], electronic chips [10], micro-systems [11], chromate-graphics [12], mixers [13], induction pumps [14, 15], microelectronics [16], and nano-wears [17].

One of the most interesting matters about the MHD is its stability. This theme followed in annular linear induction pumps[18], double-supply-frequency pressure pulsations [19], sodium flow rate measurements [20, 21], open-cycle power generation systems [22-24], advanced Tokamak [25], dumping resistors [26], subsonic disk generators [27-31], liquid metal jet flows[32], perforated and parallel walls [33], compressible and radiative flow [34], Jeffery–Hamel flows[35], toroidal devices[36], gyro-kinetics [37], torus [38], free-surfaces [39], supersonic generators and diffusers[40], Turbulence and Nonlinear Dynamics [41], and anisotropic MHD [42].

As seen from the literature review the stability of MHD for ship propulsion application is not studied. In this study the effect of electro-magnetic frequency and the gusts frequency on the stability of the sea water through the propulsion system is considered.

2. Governing equations and stability analysis

Consider unsteady, however, hydro-dynamically and thermally fully-developed, laminar incompressible fluid between two parallel plates. The both plate is assumed to be in a stationary. The magnetic and electric properties are set as constant. The momentum equation in the x-direction is described as:

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = -\frac{dP}{dx} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) + F_L
\]

in which Lorentz force can be written as:

\[
F_L = \sigma (EB - B^2 u)
\]
where $E$ is electric field intensity in $z$-direction ($E = E_{\text{max}} \sin(\omega t + \phi)$), $B$ is magnetic density in $y$-direction ($B = B_{\text{max}} \sin(\omega t + \phi)$), $\sigma$ is electrical conductivity of the fluid, $p$ is pressure, $\mu$ is kinematic viscosity, $\rho$ is density of fluid (1000 kg m$^{-3}$ is a reference density). The boundary conditions of equation (1) are:

\begin{align*}
u(y = \pm w) &= u(z = \pm L) = 0 \\
\pm \frac{\partial B(y = \pm w)}{\partial y} + B(y = \pm w) &= 0 \\
\pm \frac{\partial B(z = \pm L)}{\partial z} + B(z = \pm L) &= 0 \\
u(x = 0) &= V_A + V_w \sin(\omega_w t) \\
u(x = \infty) &= V_A
\end{align*} \tag{2}

Where $c$ is equal to zero for fully electrically insulated walls and $c$ tends to infinity at perfectly conducting boundary condition. In fully-developed flow ($v = 0$), the momentum equation based on continuity equation ($\frac{\partial u}{\partial x} = 0$), and approximate velocity profile ($u = U(t)(1 - \left(\frac{y}{w}\right)^2)(1 - \left(\frac{z}{L}\right)^2)$) as

\begin{align*}
U' &= -U \left\{ \nu \left( \frac{2}{w^2} (1 - \left(\frac{z}{L}\right)^2) + 2 \left(\frac{y}{w}\right)^2 (1 - \left(\frac{z}{L}\right)^2) \right) + \frac{\sigma B_{\text{max}}^2 \sin^2(\omega t)}{\rho} \right\} \\
&\quad + \frac{\sigma E_{\text{max}} B_{\text{max}} \sin(\omega t) \sin(\omega t + \phi)}{\rho (1 - \left(\frac{y}{w}\right)^2)(1 - \left(\frac{z}{L}\right)^2)} \tag{3}
\end{align*}

by considering the definition of Stuart number ($N = \frac{\sigma B_{\text{max}}^2 L}{\rho V_A}$), Reynolds number ($\text{Re} = \frac{V_A L}{\nu}$), electromagnetic interaction parameter ($M = \frac{\sigma E_{\text{max}} B_{\text{max}} L}{\rho V_A^2}$), dimensionless velocity ($\bar{U} = \frac{\iint U dydz}{\iint V_A dydz}$), dimensionless time ($\bar{t} = \frac{tV_A}{L}$), dimensionless angular velocity ($\bar{\sigma} = \frac{\omega L}{V_A}$), aspect ratio ($A = \frac{L}{w}$), and substitution of the average of the coordinate’s lengths in $y$ and $z$ direct, the equation (3) rewritten as
\[
\frac{d\bar{U}}{dt} = \bar{U}\left(\frac{3}{2 \text{Re}}(1 + A^2) + N \sin^2(\omega t) + M \sin(\omega t) \sin(\omega t + \varphi)\right)
\]

which has the solution in the form of

\[
q = \exp\left(\frac{3}{2 \text{Re}}(1 + A^2) + \frac{N}{2}\right) - \frac{N \sin(2\omega t)}{4\sigma}
\]

\[
\bar{U} = \frac{M}{2} (\cos(\varphi) - \cos(2\omega t + \varphi))q
\]

For high Stuart numbers (by initial condition of \( \bar{U}(t = 0) = 1 \)) the solution of equation (4) is

\[
\bar{U} = \exp\left(N\left(\frac{\sin(2\omega t)}{4\sigma} - \frac{t}{2}\right)\right)
\]

and at low Stuart numbers (by initial condition of \( \bar{U}(t = 0) = 1 \)) the solution of equation (4) is

\[
\bar{U} = \left(1 + \frac{M}{4\sigma^2}\right)\left(\frac{9}{4\text{Re}^2}(1 + A^2)^2 + 4\sigma^2\right)\exp\left(-\frac{3}{2 \text{Re}}(1 + A^2)\right)\]

at simultaneous electric and magnetic field (\( \varphi = 0 \)) and steady state solution (\( t \to \infty \)) the amplitude of harmonic to constant term is equal to \( \left(\frac{M}{4\sigma^2}\right)^{1/2} \) and so for stabilized velocity profile if the maximum value of velocity fluctuation to mean value is less than “m” then

\[
\omega > 0.75m\left(\frac{1}{L} + \frac{1}{w^*}\right)
\]

at high Reynolds numbers the inertia term cannot be avoided in the momentum equation (1) and the effect of gusts at the inlet of the duct should be considered. By considering the dimensionless coordinate \( x \) as \( \bar{x} = \frac{x}{L} \) and linearization of inertia term about the ship average velocity, the equation (1) is rewritten as
\[
\frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} = -U\left(\frac{3}{2 \text{Re}} \left(1 + A^2\right) + N \sin^2(\omega t)\right) + M \sin(\omega t) \sin(\omega t + \phi)
\]

by the method of imaginary profile \((\psi = U^* + i(U - 1) = \Psi(x)e^{i\sigma t})\) the auxiliary of equation (8) can be rewritten as

\[
\Psi'(x) = -\Psi\left(\frac{3}{2 \text{Re}} \left(1 + A^2\right) + N \sin^2(\omega t) + i \omega \sigma + e^{-i\sigma t}M \sin(\omega t) \sin(\omega t + \phi)\right)
\]

\[
\Psi(0) = \frac{V}{V_A}
\]

Then the solution of equation (8) is

\[
U = 1 + e^{-\sigma t}\left(\frac{V}{V_A} \sin(\sigma_\sigma (T - \tau)) + \frac{d - c}{\sigma_\sigma^2 + c^2}(c \sin(\sigma_\sigma x) + \sigma_\sigma \cos(\sigma_\sigma x))\right) + \frac{(c - d)\sigma_\sigma}{\sigma_\sigma^2 + c^2}
\]

\[
c = \frac{3}{2 \text{Re}} \left(1 + A^2\right) + N \sin^2(\omega t)
\]

\[
d = M \sin(\omega t) \sin(\omega t + \phi)
\]

at distances far from the entrance region, if it is required that the maximum value of velocity fluctuation to mean value should less than “m” then

\[
M \sin(\omega t) \sin(\omega t + \phi) > \left(\frac{3}{2 \text{Re}} \left(1 + A^2\right) + N \sin^2(\omega t)\right)(1 - 2m)
\]

3. Conclusions

In this study, the effect of Stuart number as well as magnetic and electrical angular frequency on the velocity distribution in a magneto-hydro-dynamic pump is scrutinized. A criterion for stability of the velocity field has been derived for the laminar, transient problem in a Poiseuille flow between plane parallel plates with the Lorentz force. Interest has been focused on the influence of gusts on stability frequency of. The effect of the Stuart, Reynolds, and interaction numbers on the transient velocity has been discussed in terms of the time and location. Results show that as Stuart number approaches zero the velocity profile becomes similar to that of fully developed flow in a pipe. Furthermore, for high Stuart number there is a frequency limit for stability of fluid flow in certain direction of flow. This stability frequency is depending on geometric parameters of channel. Furthermore stability frequency of electro-magnetic field is independent of gusts frequency and fluid thermo-physical properties.

Acknowledgement

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (NRF-2012R1A1A1042920).

References


[22] Nobuhiko Hayanose, Yoshitaka Inui, Motoo Ishikawa, Juro Uimoto, Stability of open-cycle MHD generation system connected to power transmission line, Energy Conversion and Management, Volume 39, Issue 11, 1 August 1998, Pages 1181-1192


[28] I Inoue, Y Inui, N Hayanose, M Ishikawa, Transient stability analysis of commercial scale open cycle disk MHD generator connected to power systems, Energy Conversion and Management, Volume 44, Issue 5, March 2003, Pages 731-741


[34] Yuming Qin, Xin Liu, Xinguang Yang, Global existence and exponential stability for a 1D compressible and radiative MHD flow, Journal of Differential Equations, Volume 253, Issue 5, 1 September 2012, Pages 1439-1488


